Invariance Property of Isotropic Random Walk Mobility Patterns in Mobile Ad-Hoc Networks

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***Abstract*—The class of isotropic random walk mobility models, including Random Direction mobility model, Random Walk mobility model and Brownian motion mobility model, has been widely used in the study of Mobile Ad-Hoc Networks for mobility modeling and control. In this paper, we show an important property for contact time of isotropic random walk mobility models. Specifically, we find that the mean contact time of two mobile nodes following isotropic random walk mobility models is invariant with respect to the step-length distribution under both the simplest distance-based (Boolean) interference model and the more realistic SINR-based interference model. We also provide results about the effect of system parameters on the contact and inter-meeting time of mobile nodes and discuss their higher-order statistics.**

I. Introduction

Mobility modeling and control are critical in the study of Mobile Ad-Hoc Networks (MANETs). *Mobility modeling* studies how to generate mobility patterns to capture key statistical characteristics of real life mobility behaviors. Commonly used mobility models include Random Direction mobility model (RDM) [15], [14], Random Waypoint mobility model (RWP) [4], Random Walk mobility model (RWM) [7], Brownian motion mobility model (BMM) [6], Reference Point Group Mobility (RPGM) [12] and Manhattan (MH) [1]. In contrast, *mobility control* is to design mobility pattern for controllable mobile elements (e.g., data MULE [16] in wireless sensor networks) so as to improve system performance, e.g., in terms of simple and scalable implementation, economical power consumption, etc.

Contact time and inter-meeting time are two key metrics in MANETs, as these two specify the link-level dynamics that forms a basis for any network protocol above. In sparse MANETs such as Delay Tolerant Networks, in order to transmit data, a mobile node A has to wait until it meets other node B (*inter-meeting time*), then A can transmit data to B when they are still in contact (*contact time*). Node B will keep the data if it is the intended receiver, or relay/forward it to others upon encounter if not. Several properties of the inter-meeting time (e.g., mean or tail behavior) have been widely studied. For instance, theoretical analysis on mean inter-meeting time [10], [17] are conducted to facilitate the performance analysis of forwarding algorithms.

In contrast, current studies on the contact time are mostly either case-by-case under a specific mobility model (e.g, RWP) or under oversimplified assumptions (e.g., mobile nodes always go straight (never turns around) once in contact). For example, a mobile node (human being) in MANET does not necessarily move straight just because it's under contact with other mobile nodes. In addition, all the current results on both contact and inter-meeting time are based on a very simple assumption that nodes are in contact if and only if they are within certain distance (so-called Boolean model). However, recent work [5] shows that even in a sparse network interference from other nodes are not always negligible, not to mention the case of moderate-to-dense networks. Thus, it is also important to study the characteristics of contact and inter-meeting time under a more realistic setting with interference from other nodes taken into account (this is called physical model or SINR model; see Section II-A for details).

In this paper, we consider a class of isotropic random walk mobility models that includes RDM, RWM, and BMM, and show that the *average contact time* of mobile nodes under this class of mobility models is *invariant* with respect to different choices of mobility models. Specifically, we show that the average contact time is independent of the step-length[1] distribution of mobility models. Our invariance results hold true for both Boolean model and SINR model. This immediately implies that it really doesn't matter to assume whether a mobile node moves straight or random zigzag (like Brownian motion) or whether the Boolean model or SINR-based interference model is used, as long as the average contact time is concerned. In addition, we investigate key characteristics of inter-meeting time under Boolean and SINR-based model and point out that the average inter-meeting time does not enjoy the invariance property. We also study higher-order behavior of both contact and inter-meeting time.

II. Preliminaries

A. Models and Definitions

Boolean interference model, or distance model, is widely used in the study of MANET. Let $A(t),\ B(t) \in \Omega$ be the position of node A and B at time t, respectively, in a domain Ω. Then, under the Boolean model, nodes A and B are in contact at time t if and only if:

$$\|A(t) - B(t)\| \le d, \tag{1}$$

[1] Step-length is the distance that a mobile node travels before it changes its direction.

where $\|\cdot\|$ is the Euclidian norm in 2-D and d is their communication range.

