Exploiting Heterogeneity for Improving Forwarding Performance in Mobile Opportunistic Networks: An Analytic Approach

Chul-Ho Lee and Do Young Eun

Abstract—Heterogeneity arises in a wide range of scenarios in mobile opportunistic networks and is one of the key factors that govern the performance of forwarding algorithms. While the heterogeneity has been empirically investigated and exploited in the design of new forwarding algorithms, it has been typically ignored or marginalized when it comes to rigorous performance analysis of such algorithms. In this paper, we develop an analytical framework to quantify the performance gain achievable by exploiting the heterogeneity in mobile nodes' contact dynamics. In particular, we derive a delay upper bound of a heterogeneity-aware static forwarding policy per each given number of message copies and obtain its closed-form expression, which enables our quantitative study on the benefit of leveraging underlying heterogeneity structure in the design of forwarding algorithms. In addition, we develop a dynamic forwarding policy that performs as an extension of the static forwarding policy while proven to improve the delay performance. We then demonstrate that only a small fraction of total (unlimited) message copies, via both static and dynamic forwarding policies, are enough under various heterogeneous network settings to achieve the same delay as that obtained using the unlimited message copies when the networks become homogeneous. We also show that, given the same number of message copies, our dynamic forwarding policy significantly outperforms the 'homogeneous-optimal' forwarding policy (up to about 50 percent improvement in the delay performance), especially when the number of message copies allowed in the networks is small.

Index Terms—Mobile opportunistic networks, heterogeneous contact behaviors between different mobile nodes, heterogeneity-aware forwarding policies, forwarding performance, performance analysis

1 INTRODUCTION

Mobile opportunistic networks, a.k.a. delay or disruption tolerant networks, have received much attention from the networking research community as a promising evolution of mobile ad-hoc networks toward several applications such as Pocket Switched Networks [2] or UMass Dieselnet [3]. In mobile opportunistic networks, network connectivity is changing over time and frequently disrupted due to node mobility, power limitation, limited storage, among others. To overcome this intermittent connectivity nature, mobile opportunistic networks employ a *'store-carry-and-forward'* principle in which mobile nodes can carry messages and copy and/or relay them to other nodes upon encounter, thereby rendering messages eventually delivered to their destinations.

Many empirical studies have indicated the presence of heterogeneity in a wide range of scenarios in mobile opportunistic networks. For example, Refs. [4], [5] investigate real mobility traces and disclose the characteristics of heterogeneity in mobile nodes' contact dynamics. Similarly, based on real mobility traces and survey data [5], [6], [7], [8] uncover spatially and/or socially formatted community structures in node mobility. The observed characteristics of heterogeneity structures have been mainly used for the development of new mobility models [6], [8] and empirically exploited to the design of new forwarding/routing algorithms [5], [7], [9].

There are several analytical studies on the performance of a few forwarding/routing algorithms including epidemic routing [10], [11], [12], single-copy and multicopy two-hop relay protocols [10], [13], spray and wait [14], [15], etc; however, these works are mainly based upon a homogeneous model in which *any* mobile node is making contacts with others according to a Poisson process. Other analytical studies also fully rest on the homogeneous model for their investigation on the capacity-delay tradeoff [13], the cost-delay tradeoff [16], [17], the design of forwarding policy [18], [19], [20], and the content distribution [21]. The current literature still lacks analytical studies on exploiting the underlying heterogeneity structure in order to correctly understand the resulting performance gain.

In this paper, we analytically investigate how much benefit the heterogeneity in mobile nodes' contact dynamics can bring in the forwarding performance. To this end, we employ the heterogeneous network model used in [4], [22], [23], [24] in which the pairwise inter-contact time of a given node pair is exponentially distributed but with *different* rates over different pairs. (See Section 2 for its detailed description and justification.) Under this heterogeneous setting, we then consider a class of probabilistic two-hop forwarding policies in which a source node forwards a message with probability p_i to each relay node i upon encounter. Since message delivery delay and the number of (used) message copies are both mainly

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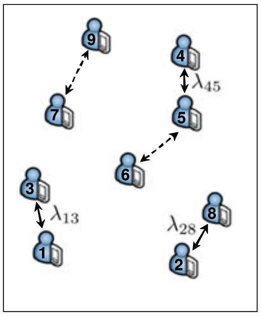
functions of p_i and the heterogeneity of contact rates over different node pairs, we are led to find an optimal forwarding policy $\{p_i^\star\}$, maximally exploiting the heterogeneity structure, to minimize the average message delivery delay under a given constraint on the number of message copies.

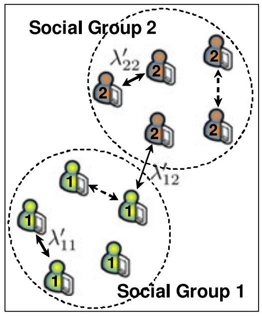
Rather than directly solving the optimization problem, as a viable alternative, we derive a delay upper bound of any given two-hop forwarding policy and find a *static forwarding policy* that minimizes the delay bound while satisfying the given constraint on the number of message copies. Although this forwarding policy is a sub-optimal solution to the original problem, we are able to derive a *closed-form* expression of its guaranteed delay bound, which in turn enables to quantify the performance gain achievable by exploiting the heterogeneity structure in contact dynamics. There is, however, still room for further improvement, since the forwarding probabilities in the static forwarding policy (and originally in the optimization problem) are not time-varying (but constant over time), thus losing the benefit of changing the relay nodes on the fly upon encounter. We thus develop a *dynamic forwarding policy* as an extension of the static policy to take advantage of dynamically changing relay nodes at each contact instant while maintaining the same number of message copies, and prove that this dynamic policy leads to better delay performance. We also provide simulation results for performance evaluation as well as to support our analytical results.

In the performance evaluation, we demonstrate that under various heterogeneous network settings only a small fraction of total (unlimited) message copies, via both static and dynamic forwarding policies, is sufficient to achieve the same delay as the optimal delay (obtained at the expense of unlimited message copies by multicopy two-hop relay protocol) when the networks become homogeneous. We also show that, given the same number of message copies, the dynamic forwarding policy outperforms the '*homogeneous-optimal*' forwarding policy, where the homogeneous-optimal policy is the optimal two-hop policy under any homogeneous network that simply makes message copies to *any first* encountered nodes up to the given number of copies. In particular, the performance improvement becomes significantly greater when the number of copies allowed in the networks is small. While there have been several empirical studies (e.g., [5], [7], [9]) that propose heuristic forwarding/routing algorithms utilizing the underlying heterogeneity structure, our analytical work provides fundamental insights on the attainable performance gain by exploiting the underlying heterogeneity structure.

The rest of this paper is organized as follows. Section 2 gives preliminaries on a heterogeneous network model and related work. Section 3 presents a class of probabilistic two-hop forwarding policies and an optimization problem to find an optimal forwarding policy. In Section 4, we provide an analysis on the achievable delay upper bound for any two-hop forwarding policy. We then present a static forwarding policy which minimizes the derived delay upper bound in Section 5, and describe a dynamic forwarding policy working on top of the static policy with its proven performance improvement in Section 6. We provide simulation results for performance evaluation and to support our analytical results in Section 7, and finally conclude in Section 8.



(a) A general case



(b) A special case with two social groups

Fig. 1. The heterogeneous network model.

2 Preliminaries

2.1 Network Model

The heterogeneous network model that we consider in this paper is used in Refs. [4], [22], [23], [24] and described as follows. There is a set of mobile nodes $\mathcal{N}$ in the whole network domain. The pairwise inter-contact time between mobile nodes i and j, denoted by T_{ij}, is independently drawn from an exponential distribution with rate $\lambda_{ij} > 0$ (i.e., contacts between nodes i and j occur according to a Poisson process with rate parameter λ_{ij}), where $i, j \in \mathcal{N}$ and $i \neq j$. Note that this contact process between nodes i and j is symmetric ($\lambda_{ij} = \lambda_{ji}$). The pairwise inter-contact times between any two node pairs are also mutually independent. In this model, the heterogeneity in mobile nodes' contact dynamics is captured by different contact rates λ_{ij}. Fig. 1a shows a general case of this heterogeneous network model.

If $\lambda_{ij} = \lambda$ for all $i, j \in \mathcal{N}$ and $i \neq j$, the heterogeneous network model reduces to the homogeneous model (a.k.a. Poisson contact model) in which contacts between *any pair* of mobile nodes occur according to a Poisson process with the same rate parameter λ. This heterogeneous model can also capture social community structures [22]. Suppose that there are M different social groups G_i $(i = 1, \ldots, M)$ forming a partition of $\mathcal{N}$, i.e., $\mathcal{N} = \bigcup_{i=1}^{M} G_i$. Let λ'_{lk} be common contact rate between any member of G_l and another member of G_k for $l, k = 1, \ldots, M$. That is, $\lambda_{ij} = \lambda'_{lk}$ for all $i \in G_l$ and $j \in G_k$ where $l, k = 1, \ldots, M$. Here, by assigning higher values to the contact rates for nodes in the same group (than those between different groups), we could emulate the human social behavior—people are more likely to meet their friends or others from the same community than some strangers. Fig. 1b shows a setting of two social groups as a special case of the heterogeneous network model.

2.2 Related Work

In the literature, many empirical studies [4], [5], [6], [7], [8] focus on the presence of heterogeneity and its characteristics from real mobility traces and survey data. First, Conan et al. [4] show heterogeneity in pairwise inter-contact time of each node pair. Hui et al. [5] also find heterogeneity structure from individual node and (social) community viewpoints. In particular, it shows that human community can be divided into several social communities, and within each community there are several socially-active people (nodes) which make more frequent contacts with others. In addition, Refs. [7], [8] observe that spatial node distributions are heterogeneous (non-uniform) and there exist clustering points with high node density in which mobile nodes have higher chance to encounter other nodes. Similarly, Hsu et al. [6] show uneven spatial distribution of mobile nodes from survey data. All these empirical studies focus more on the development of a new mobility model [6], [8] and on the design of a new heuristic forwarding algorithm exploiting the underlying heterogeneity structure [5], [7].

