Aging Rules: What Does the Past Tell About the Future in Mobile Ad-Hoc Networks?

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ABSTRACT

The study in mobile ad-hoc networks (MANET) is facing challenges brought by recent discovery of non-exponential behavior of the inter-contact time distribution of mobile nodes. In this paper, we analyze various characteristics of the relative mobility of a random pair of nodes in MANET to show that they produce inter-contact time with different *aging* properties. First, by fixing one node and resorting to the random walks on directed graphs, we mathematically prove that under four classes of stochastic mobility patterns, the resulting inter-contact times have constant/decreasing/increasing failure rate and new-better-than-used property. Then, we consider the case when both nodes are mobile and use simulation results to uncover the aging property of their inter-contact times under random waypoint models and random walk mobility models. This aging property tells us how to correctly relate the past experience of mobile nodes with their future behavior, thereby allowing tremendous opportunities brought by the memory structure in the non-exponential inter-contact time, which would be impossible under the widely assumed exponentially distributed (memoryless) inter-contact time. As an application of our results, we establish for the first time that the approach based on exponential inter-contact time assumption can either under-estimate or over-estimate the actual system performance, under different stochastic mobility patterns indexed by their aging properties. Our results on aging properties also provide theoretic guidelines on how to exploit the memory structure toward better design of protocols under general mobility.

Categories and Subject Descriptors: C.2.1 [Computer-Communication Networks]: Network Architecture and Design - *Wireless communication*

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1. INTRODUCTION

Mobility is central to applications in mobile ad-hoc networks. Mobility patterns in various scenarios can be very different. Hence, the studies in MANET have taken two systematic approaches in parallel: one is based on real traces, the other is based on synthetic mobility models that abstract out key mobility characteristics. These two approaches are complimentary to each other: the former is more realistic, while the latter facilitates tractable performance analysis as well as system design over different mobility settings in a controlled and repeatable manner. Synthetic mobility models have also been used for deducing properties of link-level metrics such as the inter-contact time of a pair of mobile nodes (the length of time period over which the two nodes are out of contact), which in turn is used as an input to the analysis and prediction of network performance [20, 13, 16, 33]. The relationship between key characteristics of mobility patterns and the resulting link-level dynamics is thus crucial to the study of MANET.

A popular and effective abstraction for this relationship has been to assume that the inter-contact time is exponentially distributed [14, 13]. This assumption of *memoryless* inter-contact time (or Poisson contacts if all nodes are *i.i.d.*), supported by numerical simulations [32, 13] based on existing synthetic mobility models [21, 6], has been heavily adopted for tractable analysis with Markovian modeling on MANET performance [20, 13, 16, 33]. However, recent measurement studies [8, 17, 18, 23, 29] clearly show the existence of mixture behavior (first power-law followed by an exponential) of the CCDF of inter-contact time. Recent work [23, 4] studies mobility models that can produce inter-contact time distribution with this mixture behavior. At the same time, due to the complexity of analyzing performnace under non-exponential inter-contact time, the classical approach based on exponential inter-contact time assumption is still very popular. The coexistence of two approaches based on exponential and non-exponential inter-contact time seems reasonable in view of two contradictory goals – realism and simplicity (or analytical tractability). Nevertheless, up till now, works based on these two approaches are conducted in separate ways, and this raises the following question: how much sacrifice have we made for simplicity?

In this paper, we analyze various characteristics of the relative mobility of a random pair of nodes in MANET to show that they produce inter-contact time with different *ag-*

ing properties. Particularly, by exploring the memory structure of the inter-contact time, we characterize how the past experience of a mobile node (e.g., the time elapsed since the latest meeting between the node and its target node, called the *age*) relates to its future behavior (e.g., how long it will take to meet the same target node again, called the *residual lifetime*). Due to the memoryless property of exponential inter-contact time, the memory structure in contact dynamics has seldom been analytically studied. In contrast to existing works on the explanation and modeling of non-exponential inter-contact time, our work here focus on *opportunities* brought by the non-exponential (memory) structure of the inter-contact time.

Specifically, by first fixing one node and changing the law of random walks on directed graphs with dynamic topologies modeling random environments, we analyze four classes of stochastic mobility patterns and show that they produce inter-contact times with constant/decreasing/increasing failure rate and new-better-than-used property. We then consider the case when both nodes are mobile and use simulation results to disclose that the inter-contact times under Random Waypoint (RWP) models [22, 6] and random walk (RW) mobility models [9] have increasing and decreasing failure rates, respectively. After that, we show *convex ordering* relationships between *exponential and non-exponential* inter-contact times with different aging properties, which allow us to establish for the first time that the approach based on exponential inter-contact time assumption can either *under-estimate* or *over-estimate* the actual network performance depending on positive or negative aging properties of underlying non-exponential mobility patterns. Our results on aging properties rigorously state how we should develop forwarding/routing algorithms that exploit the 'memory' in the right way.

The rest of this paper is organized as follows. Section 2 provides backgrounds on random walks on (directed) graphs and how to characterize aging property of a given random variable. In Section 3, we analyze aging property of inter-contact time under four classes of stochastic mobility patterns as well as some existing mobility models in terms of its failure rate. In Section 4, we show how to compare approaches based on non-exponential and exponential inter-contact time in the sense of stochastic ordering. In Section 5, we discuss how our results on aging properties can be exploited toward better design of forwarding/routing algorithms in MANET. We finally conclude in Section 6.

2. PRELIMINARIES

In this section we first give definitions of random walk on directed graph and then collect several concepts used in reliability engineering to characterize aging properties of a random variable, which will be used throughout the paper.