SINR interference model, or physical model [11], [8], is more realistic than Boolean model in the sense that the interference from other nodes $\{M_i\}$ $(i = 1, 2, \cdots, N)$ is directly taken into account. As before, if we let $M_i(t) \in \Omega$ denote the position of node M_i at time t, then under SINR model, node B can successfully receive data from node A at time t if and only if

$$\frac{P_A \|A(t) - B(t)\|^\alpha}{N_0 + \sum_{i=1}^{N} P_i \|M_i(t) - B(t)\|^\alpha} \geq \gamma_0, \qquad (2)$$

where α (usually $\alpha \in [-4.5, -2]$) is the path loss coefficient, N_0 is the noise power level, and γ_0 (usually $\gamma_0 \in [0, 20]\ dB$) is the minimum signal-to-interference noise ratio required for successful decoding at the receiver. For simplicity, we assume that $P_A = P_i = P$ for any i. We define by SNR $= P/N_0$ the ratio between power of the user and that of the background noise.

Contact-based mobility metrics are often used to statistically characterize different mobility patterns. We note that all the contact-based metrics can be defined in the following unified way:

Definition 1: The general contact time T_{GC} of nodes A and B is given by

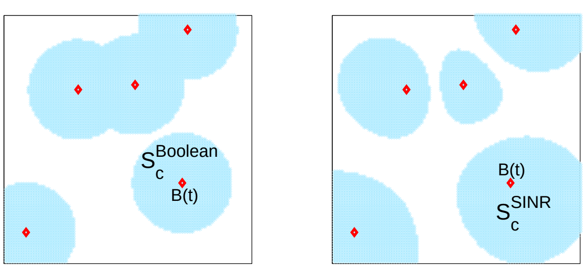
$$T_{GC} \triangleq \inf_{t>0} \{t : A(t) \notin S(B(t))\},$$

for a suitably chosen set of points S (to be specified below), given that $A(0^-) \notin S(B(0^-))$ and $A(0) \in S(B(0))$. □

For the choice of S, we use the following:

$$S_c^{Boolean}(Y) \triangleq \{x \in \Omega \mid \|x - Y\| \leq d\}$$

$$S_c^{SINR}(Y) \triangleq \left\{ x \in \Omega \;\middle|\; \frac{P\|x-Y\|^\alpha}{N_0 + P\sum_{i=1}^{N} \|M_i(t) - Y\|^\alpha} \geq \gamma_0 \right\}$$



(a) Boolean model (b) SINR model

Fig. 1. Contact sets S (possibly time-varying) under different interference models. Positions of nodes in (a) and (b) are the same.

Figure 1 shows examples of $S_c^{Boolean}(\cdot)$ and $S_c^{SINR}(\cdot)$ for a node B. Then, when $S = S_c^{Boolean}$, T_{GC} in Definition 1 gives the *contact time* of nodes A and B under Boolean model, and when $S(\cdot) = S_c^{SINR}(\cdot)$, it becomes contact time under SINR interference model. In particular, if $S(\cdot) = \bar{S}_c^{Boolean}(\cdot) = \Omega \setminus S_c^{Boolean}(\cdot)$, then T_{GC} now becomes the inter-meeting time of nodes A and B under Boolean model, and similarly for SINR model. When node B is static (i.e., $B(t) = B(0)$ for all t just like access points or stationary sensors), the contact and inter-meeting time become the *session* and *inter-session* time under the appropriate choice of Boolean or SINR models.

Throughout this paper, we do not consider nodes' pause time as our main focus lies in the *effect of mobility patterns* on the contact-based metrics.

B. A Class of Isotropic Random Walk Mobility Models

In the class of *isotropic random walk mobility models*, a mobile node first selects a random step-length L, a speed from some well-defined distribution, and a direction ϕ taken uniformly and randomly from $[0, 2\pi)$. Then, it moves according to the chosen velocity for L with angle ϕ, and upon its completion of the step, the whole procedure repeats independently of all others. This set of models includes RDM, RWM and BMM, by suitably choosing the distribution of step-length L (and that of speed if appropriate). To approximate BMM, we take very small step-lengths over a small time interval.