On the other hand, there are a few of analytical works [23], [24], [25], [26] on the performance of forwarding/routing protocols in mobile opportunistic networks under heterogeneous network settings. In our previous work [24], we address how two different sources of heterogeneity (spatial and node heterogeneities) in mobile nodes' contact dynamics impact the delay performance. In addition, Garetto et al. [25] analyze an asymptotic capacity scaling property as the number of nodes grows to infinity under the presence of node and spatial heterogeneities. In [26], the authors also study the improvement of network performance by adding infrastructures to mobile opportunistic networks under spatially heterogeneous network model. Spyropoulos et al. [23] propose a class of heuristic routing algorithms which exploit the underlying heterogeneity structure. In contrast, in this paper, we are aiming at analyzing how much benefit the heterogeneity in mobile nodes' contact dynamics can bring in the forwarding performance for any given *finite* number of mobile nodes in the network. In particular, we examine *how much better* we can do than simply assuming that the network is homogenous or the optimal forwarding policy obtained under the homogeneous network.

Lastly, the heterogeneous network model we adopt in this paper has also been used in several other papers [22], [23], [24] for the design of forwarding protocol and/or performance analysis. In addition, Refs. [4], [27] have provided empirical evidences of the exponential assumption for the pairwise inter-contact time distribution of each node pair in the heterogeneous network model. It is shown, through statistical methods, that there exist a non-negligible portion of node pairs in real contact traces (such as those collected in the MIT Reality Mining and RollerNet experiments) whose inter-contact time distributions can be well fitted by exponential distributions with different rates.

3 Probabilistic Relay Selection

In this section, we first explain a class of probabilistic two-hop forwarding policies with a given constraint on the number of message copies under the aforementioned heterogeneous model. We then formulate an optimization problem to find an optimal forwarding policy to minimize the message delivery delay while satisfying the constraint.

In the class of probabilistic two-hop forwarding policies, a source node forwards a message copy to each relay node r_i with probability $p_i \in [0, 1]$ upon encounter. Note that the source node has no benefit of forwarding a copy to each relay node upon the second or later encounter after skipping the first forwarding opportunity. Thus, the forwarding decision for each relay node is done only once upon the first encounter. Then, the forwarded message copies or an original message can be delivered to their destinations via relay nodes chosen in the forwarding decision or directly by the source, respectively. Fig. 2 depicts this operation under the probabilistic two-hop forwarding policies.

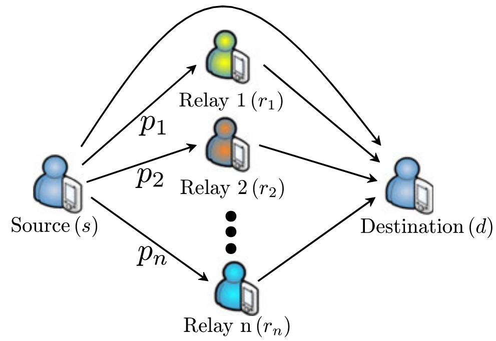


Fig. 2. A class of probabilistic two-hop forwarding policies. Source s forwards a message copy to relay node r_i with probability p_i.

Each forwarding probability p_i should be chosen to satisfy a constraint on the number of message copies, which in turn controls network cost or the amount of resource consumption incurred by additional message transfers.[1] At the same time, the message delivery delay critically depends on how we choose p_i for each relay node r_i. Thus, we can formulate the problem of finding an optimal forwarding policy $\vec{p}^\star$ under the constraint on the number of message copies in the heterogeneous model as an optimization problem. In what follows, we describe this formulation step by step.

For the optimization problem, we here do not consider two-hop forwarding policies that change relay paths or choose relay nodes on the fly upon encounter, i.e., the forwarding probability p_i for each relay node i is not changing over time but static (time-invariant). However, after obtaining a static forwarding policy from the optimization problem, we later develop a dynamic forwarding policy working on top of the static policy, which further takes advantage of changing relay nodes (or paths) dynamically upon encounter, and prove that this dynamic policy leads to better delay performance.

Throughout the rest of this paper, we assume the followings as in other analytical works [10], [12], [14], [20], [21]. The network is sparse and network traffic is light such that interference and contention [11] are not important factors. In other words, we assume that each node has infinite bandwidth and buffer. In fact, as will be shown in Section 7, exploiting the heterogeneity in mobile nodes' contact dynamics is helpful in significantly reducing the number of message copies, which in turn keeps the network traffic low and thus decreases the effect of interference/contention. In addition, we assume that a message transfer between any two nodes at their contact instant takes a negligible time with respect to their inter-contact time.

Let $\mathcal{N} = \{s, r_1, \ldots, r_n, d\}$ with source s and destination d, and n possible relay nodes $r_1, r_2, \ldots, r_n$. Let $\{Y_i\}_{1\leq i\leq n}$ be the set of independent Bernoulli random variables with

$$\mathbb{P}\{Y_i = 1\} = p_i \text{ and } \mathbb{P}\{Y_i = 0\} = 1 - p_i,$$

1. As a special case, if $p_i = 1$ for all relay nodes (no resource constraint), then the probabilistic two-hop forwarding policy reduces to the multicopy two-hop relay protocol [10], [12].

to represent the forwarding decision to relay node r_i. For each i, we define a function I_{p_i} as

$$I_{p_i} = \begin{cases} 1 & \text{if } Y_i = 1, \\ \infty & \text{otherwise.} \end{cases} \tag{1}$$

Let $M = \sum_{i=1}^{n} Y_i$ denote the random variable to represent the number of message copies except the original message at the source node in the whole network. We also define $\vec{Y} \triangleq [Y_1, \ldots, Y_n]$ and $\vec{p} \triangleq [p_1, \ldots, p_n]$.

The message delivery delay D under a probabilistic two-hop forwarding policy with $\vec{p}$ can then be written as

$$D = \min\{T_{sd}, (T_{sr_1} + T_{r_1 d})I_{p_1}, \ldots, (T_{sr_n} + T_{r_n d})I_{p_n}\}. \tag{2}$$

We want to compute $\mathbb{E}\{D\}$ in terms of forwarding policy $\{p_i\}$ and the network parameter λ_{ij}. Since all the random variables inside the minimum operator in eq. (2) are independent of each other, we have

$$\mathbb{P}\{D > t\} = \mathbb{P}\{T_{sd} > t\} \prod_{i=1}^{n} \mathbb{P}\{(T_{sr_i} + T_{r_i d})I_{p_i} > t\}. \tag{3}$$

By conditioning on I_{p_i}, we have

$$\begin{aligned} \mathbb{P}\{(T_{sr_i} + T_{r_i d})I_{p_i} > t\} &= \mathbb{E}\{\mathbb{P}\{(T_{sr_i} + T_{r_i d})I_{p_i} > t \mid I_{p_i}\}\} \\ &= \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\}\mathbb{P}\{I_{p_i} = 1\} + \mathbb{P}\{t < \infty\}\mathbb{P}\{I_{p_i} = \infty\} \\ &= \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\}p_i + (1 - p_i), \end{aligned} \tag{4}$$

where the last equality is from the definition of I_{p_i} in eq. (1).

For notational simplicity, we define $f_0(t) \triangleq \mathbb{P}\{T_{sd} > t\}$ and $f_i(t) \triangleq \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\}$, where T_{sd}, T_{sr_i}, and $T_{r_i d}$ are independent exponential random variables with rate $\lambda_{sd}, \lambda_{sr_i}$, and $\lambda_{r_i d}$, respectively. Then, from eqs. (4), (3) can be rewritten as

$$\mathbb{P}\{D > t\} = f_0(t) \prod_{i=1}^{n} [p_i f_i(t) + (1 - p_i)], \tag{5}$$

and thus, by noting that $\mathbb{E}\{D\} = \int_0^\infty \mathbb{P}\{D > t\}dt$, we have

$$\mathbb{E}\{D\}_{\vec{p}} = \int_0^\infty f_0(t) \prod_{i=1}^{n} [p_i f_i(t) + (1 - p_i)]dt, \tag{6}$$

where we use the subscript in $\mathbb{E}\{D\}_{\vec{p}}$ to clearly indicate that the average delay is a function of the forwarding policy $\vec{p} = [p_1, \ldots, p_n]$.

Now, we formally state our problem to find an optimal forwarding policy $\vec{p}^\star$ under the constraint on the average number of message copies, i.e., $\mathbb{E}\{M\} = \mathbb{E}\{\sum_{i=1}^{n} Y_i\} = \sum_{i=1}^{n} p_i$, as the following optimization problem: For $\mathbb{E}\{D\}_{\vec{p}} : [0,1]^n \to \mathbb{R}_+$,

$$(\mathbf{P1}) \qquad \begin{aligned} &\textbf{minimize } \mathbb{E}\{D\}_{\vec{p}} \\ &\textbf{subject to} \sum_{i=1}^{n} p_i \le K, \end{aligned}$$

where $\vec{p}$ denotes a forwarding policy and K is a positive integer ($1 \le K \le n$). As explicitly shown in ($\mathbf{P1}$), the average number of copies (except the original message at the source node) allowed in the network is limited up to K copies.

4 Delay Analysis

In this section, we derive an achievable upper bound on the delay in a tractable form, which leads us to find a static forwarding policy that is a sub-optimal solution to ($\mathbf{P1}$). We also explain an intuition behind the delay upper bound by considering multicopy two-hop relay protocol [10], [12] as an importance special case.

4.1 An Upper Bound of Message Delivery Delay

A difficulty in solving the optimization problem ($\mathbf{P1}$) arises, since it is not a convex optimization problem, which can be checked by showing that the Hessian matrix of $\mathbb{E}\{D\}_{\vec{p}}$ with respect to $\vec{p}$ is neither positive semidefinite nor negative semidefinite. Thus, we cannot resort to the standard convex optimization techniques [28] to find the optimal solution of ($\mathbf{P1}$). Instead, we below derive an upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ from eq. (6), which becomes mathematically more tractable.