2.1 Random Walk on Directed Graph

A finite digraph (graph with directed edges [35]) $\mathcal{G}$ is described by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ where $\mathcal{V} = \{0, 1, \ldots, N\}$ $(N < \infty)$ is a vertex set, $\mathcal{E}$ is an edge set such that $(i, j) \in \mathcal{E}$ if and only if a directed edge from vertex i to j exists $(i, j \in \mathcal{V})$, and $W = \{w_{ij}\}$ is $(N+1) \times (N+1)$ matrix with edge weight $0 < w_{ij} < \infty$ if $(i, j) \in \mathcal{E}$ and $w_{ij} = 0$ otherwise. Throughout the paper, we assume that the graph $\mathcal{G}$ has no multiple edges. A *directed* path of length m from vertex i to vertex j $(i = k_0, k_1, \ldots, k_{m-1}, k_m = j)$ $(k_r \in \mathcal{V}$ for $r = 0, \ldots, m)$ exists if for each $r = 0, \ldots, m-1$, $(k_r, k_{r+1}) \in \mathcal{E}$. An *undirected* path between i and j exists if, for each r, at least one of (k_r, k_{r+1}) and (k_{r+1}, k_r) belongs to $\mathcal{E}$. In our study of MANET, we find the following definitions useful.

Definition 1. *A graph $\mathcal{G}$ is connected if for any $i, j \in \mathcal{V}$, there exists a directed path from i to j.*

Definition 2. *A graph $\mathcal{G}$ is weakly connected if for any $i, j \in \mathcal{V}$, there exists an undirected path between i and j.*

In MANETs such as pocket switched networks [8, 18], usually there is a 'two-way' path connecting any two different sites, i.e., $\mathcal{G}$ is connected, while in vehicular ad-hoc networks, there might be only 'one-way' path connecting two different sites (e.g., the 'one-way' roads), for which $\mathcal{G}$ is weakly connected. Clearly, any connected graph $\mathcal{G}$ is also weakly connected, but the converse is generally not true.

We define a vertex weight matrix D as follows:

Definition 3. *For $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$, the vertex weight matrix $D = \{d_{ij}\}$ is $(N+1) \times (N+1)$ matrix with $d_{ii} = \sum_{k=0}^{N} w_{ik}$ (sum of i^{th} row elements of W) and $d_{ij} = 0$ for $i \neq j$.*

For a mobile node C following random walk on $\mathcal{G}$ (written as RWG), at each time step, the probability for C walking from vertex (or site) i to site j is $p_{ij} = w_{ij}/d_{ii}$, which is well defined when $\mathcal{G}$ is connected, i.e., $d_{ii} > 0$. (From now on, we will assume graph $\mathcal{G}$ is connected, unless otherwise specified.) Since D is a diagonal matrix with positive elements on the diagonal, we can set D^{-1} as the diagonal matrix with diagonal elements $1/d_{ii}$. Then, the random walk on graph $\mathcal{G}$ (or the position of the walker $C(t)$ at time step t) can be described by a Markov chain (MC) with state space $\mathcal{V} = \{0, 1, \ldots, N\}$ and transition matrix $P = D^{-1}W = \{p_{ij}\}$.

Remark 1. Conversely, for any MC with transition matrix P, we can always find a RWG model with edge weight matrix W (and thus D constructed from W) such that $P = D^{-1}W$.[1] In other words, the set of random walk on graph (RWG) mobility models is equivalent to the set of all Markov chain mobility models.

2.2 Aging Property of Random Variables

For a positive random variable denoting certain duration of an event such as the inter-contact time in MANET or the system (component) lifetime in reliability engineering [12, 27], a simple but powerful way to characterize how its past experience relates to the future behavior (memory structure) is through its aging property. For instance, for a given component with its age $t > 0$ (has been operational since t seconds ago), what is its residual life (remaining time until failure)? Similarly, given that the latest contact with a certain node took place t seconds ago, how long does it take until encounter to the same node again? To characterize the aging property, we need the following definition:

Definition 4. *[30, 27] For a discrete random variable $X > 0$ with distribution $F(t) = \mathbb{P}\{X < t\}$ $(t = 1, 2, \ldots)$, for*

[1] The simplest way is to set $D = I$ (identity matrix) and $W = P$.

each t such that $\bar{F}(t) = 1 - F(t) > 0$, the failure/hazard rate function of $F(t)$ is defined by

$$r(t) \triangleq \mathbb{P}\{X = t\}/\bar{F}(t) = \mathbb{P}\{X = t\}/\mathbb{P}\{X \geq t\}. \quad (1)$$

X is an increasing/decreasing failure rate (IFR/DFR) random variable if $r(t)$ is increasing/decreasing function of t.

The failure rate of a continuous lifetime random variable X can be similarly defined as in Definition 4. Note that from (1) the failure rate $r(t)$ can be looked as the conditional probability that a component with lifetime X fails at time t, given that it has not failed till time $t-1$. When X is geometrically distributed with $\mathbb{P}\{X = t\} = p(1-p)^{t-1}$ for some $p \in (0, 1)$, it follows that $r(t) = p$, i.e., X has constant failure rate (CFR). Conversely, if a discrete random variable X has CFR, then its distribution is necessarily geometric or equivalently memoryless (in continuous time case, exactly exponential).