There are several reasons that this class of isotropic random walk mobility models is important and has thus received much attention in the literature for the study of mobility modeling and control. First, they are easy to implement, which makes them the first choice to provide a *benchmark* for performance analysis and quick performance evaluation of forwarding/routing algorithms. Second, from recent theoretical and extensive simulation results [14], [2], [9], [3], this class of isotropic random walk mobility patterns have *uniform stationary distribution* for both node position and node direction (angle). This not only simplifies study on contact metrics[2], but also leads to scalable implementation of data MULEs [16].

C. Related work

[13] derives general results on the average session time and apply them for the case of RWP mobility model. [17] obtains the mean inter-meeting time for both RDM and RWP. Specifically, their results show that the average inter-meeting time for RDM is not affected by the step-length distribution. While this holds under the network setting of their interest, our detailed study shows that this is not always the case.[3] While our theoretical analysis is based on results in [13], our focus in this paper is on the invariance property for the contact time of isotropic random walk mobility patterns (not necessarily RWP) under both Boolean and SINR interference models.

[2]As will be shown later, our invariance result for contact time under Boolean model is direct application of this property.

[3]As will be shown in Section III, the difference in the average inter-meeting time caused by different step-length distributions of isotropic random walk mobility models, can be up to $30\% \sim 40\%$ in sparse networks.

III. Invariance Property Under Boolean Interference Model

A. Contact Metrics under Boolean Model

In this section we consider several contact-based metrics, namely, session time, inter-session time, contact time, and inter-meeting time, all under Boolean interference model. We first present numerical simulation results on these metric. Later in Section III-B we provide theoretical support on our findings in this section. Our simulation setup is as follows:

- Simulation domain Ω: $400m \times 400m$ square
- Mobility models: Since the effect of speed on contact metric is quite straightforward, we here set the speed to be $1m/s$ all the time. In this case, *time* has the same value as *length* in all mobility models. Table I summarizes mobility models under our consideration. Note that step-length distribution for the three models are widely different with different means.

Mobility models	Step length dist. (m)	Speed v (m/s)
BMM	Fixed: 8	Fixed: 1
RWM	Exponential with mean 40	Fixed: 1
RDM	Uniform over [0, 40]	Fixed: 1

TABLE I
Mobility models

- Communication range: $d = \{5, 10, 15, 20, 25, 30\}m$.
- Boundary behavior: reflecting.

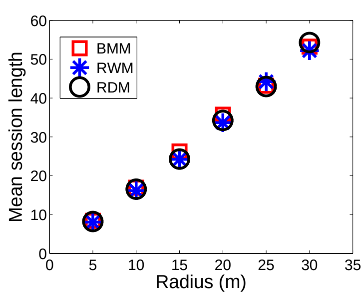
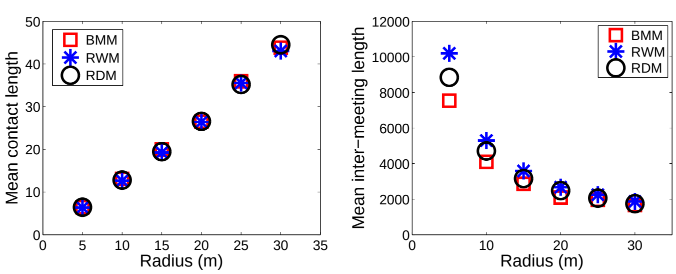


Fig. 2. Average session time under Boolean model is invariant with respect to different random walk mobility patterns.



(a) contact time (Boolean) (b) Inter-meeting time (Boolean)

Fig. 3. Average contact and inter-meeting time under Boolean model: (a) The average contact time is invariant and linearly increases with transmission range d. (b) The average inter-meeting time are different for three mobility models for smaller transmission range (sparse network), but this difference diminishes as the network becomes denser.