First, by noting that T_{sd} is an exponential random variable with rate λ_{sd}, we rewrite eq. (6) as

$$\begin{aligned} \mathbb{E}\{D\}_{\vec{p}} &= \int_0^\infty e^{-\lambda_{sd}t} \prod_{i=1}^{n} [p_i f_i(t) + (1 - p_i)]dt \\ &= \frac{1}{\lambda_{sd}} \int_0^\infty \left(\prod_{i=1}^{n} [p_i f_i(t) + (1 - p_i)] \right) \lambda_{sd} e^{-\lambda_{sd}t} dt \\ &= \frac{1}{\lambda_{sd}} \mathbb{E}\left\{ \prod_{i=1}^{n} [p_i f_i(T_{sd}) + (1 - p_i)] \right\}, \end{aligned} \tag{7}$$

where the expectation is with respect to T_{sd}.

We denote $\|X\|_q$ to be the L_q norm of a (real-valued) random variable X, i.e., $\|X\|_q \triangleq [\mathbb{E}\{|X|^q\}]^{1/q}$ for $0 < q < \infty$, and $\|X\|_\infty \triangleq \inf[c \in \mathbb{R} : \mathbb{P}\{|X| > c\} = 0]$. We also define by $\mathcal{L}^q$ a set of all random variables X for which $\|X\|_q < \infty$.

To proceed, we need the following two inequalities that will be used to derive the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ from eq. (7).

Theorem 1 [29], [30] (Generalized Hölder's Inequality). *Let $1 \le q_i \le \infty$ with $\sum_{i=1}^{n} 1/q_i = 1$. If $X_i \in \mathcal{L}^{q_i}$ for $1 \le i \le n$, then $\prod_{i=1}^{n} X_i \in \mathcal{L}^1$ and*

$$\left\| \prod_{i=1}^{n} X_i \right\|_1 \le \prod_{i=1}^{n} \|X_i\|_{q_i}.$$

Theorem 2 [29], [30] (Minkowski's Inequality). *For $X, Y \in \mathcal{L}^q$ with $1 \le q \le \infty$,*

$$\|X + Y\|_q \le \|X\|_q + \|Y\|_q.$$

Since $f_i(t) = \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\} \le 1$, it follows that $f_i(T_{sd}) \in \mathcal{L}^q$ and $p_i f_i(T_{sd}) + (1 - p_i) \in \mathcal{L}^q$ for $1 \le q \le \infty$. Thus, for any $1 \le q_i \le \infty$ with $\sum_{i=1}^{n} 1/q_i = 1$, we have

$$\mathbb{E}\{D\}_{\vec{p}} \le \frac{1}{\lambda_{sd}} \prod_{i=1}^{n} \|p_i f_i(T_{sd}) + (1 - p_i)\|_{q_i} \tag{8}$$

$$\le \frac{1}{\lambda_{sd}} \prod_{i=1}^{n} [p_i \|f_i(T_{sd})\|_{q_i} + (1 - p_i)], \tag{9}$$

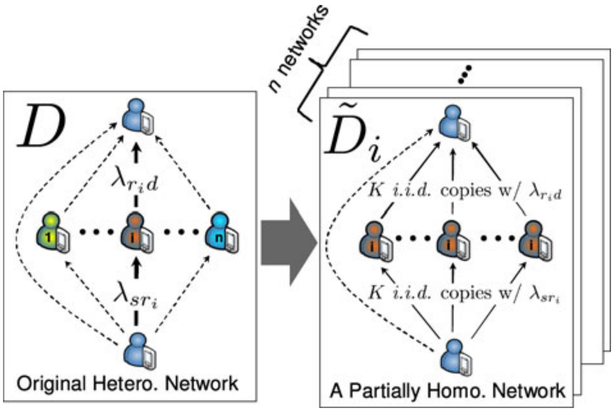


Fig. 3. Decomposition of a heterogeneous network into n partially homogeneous networks $\mathfrak{N}_i$. K two-hop relay paths in $\mathfrak{N}_i$ are *i.i.d.* copies of the two-hop relay path via relay node r_i in the original heterogeneous network.

where eq. (8) is from the generalized Hölder's inequality and eq. (9) is from Minkowski's inequality.

In contrast to the original form of $\mathbb{E}\{D\}_{\vec{p}}$ in eqs. (6) and (7), its upper bound in eq. (9) is in a much more tractable form. More important, this upper bound leads us to find an optimal solution $\vec{p}^*$ that minimizes the upper bound, which is our static forwarding policy and is good enough to show the benefit of exploiting the heterogeneity in mobile nodes' contact dynamics, as will be shown in Section 7. Note that the optimal solution $\vec{p}^*$ that minimizes the delay upper bound should be distinguished from the optimal solution $\vec{p}^\star$ of the original problem ($\mathbf{P1}$).

4.2 A Special Case: Multicopy Two-Hop Relay Protocol

We below consider the multicopy two-hop relay protocol as a special case ($K = n$) to get an intuition behind the delay upper bound in eq. (9).

Let $T_{sr_i}^j$ ($j = 1, \dots, K$) be *i.i.d.* exponential random variables with rate λ_{sr_i}, and similarly for $T_{r_i d}^j$ ($j = 1, \dots, K$) with rate $\lambda_{r_i d}$. We define

$$\tilde{D}_i \triangleq \min\{T_{sd}, T_{sr_i}^1 + T_{r_i d}^1, \dots, T_{sr_i}^K + T_{r_i d}^K\}, \tag{10}$$

for $i = 1, \dots, n$. $\tilde{D}_i$ here is defined for general K-copies which will be used in the rest of this paper. By definition, $\tilde{D}_i$ can be interpreted as the message delivery delay of multicopy two-hop relay policy over a partially homogeneous network $\mathfrak{N}_i$ (as depicted in Fig. 3) that is composed of a direct source-destination path and K *i.i.d.* two-hop relay paths, each of which has delay equal to the sum of two exponential random variables with rates λ_{sr_i} and $\lambda_{r_i d}$. In other words, K two-hop relay paths in $\mathfrak{N}_i$ are *i.i.d.* copies of the two-hop relay path via relay node i in the original heterogeneous network, and the direct path in both networks remains the same. Fig. 3 shows this decomposition procedure from an original heterogeneous network to n partially homogeneous networks $\mathfrak{N}_i$ ($i = 1, \dots, n$).

From $T_{sr_i}^j + T_{r_i d}^j \overset{d}{=} T_{sr_i} + T_{r_i d}$ and independence over $j = 1, \dots, K$, for each i, we have

$$\mathbb{E}\{\tilde{D}_i\} = \int_0^\infty f_0(t)\big[f_i(t)\big]^K dt. \tag{11}$$

By using the binomial expansion, we can obtain a close-form expression of $\mathbb{E}\{\tilde{D}_i\}$ as follows: for $\lambda_{sr_i} \neq \lambda_{r_i d}$,

$$\mathbb{E}\{\tilde{D}_i\} = \frac{1}{(\lambda_{r_i d} - \lambda_{sr_i})^K} \sum_{j=0}^{K} \binom{K}{j} \frac{(-\lambda_{sr_i})^j \lambda_{r_i d}^{K-j}}{\lambda_{sd} + j\lambda_{r_i d} + (K-j)\lambda_{sr_i}},$$

and for $\lambda_{sr_i} = \lambda_{r_i d}$,

$$\mathbb{E}\{\tilde{D}_i\} = \frac{1}{\lambda_{sr_i}} \sum_{j=0}^{K} \frac{K!}{(K-j)!(K + \lambda_{sd}/\lambda_{sr_i})^{j+1}}. \tag{12}$$

Now, consider $K = n$ and set $q_i = n$ for $i = 1, \dots, n$. Since the forwarding policy simply becomes $\vec{p} = \vec{1} = [1, \dots, 1]$, eq. (9) can be rewritten as

$$\begin{aligned} \mathbb{E}\{D\}_{\vec{1}} &\le \frac{1}{\lambda_{sd}} \prod_{i=1}^{n} \|f_i(T_{sd})\|_n \\ &= \frac{1}{\lambda_{sd}} \prod_{l=1}^{n} \bigg[\int_0^\infty \lambda_{sd} e^{-\lambda_{sd} t} \big[f_i(t)\big]^n dt\bigg]^{1/n} \\ &= \prod_{i=1}^{n} \bigg[\int_0^\infty f_0(t)\big[f_i(t)\big]^n dt\bigg]^{1/n} = \prod_{i=1}^{n} \big[\mathbb{E}\{\tilde{D}_i\}\big]^{1/n}. \end{aligned} \tag{13}$$

The delay upper bound in eq. (13) is nothing but a geometric mean of $\mathbb{E}\{\tilde{D}_i\}$, the average message delivery delay of multicopy two-hop relay protocol under $\mathfrak{N}_i$. We observe that this delay upper bound still captures the underlying heterogeneity in mobile nodes' contact dynamics, as each decomposed network $\mathfrak{N}_i$ contains each of n different two-hop relay paths of the original heterogeneous network. In addition, a closed-form solution of this delay upper bound can be immediately obtained from eq. (12).

Remark 1. When the network is homogeneous with $\lambda_{ij} = \lambda$, the upper bound in eq. (13) becomes identical to the original expression of $\mathbb{E}\{D\}_{\vec{1}}$ from eq. (6), i.e., all the intermediate inequalities that have led to eq. (13) hold with equality for this special case of multicopy two-hop relay protocol under the homogeneous network setting.