Denote by X_t the residual lifetime of X at time $t \geq 0$. Then, its ccdf $\bar{F}_t(\tau) = \mathbb{P}\{X_t \geq \tau\}$ (called survival function at time t) is given by

$$\bar{F}_t(\tau) = \mathbb{P}\{X - t \geq \tau | X \geq t\} = \bar{F}(t+\tau)/\bar{F}(t), \quad (2)$$

for all t such that $\bar{F}(t) > 0$. Then, it is well known that a distribution F has IFR/DFR if and only if $\bar{F}_t(\tau)$ is decreasing/increasing in t for any given $\tau > 0$ [27]. This means that for DFR $F(t)$ and for any $t_1 \leq t_2$, $\bar{F}_{t_1}(\tau) \leq \bar{F}_{t_2}(\tau)$ for all $\tau > 0$. (The inequality is reversed for IFR $F(t)$.) This is equivalent to $X_{t_1} \leq_{st} X_{t_2}$ [30], i.e., $\mathbb{E}\{f(X_{t_1})\} \leq \mathbb{E}\{f(X_{t_2})\}$ for any increasing (non-decreasing) function f. For this reason, a DFR random variable X is said to have 'negative aging' property. (The residual life becomes stochastically larger as the age t increases.) Similarly for an IFR random variable X, the residual life X_t at age t is stochastically decreasing in t (positive aging). The positive/negative aging property can also be defined in weaker sense:

Definition 5. *[30, 27] For a discrete random variable $X > 0$ with ccdf $\bar{F}(t) = \mathbb{P}\{X \geq t\}$ $(t = 1, 2, \ldots)$, X is called* new better than used *(written as $X \in$ NBU), if*

$$\bar{F}(t+\tau) \leq \bar{F}(t)\bar{F}(\tau). \quad (3)$$

If the inequality in (3) is reversed, X is called new worse than used *(written as $X \in$ NWU).*

When $\bar{F}(\tau) > 0$, (3) is equivalent to $\bar{F}(t+\tau)/\bar{F}(t) \leq \bar{F}(\tau)$. Also, from (2), if X has IFR, then $\bar{F}_t(\tau) = \bar{F}(t+\tau)/\bar{F}(t)$ is decreasing in t for each $\tau > 0$. Hence, from Definition 5, if X has IFR, $X \in$ NBU. Similarly, if X has DFR, $X \in$ NWU.

3. MOBILITY PATTERNS WITH CFR/DFR/IFR INTER-CONTACT TIME

As mentioned in Section 2.2, the aging property of a random variable can be properly captured by its failure rate. To show the relationship between the various characteristics of relative mobility of a pair of nodes and the corresponding inter-contact times with different aging properties, in this section, we first fix one node and show four classes of stochastic mobility patterns that give rise to inter-contact time with constant/decreasing/increasing failure rates and new-better-than-used property. We then study the aging property of the inter-contact times of a pair of mobile nodes following two popular mobility models in MANET study, i.e. RWP and RW, through simulations.

Consider a mobile node C following the class of RWG models on $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ as defined in Section 2.1. Let $C(t) \in \mathcal{V}$ be C's position at time $t \geq 0$. The *inter-contact time* T_I between node C and any given site $j \in [0, N]$ is defined by

$$T_I^{(j)} = \min_{t>0} \{t : C(t-1) \neq j,\ C(t) = j\}, \quad (4)$$

given that $C(0) = j$ and $C(1) \neq j$. In general, $T_I^{(j)} \neq T_I^{(k)}$ for $j \neq k$ in the RWG model and such distinction will be important in describing various topologies with different degrees of 'popularity' of sites. Since we can always renumber the vertex (site), without loss of generality, we consider only $T_I^{(0)}$ and omit the superscript, unless otherwise specified.

3.1 Mobility Patterns with CFR Inter-contact Time

When T_I has CFR, its distribution is exactly geometric (or exponential in continuous time case). A well-known mobility pattern producing memoryless inter-contact time is the so-called *i.i.d.* mobility pattern, as shown in Figure 1(a). In this model, the mobile node chooses any site randomly and uniformly at each time step, and then jump to it. This corresponds to RWG model with $w_{ij} = w > 0$ for all i, j (complete graph with equal edge weights). We will present a more general class of mobility patterns but still with memoryless property all the time. This class of CFR mobility patterns here will be useful in understanding the asymptotic behavior of the failure rate of any general mobility model and also constructing DFR/IFR mobility patterns later on.

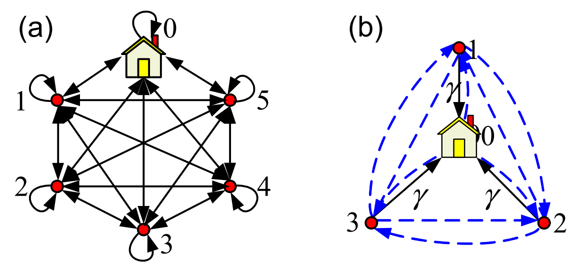


Figure 1: Examples of mobility patterns with CFR T_I (site 0 is the 'home' vertex). (a) *i.i.d.* mobility pattern, $p_{ij} = 1/6$ for all $i, j \in \mathcal{V}$; (b) $p_{10} = p_{20} = p_{30} = \gamma \in (0, 1)$ (in solid lines) and for all other $i, j \in \mathcal{V}$, $p_{ij} \in (0, 1)$ and $\sum_{j \in \mathcal{V}} p_{ij} = 1$.