Figure 2 shows that the session time (or session length since $v = 1m/s$) under Boolean model is invariant with respect to different random walk mobility patterns, i.e., invariant with respect to step-length distribution, and so is the contact time as shown in Figure 3(a).

Figure 3(b) shows that the inter-meeting time is not always invariant, and different mobility patterns lead to different mean inter-meeting time, especially in *sparse networks* where the communication range is small compared to the simulation area (e.g., $d = 5, 10$). For example, when $d = 5$, the mean inter-meeting time of RWM is at least 30% longer than that of BMM. However, when the network is denser (e.g., $d \geq 15$), the inter-meeting time is almost invariant, and the product of inter-meeting time and the radius (transmission range) seems to be constant (we will discuss this later in Section III-B.).

B. Theoretical Support on the Invariance Property

1) Session Time: The following result in [13] gives the mean session time T_S in a closed area H with boundary ∂H:

Proposition 1: [13] Let v be the node speed, $\theta(d\vec{r})$ be the direction of tangent to the differential line segment at point $\vec{r}$ in Ω, and $h(\vec{r}, \phi)$ be the normalized specific flux at point $\vec{r}$ in direction ϕ. Then, the average session time is given by

$$T_S = \frac{\mathbb{E}[1/v] \int_H \int_0^{2\pi} h(\vec{r}, \phi) d\phi dH}{\int_{\partial H} \int_0^{\pi} \sin\phi \cdot h(\vec{r}, \theta(d\vec{r}) + \phi) d\phi dr}. \tag{3}$$

To explain $h(\vec{r}, \phi)$ more clearly, we re-interpret its definition in [13] as following: there exists constant C, which does not change with respect to either $\vec{r}$ or ϕ, such that $C \cdot h(\vec{r}, \phi)$ is the expected rate of crossings over the differential line segment at point $\vec{r}$ perpendicular to ϕ per unit length of the segment and per unit angle. In other words, $h(\vec{r}, \phi)$ is the normalized stationary probability density function of both node position and node direction.

For isotropic random walk mobility models, we can derive the average session time/length from Proposition 1 as shown below.

Proposition 2: The average session time for the class of isotropic random walk mobility models under Boolean interference model, is given by

$$T_S = \mathbb{E}[1/v] \frac{\pi d}{2}, \tag{4}$$

where v is the node speed and d is the communication range.

Proof: For isotropic random walk mobility models, $h(\vec{r}, \phi) = \rho_0$, where ρ_0 is a constant, i.e., stationary pdf does not depend on $\vec{r}$ (position) or ϕ (direction). Hence, from (3), the average session time becomes

$$T_S = \mathbb{E}[1/v] \frac{2\pi\rho_0 A}{2\rho_0 L} = \mathbb{E}[1/v] \frac{\pi A}{L}, \tag{5}$$

where A and L are the area and perimeter of the closed area H, respectively. When the communication range is d, under Boolean model, H is a circle with radius d. In this case, we have $A = \pi d^2$, $L = 2\pi d$. Thus, from (5), we are done. ∎

We observe that the average session time predicted in (5) closely matches with the numerical results in Figure 2.

2) Contact Time: We now consider contact time between a pair of mobile nodes. To proceed, we pose the following assumption:

Assumption 1: Given two nodes A and B, each of which follows isotropic random walk mobility models, define the difference walker C with its position at time t given by $C(t) := A(t) - B(t)$. Then the contact time of nodes A and B is assumed to be equal to the session time of node C with respect to a circle H with radius d. □

Remark 1: In an unbounded domain (e.g., $\Omega = \mathbb{R}^2$), it is obvious that the contact time of nodes A and B is exactly the session time of their difference walker C. However, in a bounded area, when node A or B is very close to the boundary, e.g., when A is on the boundary, then its contact region is no larger than a half circle. Intuitively, when the domain size is quite large compared to the contact region, the event that A or B is very close to the boundary will be a rare event, and Assumption 1 may be well satisfied in this case. □