Remark 2. Consider two homogeneous networks $\mathfrak{N}_U$ and $\mathfrak{N}_L$ with common contact rates for any node pair given by $\lambda_{\min} = \min_{i,j \in \mathcal{N}}\{\lambda_{ij}\}$ and $\lambda_{\max} = \max_{i,j \in \mathcal{N}}\{\lambda_{ij}\}$, respectively. Also, set the average message delivery delay of multicopy two-hop relay protocol under $\mathfrak{N}_U$ and $\mathfrak{N}_L$ by $\mathbb{E}\{\hat{D}_U\}$ and $\mathbb{E}\{\hat{D}_L\}$, respectively. Then, it is straightforward to see that $\mathbb{E}\{\hat{D}_L\} \le \mathbb{E}\{D\}_{\vec{1}} \le \mathbb{E}\{\hat{D}_U\}$. It is also known [10], [12] that the average delay of multicopy two-hop relay protocol under a homogeneous network is asymptotically $\frac{1}{\lambda}\sqrt{\frac{\pi}{2(n+1)}}$ as $n \to \infty$ where $|\mathcal{N}| = n + 2$ and λ is a common contact rate for any node pair. Thus, if we were to focus on the asymptotic average delay of multicopy two-hop relay protocol, it would be $\mathcal{O}(\frac{1}{\sqrt{n+1}})$, *regardless of* whether the underlying network is homogeneous or heterogeneous.[2] This is precisely why we analyze the

2. Only the constant coefficient $1/\lambda_{\min}$ or $1/\lambda_{\max}$ will change and the order term in n remains the same.

delay performance under a heterogeneous network setting with a given *finite* number of mobile nodes.

5 A Static Forwarding Policy

In this section, we first obtain a static forwarding policy, or an optimal solution $\vec{p}^*$ that minimizes the delay upper bound derived in the previous section. We also show a closed form expression of the guaranteed delay bound under this static forwarding policy.

We fix $q_i \geq 1$, $i = 1, \dots, n$, with $\sum_{i=1}^n 1/q_i = 1$, and then derive a static forwarding policy $\vec{p}^*$ minimizing the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ in eq. (9). To this end, we consider the following optimization problem whose solution will be suboptimal to the original problem $(\mathbf{P1})$.

$$(\mathbf{P1}') \quad \begin{aligned} &\textbf{minimize} \prod_{i=1}^n [p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)] \\ &\textbf{subject to} \sum_{i=1}^n p_i \leq K. \end{aligned}$$

We define a sequence of $a_i(q_i)$ for a given q_i by

$$a_i(q_i) \triangleq \|f_i(T_{sd})\|_{q_i} = \left[\int_0^\infty \lambda_{sd} e^{-\lambda_{sd} t} [f_i(t)]^{q_i} dt\right]^{1/q_i}, \qquad (14)$$

where $f_i(t) = \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\}$. For notational simplicity, we will use a_i instead of $a_i(q_i)$ unless it is necessary to specify the given q_i. Rearrange a_i in an increasing (non-decreasing) order and set $a_{[i]}$ to be the ith smallest one among $a_1, \dots, a_n$, i.e., $a_{[1]} \leq a_{[2]} \leq \dots \leq a_{[n]}$. Also, let $c_1, \dots, c_n$ be a permutation of $\{1, \dots, n\}$ which satisfies $a_{c_l} = a_{[l]}$ for all $l = 1, \dots, n$. Then, we have the following proposition for the optimal solution $\vec{p}^*$ of $(\mathbf{P1}')$.

Proposition 1. *For any arbitrarily fixed $q_i \in [1, \infty]$ such that $\sum_{i=1}^n 1/q_i = 1$, the optimal solution $\vec{p}^*$ of $(\mathbf{P1}')$ is always of the following form:*

$$p_i^* = \begin{cases} 1 & \text{if } i \in \{c_1, \dots, c_K\}, \\ 0 & \text{otherwise.} \end{cases} \qquad (15)$$

Proof. We will find an optimal solution $\vec{p}^*$ of the optimization problem $(\mathbf{P1}')$ by obtaining a minimizer which achieves the lowest bound of the objective function in $(\mathbf{P1}')$ under the constraint $\sum_{i=1}^n p_i \leq K$. By taking $\log$ function to the objective function in $(\mathbf{P1}')$, it can be transformed as

$$\sum_{i=1}^n \log[p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)], \qquad (16)$$

since $\log$ function is a monotone increasing function. From Jensen's inequality and concavity of $\log$, eq. (16) is further lower bounded by

$$\sum_{i=1}^n \log[p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)] \geq \sum_{i=1}^n p_i \log \|f_i(T_{sd})\|_{q_i}. \qquad (17)$$

Then, we want to minimize the RHS of eq. (17) under the constraint $\sum_{i=1}^n p_i \leq K$, which in turn gives the lowest bound of eq. (16). It is equivalent to solving the following simple linear programming problem:

$$(\mathbf{P1}'') \quad \begin{aligned} &\textbf{minimize} \sum_{i=1}^n p_i \log \|f_i(T_{sd})\|_{q_i} \\ &\textbf{subject to} \sum_{i=1}^n p_i \leq K. \end{aligned}$$

Recall the definition of a_i in eq. (14), i.e., $a_i = \|f_i(T_{sd})\|_{q_i}$, and $a_{[i]}$ is the ith smallest one among $a_1, \dots, a_n$. Also, $c_1, \dots, c_n$ is a permutation over $1, \dots, n$ which satisfies $a_{c_l} = a_{[l]}$ for all $l = 1, \dots, n$. Then, since $\log$ function is monotone increasing and the objective function in $(\mathbf{P1}'')$ is linear, it is easy to see that an optimal solution of $(\mathbf{P1}'')$ is $p_i = 1$ for $i \in \{c_1, \dots, c_K\}$, otherwise $p_i = 0$. Hence, from eq. (17) and the optimal solution of $(\mathbf{P1}'')$, we have

$$\sum_{i=1}^n \log[p_i a_i + (1-p_i)] \geq \sum_{i=1}^n p_i \log a_i \geq \sum_{l=1}^K \log a_{[l]}. \qquad (18)$$

Note that the last lower bound in eq. (18) is the lowest bound of eq. (16) under the constraint $\sum_{i=1}^n p_i \leq K$, and the equality in eq. (18) holds by the optimal solution of $(\mathbf{P1}'')$. Thus, the optimal solution of $(\mathbf{P1}'')$ is indeed the optimal solution $\vec{p}^*$ of $(\mathbf{P1}')$. This completes the proof. □

Remark 3. Proposition 1 implies that, even though we start with a constraint on the average number of message copies $\mathbb{E}\{M\} = \sum_{i=1}^n p_i \leq K$, interestingly enough, the forwarding policy $\vec{p}^*$ under this average constraint actually attains $M = \sum_{i=1}^n Y_i = K$ with probability 1.

Proposition 1 says that the forwarding policy $\vec{p}^*$, or the optimal solution of $(\mathbf{P1}')$, is to choose K relay nodes $r_{c_1}, \dots, r_{c_K}$ and to forward message copies to them. Since the forwarding policy $\vec{p}^*$ in Proposition 1 holds for a given $\{q_i\}$, the choice of K relay nodes under the forwarding policy $\vec{p}^*$ still depends on $\{q_i\}$, which will be specified. In addition, from Proposition 1 and eq. (9), the guaranteed delay bound by the forwarding policy $\vec{p}^*$ becomes

$$\mathbb{E}\{D\}_{\vec{p}^*} \leq \frac{1}{\lambda_{sd}} \prod_{l=1}^K a_{[l]}. \qquad (19)$$

Since $f_i(\cdot) \leq 1$, from eq. (14), $a_i \leq 1$ for all i. Thus, we can interpret $\prod_{l=1}^K a_{[l]}$ in eq. (19) as a delay discounting factor by K additional message copies in the heterogeneous network setting, whereby $1/\lambda_{sd}$ is simply the message delivery delay of direct forwarding from source to destination.

How to choose a set of variables $\{q_i\}$. We below explain how to decide $\{q_i\}$ under constraints $\sum_{i=1}^n 1/q_i = 1$ and $q_i \geq 1$ $(i = 1, \dots, n)$. First, observe that if $p_i = 0$ for some i, then $a_i(q_i)$ will not contribute to the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ in eq. (9) (and subsequently that in eq. (19)), i.e., $p_i a_i(q_i) + (1-p_i) = 1$, *regardless of* the choice of q_i. In addition, for any random variable X, $\|X\|_q$ is monotone increasing (or non-decreasing) in q if $X \in \mathcal{L}_q$ [29], [30]. Since $f_i(T_{sd}) \in \mathcal{L}_q$ for $1 \leq q \leq \infty$ as mentioned before, $a_i(q_i)$ is then monotone increasing in $q_i \geq 1$ for $i = 1, \ldots, n$. Also, from $f_i(\cdot) \leq 1$ and the definition of L_∞ norm (i.e., $\|\cdot\|_\infty$), we have $a_i(\infty) = 1$ for $i = 1, \ldots, n$. Thus, if we can set $q_i = \infty$, jointly with $p_i^* = 0$, for some i, then we can assign the smallest possible values to the other q_i's while satisfying $\sum_{i=1}^n 1/q_i = 1$, which in turn gives smaller delay upper bound. Therefore, we arrive to the following assignment rule for $\{q_i\}$ that makes the guaranteed delay bound in eq. (19) tighter.

We set $q_i = K$ to a_i in eq. (14), i.e., $a_i(K)$, for $i = 1, \ldots, n$, temporarily. After finding $c_1, \ldots, c_n$ as mentioned above, we re-assign

$$q_i = \begin{cases} K & \text{if } i \in \{c_1, \ldots, c_K\}, \\ \infty & \text{otherwise,} \end{cases} \tag{20}$$

where $\sum_{i=1}^n 1/q_i = \sum_{i=1}^K 1/K = 1$. By noting that $a_i(\infty) = 1$ for $i \in \{c_{K+1}, \ldots, c_n\}$, $a_i(K), i \in \{c_1, \ldots, c_K\}$, now solely determines the delay upper bound in eq. (19). From Proposition 1, the optimal solution $\vec{p}^*$ in eq. (15) is still $p_i^* = 1$ for the same $i \in \{c_1, \ldots, c_K\}$, otherwise $p_i^* = 0$. Hence, the forwarding policy $\vec{p}^*$—*our static forwarding policy*, becomes to choose K relay nodes $r_{c_1}, \ldots, r_{c_K}$ based on $a_i(K)$ for all i. Note that this assignment rule for $\{q_i\}$ does not change the forwarding policy itself, but makes the delay upper bound in eq. (19) smaller.