Definition 6. *[7] The spectral radius $\rho(B)$ of $n \times n$ matrix B with its eigenvalues λ_i $(i = 1, 2, ..., N)$ is defined as*

$$\rho(B) = \max_i |\lambda_i|. \quad (5)$$

Now, we define a class of mobility models $\mathcal{M}_{C1}$ as:

Definition 7. *The class of mobility models $\mathcal{M}_{C1}$ are random walks on a 2-connected [35] digraphs $\mathcal{G}$ satisfying* either *one of the following:*

- ***In-home condition**: when mobile node C is at site $i \in \mathcal{V} - \{0\}$, it jumps to site 0 with probability $\gamma \in (0, 1)$.*
- ***Out-home condition**: when mobile node C is at site 0, it jumps to site $i \in \mathcal{V} - \{0\}$ with probability p_{0i}*

satisfying

$$\vec{p}_0^{\text{out}} P^* = \rho(P^*)\vec{p}_0^{\text{out}}, \quad (6)$$

where $\vec{p}_0^{\text{out}} = [p_{01}, \ldots, p_{0N}]$ *with* $\sum_{k=1}^{N} p_{0k} = 1$, P^* *is* $N \times N$ *matrix obtained by deleting the first row and the first column of matrix* P *defined in Section 2.1, and* $\rho(P^*)$ *is the spectral radius of* P^* *as in Definition 6.*

REMARK 2. In a 2-connected graph $\mathcal{G}$ in Definition 7, for any two different sites $i, j \in \mathcal{V}$, there exist at least two paths from i to j. This corresponds to a 'self-recovery' feature, i.e., when one vertex of 2-connected $\mathcal{G}$ becomes unavailable (due to some emergency or failure), mobile nodes are still able to bypass this site to visit other parts of $\mathcal{G}$. Moreover, $\vec{p}_0^{\text{out}}$ in (6) is called quasi-stationary distribution [24].

We now have the following theorem:

THEOREM 1. *Under the class of mobility models* $\mathcal{M}_{C1}$, *the inter-contact time* T_I *has constant failure rate.* □

PROOF. See Appendix. □

REMARK 3. As mentioned in Section 2.2, T_I has CFR is equivalent to that T_I is geometrically distributed, hence satisfies the memoryless property. Or equivalently, the survival function at time t, i.e., $\bar{F}_t(\tau) = \mathbb{P}\{T_I \geq t + \tau | T_I \geq t\}$ is invariant with respect to t. Also, mobility patterns in Figure 1(b) satisfy the in-home condition in Definition 7, hence the corresponding T_I has CFR.

The 'in-home' condition in Definition 7 implies that all other sites $\mathcal{V} - \{0\}$ are essentially the same to site 0, regardless of their internal transition probabilities. Hence, when the recurrence to site 0 is considered, there will be no difference whether the mobile node is at site i or site j as long as $i, j \in \mathcal{V} - \{0\}$. Thus, the inter-contact time is CFR or memoryless as expected. The 'in-home' condition can also be looked as an evidence for a 'small world' around site 0, where from any site other than 0, site 0 can be reached with the same probability $\gamma > 0$.

Under the 'out-home' condition in Definition 7, before returning back to site 0, the conditional distribution of {1,.., N} (given that the node has not visited site 0) always remains the same. In fact, when the graph $\mathcal{G} \setminus \{0\}$ (after removal of site 0 and all its edges from $\mathcal{G}$) is *non-bipartite* [35], it can be shown that $\vec{p}_0^{\text{out}}$ is the stationary distribution of random walk on this resulting graph $\mathcal{G} \setminus \{0\}$. In other words, if $\pi_i(t)$ is the probability that the node is at site i at time t, $[\pi_1(t), \ldots, \pi_N(t)]$ converges to $\vec{p}_0^{\text{out}}$ under a conditioning event that the node never visits site 0. Consequently, Theorem 1 also implies that for general mobility patterns (not limited to $\mathcal{M}_{C1}$), the failure rate of the inter-contact time eventually converges to a constant,[2] i.e., the inter-contact time is asymptotically CFR. This is also in consistent with previous studies showing the exponential tail behavior of the inter-contact time distribution [23, 4, 29].

3.2 Mobility Pattern with DFR Inter-contact Time

In this section we show that a *time reversibility* property in mobility patterns is a main factor that leads to inter-contact time with DFR. Since most components are from existing studies [25, 1] (and references therein), we will just briefly explain key steps of existing results and add some missing links to show the relationship between the time reversibility property in mobility patterns and inter-contact time with DFR.

First, consider a class of random walks on an *undirected*, connected, and non-bipartite graph $\mathcal{G}$. The edge weight matrix $W = \{w_{ij}\}$ of an undirected graph $\mathcal{G}$ is symmetric, i.e., $w_{ij} = w_{ji}$. When this graph $\mathcal{G}$ is connected and non-bipartite, a mobile node B's position at time t (i.e., $B(t)$) is irreducible and positive recurrent MC and thus it has unique stationary distribution $[\pi_0, \ldots, \pi_N] = [d_{00}/d, \ldots, d_{NN}/d]$, where d_{ii} is given in Definition 3 and $d = \sum_{i=0}^{N} d_{ii}$. Further, from $w_{ij} = w_{ji}$, it is easy to see that for any $i, j \in \{0, 1, \ldots, N\}$, $\pi_i p_{ij} = \pi_j p_{ji}$, i.e., $B(t)$ is a time reversible MC [30]. The time reversible MC can also be similarly defined in a continuous time setting.