We now have the following theorem on the contact time:

Theorem 1: Under Assumption 1, the average contact time of two independent nodes $A,\ B$ following any isotropic random walk mobility models is invariant with respect to the step-length distribution of the chosen mobility models. □

Proof: It is easy to see that for the difference random walker C of nodes A and B, the stationary distribution of node *direction* is still uniform in $[0, 2\pi)$. However, its stationary distribution of node *position* is not uniform anymore. Suppose that in the steady-state, $A(t)$ and $B(t)$ are uniformly distributed on $[-a, a] \times [-a, a]$ $(a > 0)$, then the stationary pdf of $C(t)$ is given by

$$f_c(x, y) = \begin{cases} (\frac{1}{2a} - |x|)(\frac{1}{2a} - |y|), & \text{for } |x|, |y| \in [0, 2a] \\ 0, & \text{otherwise.} \end{cases}$$

Since the stationary distribution of node direction is uniform, $h(\vec{r}, \phi)$ is invariant with respect to ϕ, i.e., $h(\vec{r}, \phi) = h(\vec{r})$. Then, from Proposition 2, the average session time for C becomes

$$\frac{\mathbb{E}[1/v_R] \cdot 2\pi \int_H h(\vec{r}) dH}{2 \int_{\partial H} h(\vec{r}) dr}, \tag{6}$$

where v_R is the speed of C, i.e., the relative speed of A and B. Note that no matter what the step-length distribution is, the stationary distribution of $A,\ B$, as well as that of C, does not change, hence $h(\vec{r})$ remains the same. In other words, $h(\vec{r})$ is invariant with respect to the step-length distribution. Thus, the average session time for C is also *invariant* with respect to step-length distribution. In view of Assumption 1, this completes the proof. ■

In fact, the relative speed (v_R) of A and B is not fixed at $1m/s$ any more. This is why the average contact time in Figure 3(a) is smaller than the average session time in Figure 2.

3) Inter-meeting Time: As long as the inter-meeting time of two nodes A and B can be well approximated by the inter-session time of their difference walker C with respect to a circle H with radius d, the inter-meeting (or inter-session, when one node is static) time can be derived similarly as in Theorem 1, since Proposition 1 does not require any specific form of the shape of bounded area H under study. This leads to the estimates of the mean inter-session time as $\mathbb{E}[1/v]\frac{A-\pi d^2}{2d}$. Note that [17] derives the mean inter-session time as $\frac{A}{2d\mathbb{E}[v]}$, which is very close to $\mathbb{E}[1/v]\frac{A-\pi d^2}{2d}$.

However, Figure 3(b) shows the clear difference in the mean inter-meeting time when the communication range d (e.g., $d = 5, 10$) is very small compared to the simulation area (square with width 400), i.e., the inter-meeting time of two nodes A and B cannot be well approximated by the inter-session time of their difference walker C, at least for sparse networks.[4].

C. Effective Contact and Inter-meeting Time - Beyond the First-order Behavior

In Sections III-A and III-B, the first-order behavior of the contact and inter-meeting time is discussed. However, higher-order behavior of contact and inter-meeting time may also be critical. For example, consider the following two distributions of inter-meeting time T_I (unit in seconds): (i) T_I is uniformly distributed in (1, 10000); (ii) T_I is either 1 or 10000 with equal probability. Obviously, T_I has the same mean in both cases. However, for effective communication, we would prefer T_I in the first case to that in the second case, since if two nodes meet again 1 second after they just depart from each other, most likely they do not have any new information to exchange. Similarly, too short contact time essentially leads to 'ineffective meeting'.