We now derive a closed-form expression of the guaranteed delay bound under the static forwarding policy $\vec{p}^*$. Observe that from the definitions of $\mathbb{E}\{\tilde{D}_i\}$ in eq. (11) and $a_i(K)$ in eq. (14), we have

$$a_i(K) = \|f_i(T_{sd})\|_K = \lambda_{sd}^{1/K} [\mathbb{E}\{\tilde{D}_i\}]^{1/K}. \tag{21}$$

By noting that $g(x) = x^{1/K}$ is monotone increasing in $x \geq 0$, one can see that the static forwarding policy $\vec{p}^*$ is equivalent to choosing K relay nodes $r_{c_1}, \ldots, r_{c_K}$ based on $\mathbb{E}\{\tilde{D}_i\}$ ($i = 1, \ldots, n$). Let $\mathbb{E}\{\tilde{D}_{[l]}\}$ be the lth smallest one among $\mathbb{E}\{\tilde{D}_i\}$ ($i = 1, \ldots, n$). Then, from eqs. (19) and (21), the guaranteed delay bound under the static forwarding policy $\vec{p}^*$ becomes

$$\mathbb{E}\{D\}_{\vec{p}^*} \leq \prod_{l=1}^K [\mathbb{E}\{\tilde{D}_{[l]}\}]^{1/K}. \tag{22}$$

Similar to a special case $K = n$ in Section 4.2, as in eq. (22), the guaranteed delay bound by the static forwarding policy $\vec{p}^*$ under K-copies constraint is nothing but a geometric mean of the K smallest ones among n message delivery delays of multicopy two-hop relay protocol, each of which is obtained under each decomposed network $\mathfrak{N}_i$ (as shown in Fig. 3) that consists of a direct path and K *i.i.d.* two-hop relay paths with parameters λ_{sr_i} and λ_{r_id}. Then, from the closed-form expression of $\mathbb{E}\{\tilde{D}_i\}$ in eq. (12), we immediately have a closed-form expression of the guaranteed delay upper bound in eq. (22), which will be useful in evaluating the performance gain achieved through exploiting the heterogeneity in mobile nodes' contact dynamics.

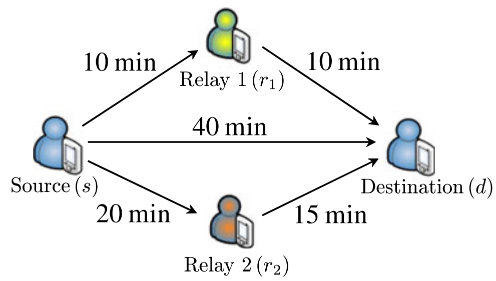


Fig. 4. An example scenario for a source-destination pair with two possible relay nodes. The average inter-contact time for each node pair is shown along the arrow.

Remark 4. When $K = 1$, the guaranteed delay bound under the static forwarding policy $\vec{p}^*$ in eq. (22) becomes identical to the original expression of $\mathbb{E}\{D\}_{\vec{p}^*}$ from eq. (6). By the forwarding policy $\vec{p}^*$, and from eqs. (6) and (11), one can see that $\mathbb{E}\{D\}_{\vec{p}^*} = \mathbb{E}\{\tilde{D}_{[1]}\} = \min_i \mathbb{E}\{T_{sd}, T_{sr_i} + T_{r_id}\}$, i.e., the equality in eq. (22) holds.

Remark 5. Similar to Remark 1, for $K < n$, when the network is homogeneous with $\lambda_{ij} = \lambda$, the delay upper bound by the static forwarding policy $\vec{p}^*$ in eq. (22) becomes equal to the expression of $\mathbb{E}\{D\}_{\vec{p}^*}$ from eq. (6). Since the expression of $\mathbb{E}\{\tilde{D}_i\}$ in eq. (11) is the same for all i, the forwarding policy $\vec{p}^*$ (in the form of $[1, \ldots, 1, 0, \ldots, 0]$) becomes to choose *any* K relay nodes among n possible nodes and thus the equality in eq. (22) holds.

6 A Dynamic Forwarding Policy

We next develop a dynamic forwarding policy leveraging the benefit of changing relay nodes (or paths) on the fly upon encounter as an extension of the static forwarding policy $\vec{p}^*$, in order to further improve the delay performance. While the static forwarding policy exploits the heterogeneity in contact dynamics, it does not capture such a benefit since it works based on a predetermined set of relay nodes. The expected utility of each mobile node as a relay node is changing over time depending on *when* and *who* a source node encounters, and thus the opportunistic nature of node mobility can be exploited in improving the static forwarding policy $\vec{p}^*$ in order to achieve smaller delay.

Consider an example scenario depicted in Fig. 4 with $K = 1$. Then, observe that

$$\mathbb{E}\{\tilde{D}_i\} = \mathbb{E}\{\min\{T_{sd}, T_{sr_i} + T_{r_id}\}\} = \frac{\lambda_{sd} + \lambda_{sr_i} + \lambda_{r_id}}{(\lambda_{sd} + \lambda_{sr_i})(\lambda_{sd} + \lambda_{r_id})},$$

and hence $\mathbb{E}\{\tilde{D}_1\} = 14.4$ minutes and $\mathbb{E}\{\tilde{D}_2\} \approx 20.61$ minutes. By the static forwarding policy $\vec{p}^*$, source s will only forward a message copy to relay node r_1 (in addition to direct forwarding of the message to destination d), *regardless of* whether encountering node r_2 earlier than node r_1. Suppose that source s first encounters node r_2 at time $t > 0$. If the source forwards a message copy to node r_2 instead of waiting the predetermined relay node r_1, then the expected delay from that time t, say $\mathbb{E}\{D_t\}$, will be $\mathbb{E}\{D_t\} = \mathbb{E}\{\min\{T_{sd}, T_{r_2d}\}\} = 1/(\lambda_{sr_2} + \lambda_{sd}) = 8$ minutes. However, if the source ignores this forwarding opportunity and keeps waiting until to encounter node r_1 as in the static forwarding policy, then the expected delay (from that time t) will still be $\mathbb{E}\{D_t\} = 14.4$ minutes due to the memoryless property of exponential inter-contact time distribution for each node pair. By properly re-evaluating the expected utility of each node as a relay node upon encounter, the static forwarding policy can be modified to opportunistically utilize relay nodes *outside* the predetermined set of relay nodes, thus leading to smaller delay.

Per-contact routing proposed in Ref. [31] already exploits such a benefit of changing (or re-computing) a routing path for single-copy message forwarding, i.e., a node having a message re-computes the next hope for the message each time a contact arrives. In single-copy message forwarding, the average delay over each routing path is nothing but the sum of average inter-contact times of nodes pairs in the path, and thus it is easy to compare each routing path with previously computed routing path(s). In contrast, it is non-trivial to find a forwarding rule—how to change relay paths *on the fly* for *multi-copy* message forwarding, theoretically guaranteeing better delay performance. Thus motivated, we below propose a dynamic forwarding policy as an extension of the static forwarding policy $\vec{p}^*$ and prove that the dynamic forwarding policy leads to smaller (or equal) average delay than the static policy.

First, suppose that, after a message arrives at source s at time 0 to be delivered to destination d, the source has not encountered the destination until time $t > 0$. We define by $\mathcal{C}_t$ an index set of relay candidates that were chosen to receive message copies from the source upon encounter, but have not received the message copies till time t. We also define by $\mathcal{F}_t$ another index set of relay nodes that have received message copies from the source by time t. Then, one can see that under the static forwarding policy $\vec{p}^*$ for a given K, the index set $\mathcal{C}_0$ (at time 0) is set to $\mathcal{C}_0 = \{c_1, c_2, \ldots, c_K\}$, where $\{c_1, c_2, \ldots, c_n\}$ is a permutation of $\{1, 2, \ldots, n\}$ while satisfying $a_{c_l}(K) = a_{[l]}(K)$, or equivalently, $\mathbb{E}\{\tilde{D}_{c_l}\} = \mathbb{E}\{\tilde{D}_{[l]}\}$, for all $l = 1, 2, \ldots, n$. Note that $a_i(K) \geq a_j(K)$ if, and only if, $\mathbb{E}\{\tilde{D}_i\} \geq \mathbb{E}\{\tilde{D}_j\}$. Also, under the static forwarding policy, $\mathcal{C}_t \bigcup \mathcal{F}_t = \{c_1, \ldots, c_K\} = \mathcal{C}_0$ for all $t \geq 0$. That is, the static policy predetermines a set of relay nodes $\{r_{c_1}, \ldots, r_{c_K}\}$, and use them only in delivering message copies to destination d. However, as seen from the above motivating example, it is better to opportunistically utilize relay nodes upon encounter that do not belong to the set $\{r_{c_1}, \ldots, r_{c_K}\}$ instead of relying solely on the predetermined relay nodes, while satisfying the constraint K on the number of message copies. Our proposed dynamic forwarding policy specifies such an opportunistic relay selection and is given as follows. Here the only difference between the dynamic and static policies is the addition of the nested 'if' statement with its sub-routines (3)-(4).

We below show that the dynamic forwarding policy leads to smaller average delay than the static policy. For this performance comparison, let D^{dp} and D^{sp} be the delay under the dynamics and static forwarding policies with K message copies, respectively. Note that $\mathbb{E}\{D^{\mathsf{sp}}\} = \mathbb{E}\{D\}_{\vec{p}^*}$.