Now, consider a time-reversible continuous-time MC $B(t)$ on states $\{0, 1, \ldots, N\}$ with transition rate matrix $Q = \{q_{ij}\}$. Define $(N+1) \times (N+1)$ matrix $U = \{u_{ij}\}$ with $u_{ii} = \sqrt{\pi_i}$ and $u_{ij} = 0$ for $i \neq j$. Since the graph $\mathcal{G}$ is connected, $\pi_i > 0$, and U^{-1} is well-defined. Note that the matrix $S = UQU^{-1}$ is a real symmetric matrix from the time reversibility. Thus, the spectral representation theorem [7] allows to write $S = V\Lambda V^T$, where V is orthonormal (i.e., $VV^T = I$) and $\Lambda = \{\lambda_{ij}\}$ is a diagonal matrix with λ_{ii} being the eigenvalues of matrix S (also eigenvalues of matrix Q). In consequence, $S^{(t)} \triangleq \exp(St) = V \exp(\Lambda t) V^T$ and $Q^{(t)} \triangleq \exp(Qt) = U^{-1} S^{(t)} U = U^{-1} V \exp(\Lambda t) V^T U$. This implies that the transition function [25] can be written as

$$\begin{aligned} f_{ij}(t) &= \mathbb{P}\{B(t) = j | B(0) = i\} = Q_{ij}^{(t)} \\ &= \sum_{k=0}^{N} a(i,k) a(j,k) \exp(-\lambda_{kk} t), \end{aligned} \quad (7)$$

where $a(i,k), a(j,k)$ are real numbers. Thus, $f_{ii}(t)$ is complete monotone. Here, a function $f : [0, \infty) \to [0, \infty)$ is said to be *completely monotone* (CM) [25] if $f(t) = \int_0^\infty e^{-t\tau} \gamma(d\tau)$, where γ is a probability measure on $[0, \infty)$. This complete monotonicity of $f_{ii}(t)$ can be extended to a set $\mathcal{S}$, i.e., $f_{\mathcal{SS}}(t) = \mathbb{P}\{B(t) \in \mathcal{S} | B(0) \in \mathcal{S}\}$ is also complete monotone. In this set-up, it is shown [1] that the first hitting time T_F of time-reversible MC $B(t)$ to the state 0, starting from the initial distribution $\mathbb{P}\{B(0) = j\} = q_{0j} / \sum_{k \neq 0} q_{0k}$ $(j \neq i)$, is completely monotone. Specifically, [1] modified the original MC by making the state 0 as an absorbing state and setting $\mathcal{S} = \{1, \ldots, N\}$. Then, $\mathbb{P}\{T_F > t\} = f_{\mathcal{SS}}(t)$, i.e., T_F has CM distribution.

We now return to the case of inter-contact time. Let $\mathcal{M}_{C2}$ denote the class of random walks on an *undirected*, connected, and non-bipartite graph $\mathcal{G}$. Consider a mobile node C under $\mathcal{M}_{C2}$ in continuous time setting[3] with transition rate matrix $Q' = \{q'_{ij}\}$. Then, upon leaving state 0, node C arrives at state j $(j \neq 0)$ with probability $q'_{0j} / \sum_{k \neq 0} q'_{0k}$. Therefore, the inter-contact time T_I of node C has the same distribution as the aforementioned T_F. Hence, T_I is also CM. The failure rate function of a positive random variable

[2]Here, conditions such as 2-connected and non-bipartite graph $\mathcal{G} \setminus \{0\}$ are sufficient to ensure the convergence.

[3]Our focus here is to find characteristics of the relative mobility that produce contact dynamics with certain aging property (here, DFR). We choose the continuous time setting only for convenience borrowing results in [1]. Same results can be also stated in the discrete time setting but with more involved notations.

with complete monotone density (or ccdf) is known to be decreasing [27]. In other words, T_I has DFR.

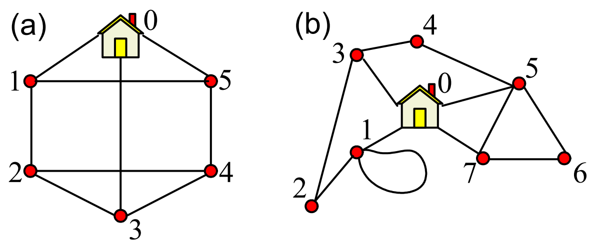


Figure 2: Examples of mobility models belonging to $\mathcal{M}_{C2}$. In (a) and (b), the weight of each edge can be any arbitrary positive number.

The class of stochastic mobility patterns $\mathcal{M}_{C2}$ is quite large, as it includes random walks on any connected, non-bipartite and undirected graph $\mathcal{G}$. Figure 2(a) and (b) show examples of RWG mobility models belonging to $\mathcal{M}_{C2}$. Clearly, the graphs in both Figure 2(a) and (b) are connected with undirected edges with arbitrary positive weights. The non-bipartiteness follows from the existence of a closed path with odd length [35] (e.g., $0-1-5-0$) in Figure 2(a) and of a self-loop (site 1) in Figure 2(b).

3.3 Mobility Pattern with IFR Inter-contact Time

As mentioned in Section 2.1, some MANETs may have 'one-way' feature, i.e., while there is a (directed) path connecting site i to site j, a (directed) path from site j to i may not exist. Reasons for this 'one-way' feature are not only from geographical constraints (e.g., one-way street or bus route) [3, 36], but also from social behavior. For example, in a park with several attractions/sites (e.g., Magic Kingdom park at Walt Disney World), although there are likely to be two-way connections between any two sites in the park, visitors usually take 'streamlined' path (tour-like) over multiple sites one by one, often bypassing less popular sites for efficiency. We first show a class of stochastic mobility patterns that formalize this 'tour'–like path, leading to IFR inter-contact time.