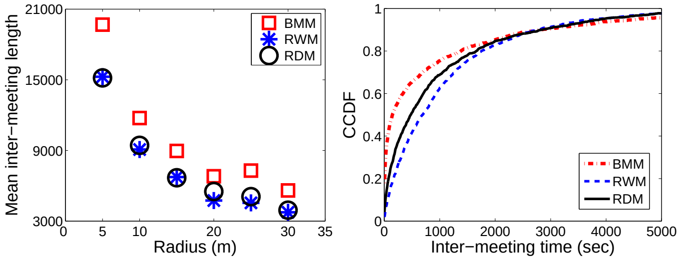
For each contact/inter-meeting sample we have collected (we collect 7500 samples for each point in Figures 3(a) and (b)), we use the following rules to decide whether it is 'effective' or not: (i) Effective contact time samples should be at least 3 seconds long[5] or larger than $1/10$ of the mean contact time. (ii) Similarly, effective inter-meeting time sample should be at least 2 minutes (120 seconds). This threshold is also very small compared to the mean inter-meeting time shown in Figure 3(b).

Under these two rules, we find that the percentage of 'ineffective' contact samples is always less than 3%. So, even if we plot the mean contact time using only 'effective' samples, the invariance results still hold. However, the percentage of 'ineffective' inter-meeting meeting samples is rather high, e.g., more than 40% of the inter-meeting time samples collected for BMM is smaller than 120 seconds.

Figure 4(a) shows the average effective inter-meeting time. Now, BMM has the largest mean effective inter-meeting time among these models, which is in stark contrast to Figure 3(b), where the mean inter-meeting time for BMM is the smallest. This is because BMM produces too many ineffective (too small) inter-meeting time samples, which contributes to the smallest mean inter-meeting time in Figure 3(b). To see this

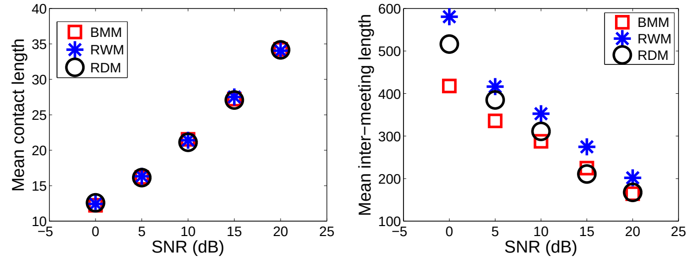
[4]Another example where the inter-meeting time of nodes A and B cannot be well approximated by the inter-session time of C, is the case in Section IV where the contact between two nodes is not uniquely determined by their distance.

[5]Note that in our simulation setting, the contact time is very small, which is why the threshold is also chosen to be small.



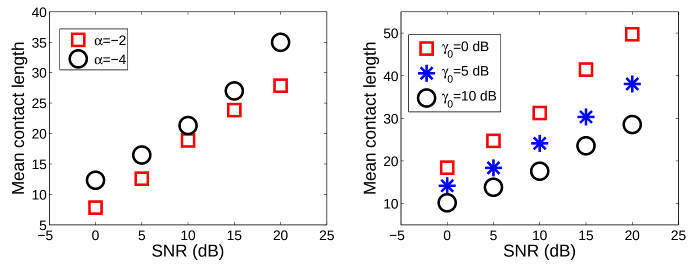
(a) Effective inter-meeting time (b) Inter-meeting time CCDF

Fig. 4. This figure use the same data set as Figure 3(b). (a): Average 'effective' inter-meeting time, i.e., the mean of all inter-meeting time samples no less than 120 seconds. (b) CCDF of all inter-meeting time samples: BMM model produces much larger percentage of 'ineffective' inter-meeting time than other two models.



(a) contact time (SINR) (b) Inter-meeting time (SINR)

Fig. 5. Average contact and inter-meeting time under SINR model. Similar invariance property as that under Boolean model (Figure 3) is observed: (a) The mean contact time is invariant with respect to mobility patterns; (b) The mean inter-meeting time is not invariant.



(a) contact time vs. α (b) contact time vs. γ_0

Fig. 6. The effect of path loss exponent α (a) and the SINR threshold γ_0 (b) on the mean contact time under SINR interference model.

more clearly, we plot the Complementary Cumulative Distribution Function (CCDF) of all inter-meeting time samples (including both effective and ineffective ones) in Figure 4(b). We can see clearly that BMM produces much larger percentage of ineffective inter-meeting time samples than RWM and RDM do. Intuitively, this is because BMM generates many short steps in its mobility pattern. Thus, when a pair of nodes meet and depart under BMM, they tend to 'hang around' nearby and meet again shortly.