A Dynamic Forwarding Policy (with a given K)

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1. Initialization (at time $t = 0$):
   $\mathcal{C}_0 \leftarrow \{c_1, c_2, \ldots, c_K\}$ and $\mathcal{F}_0 \leftarrow \emptyset$
2. For node r_i upon encounter (at time $t > 0$):
   If $i \in \mathcal{C}_{t^-}$,
   (1) Forward a message copy to node r_i.
   (2) $\mathcal{C}_t \leftarrow \mathcal{C}_{t^-} \setminus \{i\}$ and $\mathcal{F}_t \leftarrow \mathcal{F}_{t^-} \bigcup \{i\}$
   Else
   If there exists $j \in \mathcal{C}_{t^-}$ such that $\mathbb{E}\{T_{r_i d}\} \leq \mathbb{E}\{T_{s r_j}\} + \mathbb{E}\{T_{r_j d}\}$,
   (3) Forward a message copy to node r_i.
   (4) $\mathcal{C}_t \leftarrow \mathcal{C}_{t^-} \setminus \{j\}$ and $\mathcal{F}_t \leftarrow \mathcal{F}_{t^-} \bigcup \{i\}$
   Else
   (5) Skip node r_i.

To proceed, we need the following definitions and lemmas. In the definition for stochastic orderings between two random variables X and Y, we denote $X \geq_{(\cdot)} Y$, if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ for a class of functions ϕ for which the expectation exists. In the following definitions, 'increasing' means 'non-decreasing'.

Definition 1 ([32], [33]). *For a nonnegative random variable X with density function f, if the failure rate $\gamma(t) \triangleq f(t)/\,\mathbb{P}\{X > t\}$ is increasing in $t \geq 0$, then X is said to be an increasing failure rate (IFR) random variable.*

Definition 2 ([32], [33]). *(i) X is said to be stochastically larger than Y (denoted by $X \geq_{st} Y$) if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ holds for any increasing function ϕ, or equivalently if $\mathbb{P}\{X > u\} \geq \mathbb{P}\{Y > u\}$ for all $u \in \mathbb{R}$. (ii) We also define the convex (resp. concave, and increasing concave) order, written $X \geq_{cx} Y$ (resp. $X \geq_{cv} Y$, and $X \geq_{icv} Y$), if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ holds for any convex (resp. concave, and increasing concave) function ϕ.*

Lemma 1. *Let X_1, X_2, Y be independent exponential random variables. Then, if $\mathbb{E}\{X_1\} + \mathbb{E}\{X_2\} \geq \mathbb{E}\{Y\}$, then $X_1 + X_2 \geq_{icv} Y$.*

Proof. X_1 and X_2 are IFR, as their failure rates are constant. It is also known that $Z \triangleq X_1 + X_2$ is IFR (closure under convolution) [32], [33]. Let Z_e be an independent exponential variable with mean $\mathbb{E}\{Z_e\} = \mathbb{E}\{Z\}$. It is known that for the IRF random variable Z, we have $Z \leq_{cx} Z_e$ [32], [33]. Also, by noting that ϕ is concave if $-\phi$ is convex, from Definition 2, $Z \leq_{cx} Z_e$ implies that $Z \geq_{cv} Z_e$ and so $Z \geq_{icv} Z_e$. In addition, it is not difficult to see that if $\mathbb{E}\{Z_e\} \geq \mathbb{E}\{Y\}$, then $Z_e \geq_{st} Y$, and so by Definition 2, $Z_e \geq_{icv} Y$. Thus, $Z \geq_{icv} Z_e \geq_{icv} Y$, which completes the proof. □

Lemma 2. *Let $X_1, X_2, \ldots, X_m$ be independent random variables. If we define another independent random variable Y_j satisfying $X_j \geq_{icv} Y_j$ for some j, then*

$$\mathbb{E}\{\min\{X_1, X_2, \ldots, X_j, \ldots, X_m\}\} \geq \mathbb{E}\{\min\{X_1, X_2, \ldots, Y_j, \ldots, X_m\}\}.$$

Proof. Let $\mathcal{I} = \{1, 2, \ldots, m\} \setminus \{j\}$ be an index set. Observe that

$$\begin{aligned}&\mathbb{E}\{\min\{X_1, X_2, \ldots, X_j, \ldots, X_m\} \mid X_i = x_i, i \in I\}\\ &\quad= \mathbb{E}\{\min\{x_1, x_2, \ldots, X_j, \ldots, x_m\} \mid X_i = x_i, i \in I\}\\ &\quad= \mathbb{E}\{\min\{x_1, x_2, \ldots, X_j, \ldots, x_m\}\}\\ &\quad\geq \mathbb{E}\{\min\{x_1, x_2, \ldots, Y_j, \ldots, x_m\}\}\\ &\quad= \mathbb{E}\{\min\{X_1, X_2, \ldots, Y_j, \ldots, X_m\} \mid X_i = x_i, i \in I\},\end{aligned}$$

where the second equality is from the independence of $X_1, \ldots, X_m$, the inequality is from $Y_j \leq_{icv} X_j$ as $\min\{z_1, z_2, \ldots, z_n\}$ is increasing and concave in each argument while the other arguments are held fixed, and the last equality is from the independence of $X_1, \ldots, Y_j, \ldots, X_m$. Taking expectations in both sides gives the result. □

We then present the following.

Proposition 2. *For a given* K, $\mathbb{E}\{D^{\mathsf{dp}}\} \leq \mathbb{E}\{D^{\mathsf{sp}}\}$.

Proof. Recall that the only difference between the static and dynamic forwarding policies is the nested 'if' statement with sub-routines (3)–(4). It is thus enough to show that performing this process upon encounter with each node r_i leads to smaller average delay. Consider $n-1$ variants of the dynamic forwarding policy, each of which performs such a routine *only up to the first* k *encountered relay nodes* among n relay nodes $\{r_1, r_2, \ldots, r_n\}$, where $k = 1, 2, \ldots, n-1$. Let $D^{\mathsf{dp}}_{(k)}$ be the resulting delay under the kth variant of the dynamic forwarding policy. Then, we below show that

$$\mathbb{E}\{D^{\mathsf{sp}}\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)}\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(2)}\} \geq \cdots \geq \mathbb{E}\{D^{\mathsf{dp}}_{(n-1)}\} \geq \mathbb{E}\{D^{\mathsf{dp}}\}.$$

Let $R_{(k)}$, $k = 1, 2, \ldots, n$, be the kth relay node that source s meets, in the encounter order, among n relay nodes. Let also $T_{(k)}$ be the kth order statistic of $T_{sr_1}, T_{sr_2}, \ldots, T_{sr_n}$, representing the time when source s encounters node $R_{(k)}$. For notational simplicity, we define

$$\begin{aligned}A_t &\triangleq \min\{T_{r_f d} | f \in \mathcal{F}_{t^-}\}\\ B_t &\triangleq \min\{T_{sr_c} + T_{r_c d} | c \in \mathcal{C}_{t^-} \setminus \{j\}\}.\end{aligned}$$

We first show that $\mathbb{E}\{D^{\mathsf{sp}}\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)}\}$. Observe that

$$\begin{aligned}\mathbb{E}\{D^{\mathsf{sp}} \mid D^{\mathsf{sp}} < T_{(1)}\} &= \mathbb{E}\{T_{sd} \mid T_{sd} < T_{(1)}\}\\ &= \mathbb{E}\{D^{\mathsf{dp}}_{(1)} \mid D^{\mathsf{dp}}_{(1)} < T_{(1)}\}.\end{aligned} \tag{23}$$

This follows since the events $\{D^{\mathsf{sp}} < T_{(1)}\}$ and $\{D^{\mathsf{dp}}_{(1)} < T_{(1)}\}$ under both policies are equivalent to the event that source s encounters destination d before any of n relay nodes, i.e., $\{T_{sd} < T_{(1)}\}$. On the other hand, assuming that the condition of the 'if' statement is satisfied when source s encounters node $R_{(1)} = i$ (i.e., there exists $j \in \mathcal{C}_{T^-_{(1)}}$ such that $\mathbb{E}\{T_{r_i d}\} \leq \mathbb{E}\{T_{sr_j}\} + \mathbb{E}\{T_{r_j d}\}$), we have

$$\begin{aligned}&\mathbb{E}\{D^{\mathsf{sp}} \mid D^{\mathsf{sp}} > T_{(1)}, R_{(1)} = i, T_{(1)} = t\}\\ &\quad= t + \mathbb{E}\{\min\{T_{sd}, B_t, T_{sr_j} + T_{r_j d}\}\}\\ &\quad\geq t + \mathbb{E}\{\min\{T_{sd}, B_t, T_{r_i d}\}\}\\ &\quad= \mathbb{E}\{D^{\mathsf{dp}}_{(1)} \mid D^{\mathsf{dp}}_{(1)} > T_{(1)}, R_{(1)} = i, T_{(1)} = t\},\end{aligned}$$

where the equalities are from the memoryless property of exponential random variables $\{T_{ij}\}$, and the inequality is from Lemmas 1-2. In addition, if the condition of the 'if' statement is not satisfied, both conditional expectations become identical. We thus have

$$\begin{aligned}&\mathbb{E}\{D^{\mathsf{sp}} \mid D^{\mathsf{sp}} > T_{(1)}, R_{(1)} = i, T_{(1)} = t\}\\ &\quad\geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)} \mid D^{\mathsf{dp}}_{(1)} > T_{(1)}, R_{(1)} = i, T_{(1)} = t\}.\end{aligned} \tag{24}$$

Then, we define by $f_{T_{(1)}|D^{\mathsf{sp}}>T_{(1)},R_{(1)}=i}(t)$ the conditional density function of $T_{(1)}$ given that $D^{\mathsf{sp}} > T_{(1)}$ and $R_{(1)} = i$, so that

$$\begin{aligned}&f_{T_{(1)} \mid D^{\mathsf{sp}}>T_{(1)},R_{(1)}=i}(t)dt\\ &\quad\approx \frac{\mathbb{P}\{T_{(1)} \in (t, t+dt), D^{\mathsf{sp}} > T_{(1)}, R_{(1)} = i\}}{\mathbb{P}\{D^{\mathsf{sp}} > T_{(1)}, R_{(1)} = i\}}.\end{aligned}$$