To proceed, we need the following definitions:

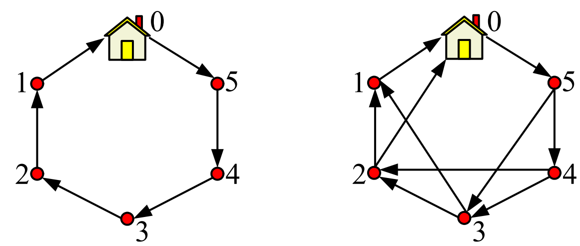
DEFINITION 8. *[26] A nonnegative $N \times N$ matrix $R = \{r_{ij}\}$ $(N \geq 2)$ is called* totally positive of order 2*, denoted by $R \in \mathrm{TP}_2$, if determinants of all 2×2 submatrices of R, $\begin{bmatrix} r_{ij} & r_{ij'} \\ r_{i'j} & r_{i'j'} \end{bmatrix}$, where $i < i'$ and $j < j'$ $(i, j, i', j' \in [1, N])$, are nonnegative.*

We define a class of models $\mathcal{M}_{C3}$ as follows:

DEFINITION 9. *The class of mobility models $\mathcal{M}_{C3}$ are random walks on weakly connected digraph $\mathcal{G}$ satisfying $p_{0N} = 1$ and* either *one of the following:*

- ***Traffic-circle condition**: when mobile node C is at site $i \in \mathcal{V} - \{0\}$, it chooses one of sites in $\{0, 1, \ldots, i-1\}$ randomly and uniformly, and then jumps to it.*
- ***Total-positivity condition**: $Q \in \mathrm{TP}_2$, where Q is obtained by replacing the first row of matrix P with $[1, 0, \ldots, 0]$.*

REMARK 4. The condition $p_{0N} = 1$ sets a primary direction for the mobility patterns in $\mathcal{M}_{C3}$. In particular, define a directed route $\mathcal{R}$ starting at site 0. Since $p_{0N} = 1$, the route $\mathcal{R}$ visits sites $N, N-1, \ldots, 1$ sequentially, often skips some sites in the middle, and finally returns to site 0. In this 'traffic-circle' condition, mobile nodes following $\mathcal{R}$ generally take a tour and never go backward between two adjacent visits to the site 0, but they can take short-cuts. As will shown later in the proof of Theorem 2, this 'traffic-circle' condition is a special case of the total-positivity condition. However, we list the former separately, since the characteristics of mobility patterns in this case is more intuitive.



(a) Periodic walk (b) Tour walk

Figure 3: Examples of mobility patterns belonging to $\mathcal{M}_{C3}$ in Definition 9.

Figure 3 shows two examples of mobility patterns satisfying the total-positivity condition in Definition 9. Specifically, the Q matrix of mobility patterns in Figure 3(b) is:

$$Q^{(b)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ p_2 & 1-p_2 & 0 & 0 & 0 & 0 \\ 0 & p_3 & 1-p_3 & 0 & 0 & 0 \\ 0 & 0 & p_4 & 1-p_4 & 0 & 0 \\ 0 & 0 & 0 & p_5 & 1-p_5 & 0 \end{bmatrix}, \quad (8)$$

where $0 < p_i < 1$ $(i = 2, \ldots, 5)$. The Q matrix of mobility patterns in Figure 3(a) equals to $Q^{(b)}$ with $p_i = 0$. It follows that the matrix $Q^{(b)}$ is TP_2 for all choices of $p_i \in [0, 1]$ (see Definition 8).

We now have the following theorem:

THEOREM 2. *Under the class of mobility models $\mathcal{M}_{C3}$, the inter-contact time T_I has increasing failure rate.* □

PROOF. See Appendix. □

In fact, mobility patterns in Figure 3(a) and (b) as well as those satisfying the 'traffic-circle' condition in Definition 9 have a common 'self-avoidance' property, i.e., mobile nodes following them never go backward to the sites it has already visited between two consecutive visits to site 0. However, $\mathcal{M}_{C3}$ (specifically, TP_2) also includes mobility patterns that are not self-avoidance type. A simple example can be seen by slightly modifying the tour walk in Figure 3 such that when mobile node C is at site 1, it can choose to jump to site 0 with probability $p_1 \in (0, 1)$, or stay at site 1 with probability $1 - p_1$. After this change, the second row of $Q^{(b)}$ in (8) becomes $[p_1, 1-p_1, 0, \ldots, 0]$ and all other rows remain the same. As long as p_1 and p_2 are chosen such that $p_1 \geq p_2$, the modified $Q^{(b)}$ is still TP_2, thereby giving IFR inter-contact time from Theorem 2. In what follows, we will present an even more general class of mobility models $\mathcal{M}'_{C3}$, which includes $\mathcal{M}_{C3}$ as a subset and allow 'going backwards' as long as its tendency to go backwards is getting weaker. To proceed, we need the following definitions:

Definition 10. *[26] For any two nonnegative vectors* $\vec{x} = [x_1, x_2, \ldots, x_n]$ *and* $\vec{y} = [y_1, y_2, \ldots, y_n]$, *we write* $\vec{x} \leq_{st} \vec{y}$ *if* $\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$ *for any* $1 \leq k \leq n-1$ *and* $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$.

Definition 11. *[26] A matrix* $R = \{r_{ij}\}$ *(*$i, j \in \{1, \ldots, n\}$*) is* stochastically monotone *if*

$$\vec{r}_1 \leq_{st} \vec{r}_2 \leq_{st} \cdots \leq_{st} \vec{r}_n, \tag{9}$$

where $\vec{r}_k = [r_{k1}, r_{k2}, \ldots, r_{kn}]$ *(*$k = 1, 2, \ldots, N$*).*

Any matrix $R \in \mathrm{TP}_2$ is stochastically monotone [26]. For example, the matrix $Q^{(b)}$ in (8) is TP_2, hence is stochastically monotone.