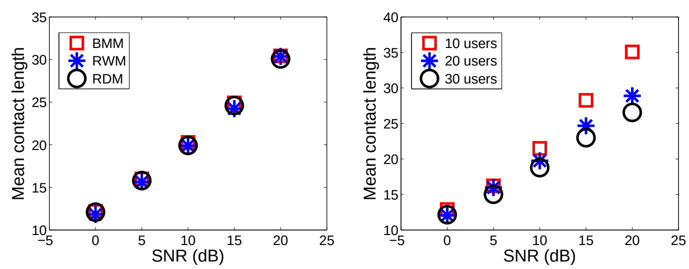
IV. Invariance Property under SINR Interference Model

As Figure 1 shows, the contact region of a node B at any time t under SINR model is affected by not only the parameters related to SINR criterion (e.g., α, SNR, γ_0), but also the existence and locations of other mobile nodes in the same system. SINR model considers more realistic situations than Boolean model, but at the same time poses challenge to our analysis due to many interferences from other nodes and more complicated geometric structure of the contact set. Thus, the question is: "Does the invariance property we have under Boolean model still hold under SINR model?" In this section, we numerically study the contact and inter-meeting time under SINR interference model. Our simulation set-up is as follows.

- Simulation area, mobility models, boundary behavior are the same as in Section III.
- Unless otherwise specified (e.g., in Figures 6, 7 where we study the effect of different α, γ_0 and the number of nodes), we set $\alpha = -4$, $\gamma_0 = 5$ $(\approx 7dB)$ and the number of nodes $= 10$.
- SNR $\in \{0, 5, 10, 15, 20\}$ dB, here SNR $= P/N_0$ is the value measured at distance $d_0 = 15(m)$. Note that d_0 may vary according to the system requirement on the quality of communication channel, e.g. when higher quality communication is required (e.g., lower bit error rate), larger d_0 can be used.

Figure 5 shows the average contact and inter-meeting time under SINR model. Similar to Figure 3, the average contact time is still invariant with respect to different isotropic random walk mobility models with different step-length distributions, while the average inter-meeting time is not.

Figure 6 shows the effect of different values of the path loss exponent α (in (a)) and the SINR threshold γ_0 (in (b)). As $|\alpha|$ increases, the mean contact time also increases, as larger $|\alpha|$ means smaller interference from other nodes. In addition, larger γ_0 leads to smaller mean contact time, as larger γ_0 effectively decreases the size of the contact region.



(a) contact time (20 users) (b) contact time vs. # of nodes

Fig. 7. The effect of the number of nodes on the mean contact and inter-meeting time under SINR model: (a) The invariance property for the average contact time still holds when the number of nodes is increased to 20 (compared to Figure 5(a) with 10 nodes). (b) We use RWM for the mobility model and change the number of nodes in the system.

Figure 7(a) shows that the mean contact time is still invariant with respect to mobility patterns when the number of users in the system is increased from 10 to 20. (Of course the contact time is smaller due to the interference from more nodes.) In Figure 7(b), we use RWM for all nodes and vary the number of nodes from 10 to 30. Clearly, the larger the number of nodes, the more interference on any of the nodes. Thus, the mean contact time decreases with the increase in the number of nodes.

V. Conclusion

In this paper, we show the invariance property for the average contact time of isotropic random walk mobility models. We find that the mean contact time is invariant with respect to different isotropic random walk mobility models under both Boolean and SINR interference models. In addition, we study the higher-order behavior (more than just the mean) of both contact and inter-meeting time. To the best of our knowledge, this is the first work reporting such a clean-cut invariance property for a large class of mobility models under both Boolean and SINR interference models. We expect that our invariance results can help simplify a number of system optimization and protocol design in the area of wireless sensor networks and MANETs.

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