Similarly for $f_{T_{(1)}|D^{\mathsf{dp}}_{(1)}>T_{(1)},R_{(1)}=i}(t)$. Here we can see that $f_{T_{(1)}|D^{\mathsf{sp}}>T_{(1)},R_{(1)}=i}(t) = f_{T_{(1)}|D^{\mathsf{dp}}_{(1)}>T_{(1)},R_{(1)}=i}(t)$. By noting that $\{D^{\mathsf{sp}} < T_{(1)}\} \equiv \{D^{\mathsf{dp}}_{(1)} < T_{(1)}\}$, this can be easily seen from

$$\begin{aligned}&\mathbb{P}\{D^{\mathsf{sp}} < T_{(1)} \mid T_{(1)} \in (t, t+dt), R_{(1)} = i\}\\ &\quad= \mathbb{P}\{D^{\mathsf{dp}}_{(1)} < T_{(1)} \mid T_{(1)} \in (t, t+dt), R_{(1)} = i\},\end{aligned}$$

and $\quad \mathbb{P}\{D^{\mathsf{sp}} < T_{(1)} \mid R_{(1)} = i\} = \mathbb{P}\{D^{\mathsf{dp}}_{(1)} < T_{(1)} \mid R_{(1)} = i\}$. Thus, from eq. (24), we have

$$\mathbb{E}\{D^{\mathsf{sp}} \mid D^{\mathsf{sp}} > T_{(1)}, R_{(1)} = i\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)} \mid D^{\mathsf{dp}}_{(1)} > T_{(1)}, R_{(1)} = i\}.$$

Since

$$\begin{aligned}&\mathbb{P}\{R_{(1)} = i \mid D^{\mathsf{sp}} > T_{(1)}\}\\ &\quad= \frac{\mathbb{P}\{R_{(1)} = i\}(1 - \mathbb{P}\{D^{\mathsf{sp}} < T_{(1)} \mid R_{(1)} = i\})}{\mathbb{P}\{D^{\mathsf{sp}} > T_{(1)}\}}\\ &\quad= \frac{\mathbb{P}\{R_{(1)} = i\}(1 - \mathbb{P}\{D^{\mathsf{dp}}_{(1)} < T_{(1)} \mid R_{(1)} = i\})}{\mathbb{P}\{D^{\mathsf{dp}}_{(1)} > T_{(1)}\}}\\ &\quad= \mathbb{P}\{R_{(1)} = i \mid D^{\mathsf{dp}}_{(1)} > T_{(1)}\},\end{aligned}$$

it then follows that

$$\mathbb{E}\{D^{\mathsf{sp}} \mid D^{\mathsf{sp}} > T_{(1)}\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)} \mid D^{\mathsf{dp}}_{(1)} > T_{(1)}\}. \tag{25}$$

Therefore, from eqs. (23) and (25), together with $\mathbb{P}\{D^{\mathsf{sp}} < T_{(1)}\} = \mathbb{P}\{D^{\mathsf{dp}} < T_{(1)}\}$, we have $\mathbb{E}\{D^{\mathsf{sp}}\} \geq \mathbb{E}\{D^{\mathsf{dp}}_{(1)}\}$.

Similarly, we can show that $\mathbb{E}\{D^{\text{dp}}_{(k-1)}\} \geq \mathbb{E}\{D^{\text{dp}}_{(k)}\}$, $k = 1, 2, \ldots, n-1$. The only difference between the $(k-1)$th and kth variants of the dynamic policy is whether to perform the process of changing a relay path (or node) upon encounter with the kth encountered relay node, implying that both policies are identical in their operations up to the first $k-1$ encountered nodes. Hence, we have

$$\mathbb{E}\{D^{\text{dp}}_{(k-1)} \mid D^{\text{dp}}_{(k-1)} < T_{(k)}\} = \mathbb{E}\{D^{\text{dp}}_{(k)} \mid D^{\text{dp}}_{(k)} < T_{(k)}\}.$$

In addition, as was done before, if the condition of the 'if' statement holds for the kth encountered node $R_{(k)} = i$, then

$$\begin{aligned}
&\mathbb{E}\{D^{\text{dp}}_{(k-1)} \mid D^{\text{dp}}_{(k-1)} > T_{(k)}, R_{(k)} = i, T_{(k)} = t\} \\
&= t + \mathbb{E}\{\min\{T_{sd}, A_t, B_t, T_{sr_j} + T_{r_j d}\}\} \\
&\geq t + \mathbb{E}\{\min\{T_{sd}, A_t, B_t, T_{r_i d}\}\} \\
&= \mathbb{E}\{D^{\text{dp}}_{(k)} \mid D^{\text{dp}}_{(k)} > T_{(k)}, R_{(k)} = i, T_{(k)} = t\}.
\end{aligned}$$

If otherwise, the conditional expectations remain the same. Thus, following the same lines as above, we can arrive at

$$\mathbb{E}\{D^{\text{dp}}_{(k-1)}\} \geq \mathbb{E}\{D^{\text{dp}}_{(k)}\}.$$

In a similar way, we can also show $\mathbb{E}\{D^{\text{dp}}_{(n-1)}\} \geq \mathbb{E}\{D^{\text{dp}}\}$, which completes the proof. □

7 Performance Evaluation

In this section, we provide simulation results for performance evaluation of static and dynamic forwarding policies and to support our analytical results. In particular, we present our quantitative study on the benefit of exploiting the heterogeneity in mobile nodes' contact dynamics through comparisons with the case of assuming that the network is homogenous and the optimal forwarding policy obtained under the homogeneous network.

We first consider the following heterogeneous network setting (labeled 'uniform') with $|\mathcal{N}| = 22$. We set the average inter-contact time between nodes i and j, or $1/\lambda_{ij}$, to be chosen from a continuous *uniform* distribution over a range from 200 seconds to 39,800 seconds (with mean 20,000 seconds). Let $\bar{\mu} := \sum_{i\in\mathcal{N}}\sum_{j\neq i} 1/\lambda_{ij}/(|\mathcal{N}|(|\mathcal{N}|-1))$ be the overall average inter-contact time over all node pairs. Then, in the corresponding homogeneous network setting, we set $1/\lambda_{ij} = \bar{\mu}$ for all i, j $(i \neq j)$, so that the overall average inter-contact time over all node pairs is the same for both settings.

Fig. 5 depicts analytical and simulation results for the average delay obtained via the static forwarding policy per each given number of message copies under the heterogeneous and corresponding homogeneous network settings. Given the number of message copies, we obtain the analytical result for the average delay of the static forwarding policy for the uniformly chosen source-destination pair (a statistical average of the average delays for all possible source-destination pairs) by computing the guaranteed delay bound of the static policy in eq. (22). Note that this delay bound is also the delay upper bound for the dynamic forwarding policy as can be seen from Proposition 2. We also implement a custom event-driven simulator using C++, where random contacts of each node pair occur according to a Poisson process with its contact rate, and provide simulation results for the actual average delay of the static forwarding policy for the uniform source-destination pair per each given number of message copies. In addition, we provide simulation results for the average delay of the dynamic forwarding policy under the heterogeneous network setting.

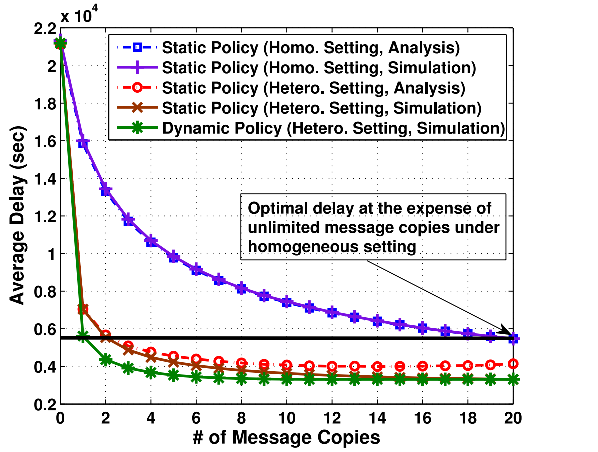

Fig. 5. Average delay achieved via the static and dynamic forwarding policies per each given number of message copies under the heterogeneous network setting ('uniform') and its corresponding homogeneous setting.

Fig. 5 shows that very few message copies with the static policy (*2-3 copies*) and dynamic policy (*1-2 copies*) under the heterogeneous network setting are only needed to achieve the same delay performance as the performance limit of *any* two-hop relay forwarding policy under its homogeneous counterpart. Here, the performance limit is the *minimum* delay achieved at the expense of unlimited message copies (20 message copies) by multicopy two-hop relay protocol, and is represented by the horizontal thick line in Fig. 5. This implies that we can save *more than 85 percent* of message copies under the heterogeneous network setting, which is significantly helpful in reducing overall resource consumption over the network. From Fig. 5, we also observe that the dynamic policy leads to *smaller* (or equal) average delay than the static policy, confirming Proposition 2.

In addition, Fig. 5 exhibits that the delay upper bound of the static policy (i.e., the RHS of eq. (22)) is closed to its actual performance under the heterogeneous network setting, while it is the exact delay under the homogeneous network setting as in Remarks 1 and 5. We can also see a slightly increasing behavior of the delay bound, which can be explained as follows: Recall that the RHS of eq. (22) can be written in a form of the product of L_q norms as in eq. (19), i.e., $\prod_{l=1}^{m} a_{[l]}(m)$, where $a_i(m) = \|f_i(T_{sd})\|_m$ for $i = 1, \ldots, n$. Also, since $a_i(m)$ is monotone increasing in $m \geq 1$ and $a_i(m) \leq 1$ as mentioned in Section 5, it is possible that $\prod_{l=1}^{m} a_{[l]}(m) \leq \prod_{l=1}^{m} a_{[l]}(m+1) \cdot a_{[m+1]}(m+1)$ when delay reduction by adding $a_{[m+1]}(m+1)$ becomes not significant. However, as shown in Fig. 5, the increasing behavior of the delay bound is not problematic and the delay bound is still useful when compared to its actual performance.