Consider route $\mathcal{R}$ as in Remark 4, i.e., $\mathcal{R}$ starts at site 0, goes over sites $N, N-1, \ldots, 1$ sequentially and occasionally skips sites in the middle, and ends at site 0. Consider a stochastic monotone Q and a mobile node at site i ($i \in \{N, N-1, \ldots, 1\}$), occasionally going backward with probability $p_i^{(back)} = \sum_{j=i}^{N} p_{ij}$ on $\mathcal{R}$. From Definitions 10 and 11, $\sum_{j=i}^{N} p_{ij} \leq \sum_{j=i}^{N} p_{(i+1)j}$. In other words, $p_i^{(back)} \leq p_j^{(back)}$ for any $i < j$. Since site i is more 'forward' on $\mathcal{R}$ than site j when $i < j$, under the stochastic monotone matrix Q, the mobile node C's tendency to go backward becomes weaker and weaker as it proceeds on $\mathcal{R}$.

We below show that if the Q matrix in Definition 9 is stochastically monotone but not necessarily TP_2 (thus a weaker condition), then the inter-contact time T_I is not necessarily IFR, but NBU.

Now we define a class of models $\mathcal{M}'_{C3}$ as follows:

Definition 12. *The class of mobility models* $\mathcal{M}'_{C3}$ *are random walks on weakly connected digraph* $\mathcal{G}$ *satisfying* $p_{0N} = 1$ *and* either *one of the following:*

- ***Progressive-route condition:*** *when mobile node* C *is at site* $i \in \mathcal{V} - \{0\}$, *it chooses one of sites in* $0, 1, \ldots, i-1$ *randomly with probability* $\big(1-(N-i+1)p_i\big)/i$, *or one of sites in* $i, i+1, \ldots, N$ *randomly with probability* p_i, *and then jump to it, where* p_i *is any number satisfying* $p_i \in [0, 1/(N-i+1))$ *and*

$$Np_1 \leq (N-1)p_2 \leq \ldots \leq 2p_{N-1} \leq p_N. \tag{10}$$

- ***Stochastic-monotone condition:*** Q *is stochastically monotone, where* Q *is from replacing the first row of matrix* P *by* $[1, 0, \ldots, 0]$.

Note that the 'progressive-route' condition in Definition 12 allows some probability of going backwards and as a special case, it becomes the traffic-circle condition in Definition 9 when $p_i = 0$ for all $i \in [1, N]$. Note however that it is not a special case of the total-positivity condition in Definition 9. Still, as will be shown later in the proof of Theorem 3, the progressive-route condition is a special case of the stochastic-monotone condition. We then have the following theorem:

Theorem 3. *Under* $\mathcal{M}'_{C3}$, $T_I \in \mathrm{NBU}$. □

Proof. See Appendix. □

Remark 5. Since the class of mobility models $\mathcal{M}'_{C3}$ is a superset of $\mathcal{M}_{C3}$, Theorem 3 implies that under $\mathcal{M}_{C3}$, $T_I \in \mathrm{NBU}$.

In Section 3.1 to 3.3, we have proposed three classes of stochastic mobility models $\mathcal{M}_{C1}$, $\mathcal{M}_{C2}$, and $\mathcal{M}_{C3}$, each of which abstracts out common characteristics such as 'small world' property for $\mathcal{M}_{C1}$, time reversibility' for $\mathcal{M}_{C2}$, or 'traffic-circle' property for $\mathcal{M}_{C3}$. In addition, each of these classes of models possesses very strong aging property in that the failure rate is monotone for all $t > 0$. There are cases whose failure rates are non-monotone and hence clearly the union of the aforementioned four classes cannot cover the entire spectrum of all possible mobility patterns. Still, we point out that each of these classes is general enough. For instance, the class of mobility models $\mathcal{M}_{C2}$ that include all random walks on undirected graphs, share the property of time-reversibility – which by no means is a small class, thereby leading to DFR inter-contact time.

3.4 Aging Properties in Existing Models

In this section, we consider the inter-contact times between a pair of independent mobile nodes following RWP models [22, 6] and RW mobility models [9], which are widely used in protocol design and performance analysis/comparison in MANET. The relative mobility of a pair of mobile nodes, even when they both follow simple mobility patterns, can become unwieldy for rigorous stochastic analysis. Nevertheless, based on our analytical results in Sections 3.1–3.3, we will discuss the similar features between the relative mobility under RWP/RW model and one of the four classes of stochastic mobility models (e.g., $\mathcal{M}_{C2}$ or $\mathcal{M}_{C3}$) to establish the aging property of the corresponding inter-contact times.

Under RWP model, a mobile node first selects a random waypoint as its destination uniformly in the domain, and then moves to its destination. After it reaches the destination, it selects a new destination, and repeat the whole procedure independently. Consider two independent mobile nodes A and B following RWP model on the same domain. The inter-contact time between A and B is equivalent to the inter-contact time of mobile node C satisfying $C(t) = A(t) - B(t)$ to a static reference site 0.[4] The farther nodes A and B are away from each other, the higher the chance that they are taking opposite directions (away from the center) at the current step. Thus, in the next step, they are likely to choose their destinations in each other's side and cross each other's path. In other words, the farther node C is away from site 0, the stronger tendency it has to visit site 0. This is to some extent similar to the *traffic circle* property of $\mathcal{M}_{C3}$, and we expect that the inter-contact time under RWP model would have IFR.