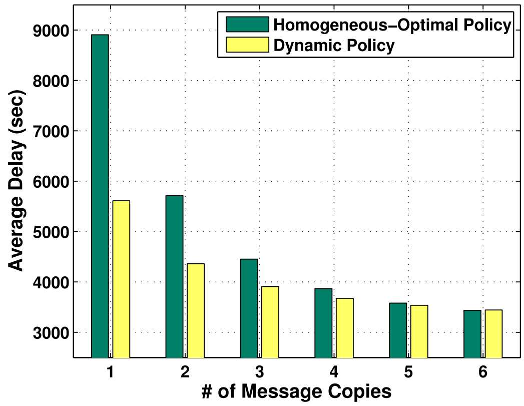

Fig. 6. Performance comparison between the dynamic forwarding policy and homogeneous-optimal forwarding policy, while varying the number of message copies.

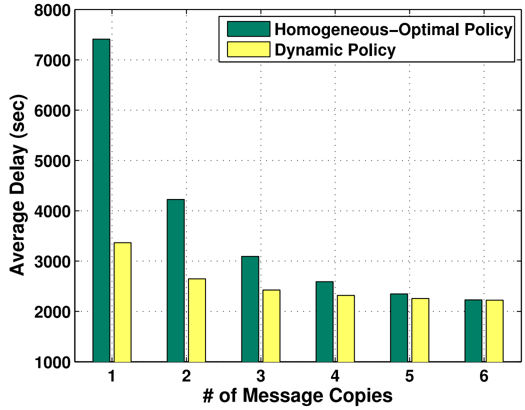

Fig. 8. Performance comparison between the dynamic forwarding policy and homogeneous-optimal forwarding policy, with different number of message copies.

Fig. 6 presents a performance comparison, in terms of average delay, between the dynamic forwarding policy and '*homogeneous-optimal*' forwarding policy under the same heterogeneous setting, when varying the number of message copies. Here, the homogeneous-optimal policy is the optimal two-hop policy under any homogeneous network that simply makes message copies to *any first* encountered nodes up to the given number of copies. As can be seen from Fig. 6, we observe that our dynamic policy leads to considerable performance improvement over the homogeneous-optimal policy (up to about *40 percent improvement* with one message copy) when the number of message copies allowed in the network is small. On the other hand, for the large number of message copies, the performance of both policies become almost the same and close to the performance of multicopy two-hop relay protocol (with unlimited message copies). To see this, we also refer to Fig. 5. Nonetheless, we emphasize that the performance of our dynamic policy with small message copies (2-3 copies) is already comparable to that of the multicopy two-hop relay protocol.

We next repeat the same procedure as above for another heterogeneous network setting (labeled 'exponential') with $|\mathcal{N}| = 22$. In this heterogeneous setting, we set the average inter-contact time between nodes i and j, or $1/\lambda_{ij}$, to be chosen from an *exponential* distribution with mean 20,000 seconds. For its homogeneous network counterpart, we also set $1/\lambda_{ij} = \bar{\mu}$ for all $i, j \in \mathcal{N}$ and $i \neq j$, so that the overall average inter-contact time over all node pairs remains the same for both settings.

Fig. 7 shows the average delay achieved via the static and dynamic forwarding policies under the heterogeneous network setting and its homogeneous counterpart, while varying the number of message copies. We observe that *only one message copy* via both policies is sufficient to achieve (or even better than) the optimal delay at the expense of unlimited message copies under the homogeneous setting. We also see that the dynamic policy gives *no worse* performance than the static policy, which corroborates Proposition 2, although the performance difference is not noticeable. In addition, Fig. 8 presents the performance comparison between the dynamic policy and homogeneous-optimal policy under the same heterogeneous setting, and again exhibits the remarkable performance improvement by the dynamic policy (up to about *50 percent improvement* with one message copy) for the small number of message copies.

We further continue our investigation over a real Bluetooth contact trace (*Infocom*) [2] which was collected in a conference environment. It contains 41 nodes' contact

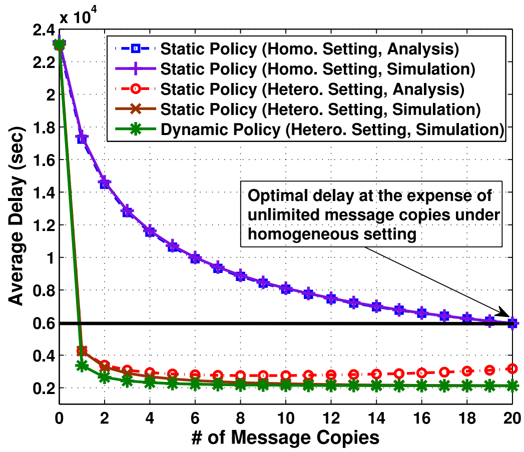

Fig. 7. Average delay achieved via the static and dynamic forwarding policies under the heterogeneous network setting ('exponential') and its homogeneous counterpart, when the number of message copies changes.

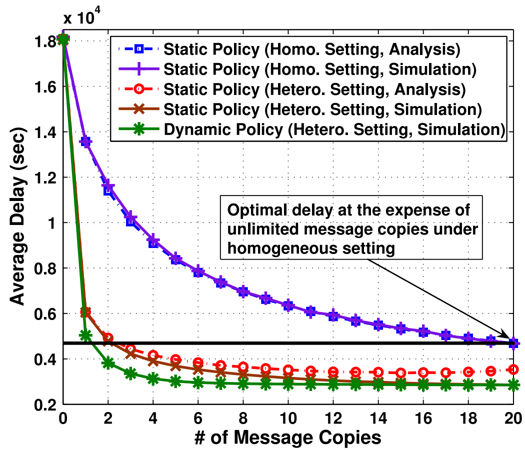

Fig. 9. Average delay performance of the static and dynamic forwarding policies under the heterogeneous network setting ('trace-based') and its corresponding homogeneous setting, when varying the number of message copies.

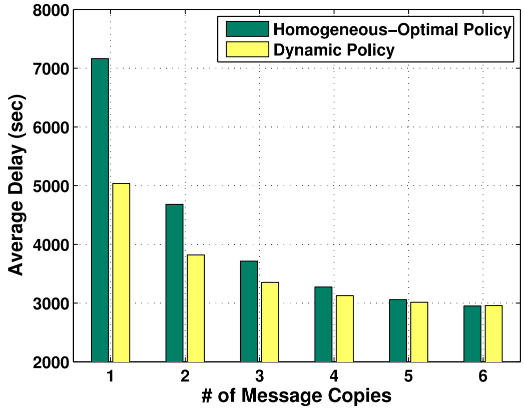

Fig. 10. Performance comparison between the dynamic forwarding policy and homogeneous-optimal forwarding policy per each given number of message copies.

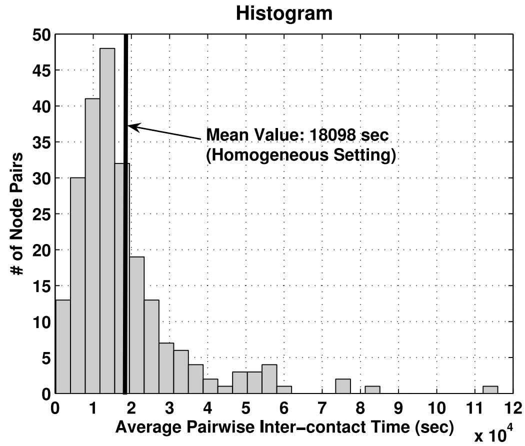

Fig. 11. Histogram of average *pairwise* inter-contact times over all considered node pairs in the trace.

information over three days, from which we here use 22 nodes' information ($|\mathcal{N}| = 22$). The 22 nodes were randomly chosen out of the total 41 nodes, each of which has non-zero contact records (histories) with the other 21 nodes. In order to adopt this trace under the heterogeneous network model, we extract the average pairwise inter-contact times of all node pairs and use them under the heterogeneous network model. A histogram of the average pairwise inter-contact times over all considered node pairs is presented in Fig. 11. Similar to the above, for the homogeneous network setting, we set $\bar{\mu} = 18{,}098$ seconds—the overall average inter-contact time over all considered node pairs. In the event-driven simulation, random contacts of each node pair are generated according to a Poisson process with its contact rate extracted from the trace for the heterogeneous network setting (labeled 'trace-based') and with rate $1/\bar{\mu}$ for its corresponding homogeneous setting.

Fig. 9 shows the average delay performance of the static and dynamic forwarding policies per each given number of message copies under the heterogeneous network setting and its homogeneous counterpart. We again achieve the significant performance improvement (*2-3 copies* are enough) as similarly observed in the previous cases. This performance gain becomes apparent, since the average inter-contact times are quite heterogeneous over different node pairs as depicted in Fig. 11. As predicted from Proposition 2, the dynamic policy also leads to *better* (or equal) delay performance than the static policy. In addition, Fig. 10 depicts the performance comparison between the dynamic policy and homogeneous-optimal policy, and again demonstrates the performance improvement by the dynamic policy (up to about *30 percent improvement* with one message copy) for the small number of message copies.

It is worth noting that there is a caveat for the significant performance gain by exploiting the heterogeneity of contact behaviors among different nodes. In fact, the gain does not come for free, but still requires that each node has to estimate its contact rates to other nodes and also share the information with others. On the other hand, it is known that the contact rate for each node pair is highly related to their social relationship in a community (or network) [5]. The social relationship also normally remains stable over time. Thus, we expect that such performance gain can still be achievable in real scenario.

8 Conclusion

We have demonstrated the significant performance gain obtained from exploiting the heterogeneity in mobile nodes' contact dynamics. In particular, we showed that under various heterogeneous network settings only a small fraction of total (unlimited) message copies, via our static and dynamic forwarding policies, are sufficient to achieve the same delay as the optimal delay using unlimited copies when the networks become homogeneous. We also showed that our dynamic forwarding policy exhibits considerable performance improvement over the homogeneous-optimal forwarding policy when the number of message copies allowed in the networks is small. We expect that our analytical work provides fundamental insights on the attainable performance gain by exploiting the underlying heterogeneity structure and complements the existing empirical studies on the design of heterogeneity-aware forwarding/routing algorithms.

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