Under RW model, a node chooses a direction randomly and uniformly from $[0, 2\pi)$, and then moves on a straight line for a time period S (random variable) with constant speed. In this case, the *uniform* stationary distribution for both node position (i.e., $\pi_i = \pi_j$, where i, j are different positions on the domain) and node direction (i.e., $p_{ij} = p_{ji}$) [28, 11, 2] readily leads to the time reversibility property (i.e., $\pi_i p_{ij} = \pi_j p_{ji}$). This property is preserved under two independent mobile nodes following RW model. Thus, we expect that the inter-contact time under RW model would have DFR.

Figure 4 shows the survival function at time t (i.e., $\bar{F}_t(\tau)$) in (a) and failure rate function $r(t)$ in (b) of the inter-contact time under RWP and RW models. Under each model, 10^5 inter-contact time samples between a pair of i.i.d. mobile nodes with constant speed 1m/s on a $(1000m)^2$ domain are

[4] $A(t), B(t), C(t)$ are the positions of nodes A, B, C at time t.

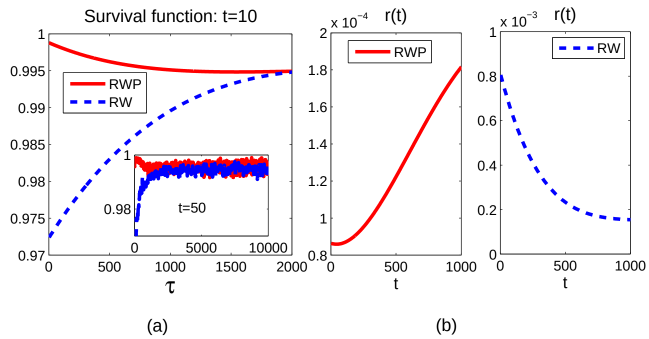


Figure 4: Survival function at time t (a) and failure rate function $r(t)$ (b) of the inter-contact time under RWP and RW models. (a) shows $\bar{F}_t(\tau)$ with $t = 10$ and the case of $t = 50$ is plotted in the inset. (Small fluctuations in the curve caused by the sampling interval effect are eliminated when $t = 10$.)

collected. The communication range is $50m$. The time period S in RW model is exponentially distributed with average 100 seconds. Under both RWP and RW models, the tail of inter-contact time ccdf is known to decay exponentially [23, 32, 13, 4], which implies that the failure rate of the inter-contact time eventually converges to a constant, or equivalently, as τ increases, $\bar{F}_t(\tau)$ converges to the same constant under different t (See also Section 3.1). This can be see in the inset in Figure 4(a). To better capture the aging property in smaller time scale, we plot $\bar{F}_{10}(\tau)$ and $r(t)$ for $\tau \in (0, 2000)$, to show the difference between two curves more clearly. As expected, Figures 4(a) and (b) show that the inter-contact time under RWP/RW model has IFR/DFR.

4. PERFORMANCE COMPARISON UNDER EXPONENTIAL AND NON-EXPONENTIAL INTER-CONTACT TIME

So far we have considered various classes of stochastic mobility patterns that lead to different aging properties. In this section, we show that our results can be applied to compare approaches based on exponential and those based on non-exponential inter-contact time in MANET study.

When T_I has DFR, for larger age (time elapsed since the most recent meeting between a pair of mobile nodes A and B under consideration), the residual lifetime (the remaining time till A and B meet again) of T_I is stochastically larger. In this case, T_I has 'negative aging' property. Correspondingly, IFR/CFR inter-contact time T_I has positive/zero aging property. The stochastically increasing/decreasing property of the remaining contact time is indeed very strong as it requires *monotone* increasing/decreasing failure rate for *all* time t. Sometimes, we only need negative/positive aging property in a weaker sense. For instance, under the class of stochastic mobility patterns $\mathcal{M}'_{C3}$ in Section 3.3, the inter-contact time $T_I \in$ NBU. From Definition 5, the residual lifetime of T_I at age $t > 0$ is stochastically smaller than T_I itself (at age $t = 0$). Note that this corresponds to the stochastic ordering relationship between the residual lifetime at age t and 0, while for the case of IFR/DFR, the relationship is between the residual lifetime at any age t_1 and t_2 ($t_1, t_2 \geq 0$) with monotone stochastic ordering indexed by age t. In fact, for any positive random variable $X \in$ NBU, the following result proves to be very useful:

PROPOSITION 1. *[30] For a new better than used (NBU) discrete positive random variable X, define its* exponential counterpart *X_e as a positive random variable satisfying $\mathbb{P}\{X_e > x\} = e^{-x/\mathbb{E}\{X\}}$ ($x \geq 0$). Then, for any convex function $\psi(\cdot)$,*

$$\mathbb{E}\{\psi(X)\} \leq \mathbb{E}\{\psi(X_e)\}, \text{ denoted by } X \leq_{cv} X_e, \quad (11)$$

i.e., X is smaller than X_e (exponential random variable with the same mean as X) in convex ordering. The inequality in (11) is reversed if X is new worse than used (NWU). □

Considering that IFR/DFR implies NBU/NWU, the result in Proposition 1 can be used to establish a convex ordering relationship between the IFR/DFR inter-contact time T_I and its exponential counterpart. For example, denote T_I under $\mathcal{M}_{C2}, \mathcal{M}_{C3}, \mathcal{M}'_{C3}$ by $T_I^{\mathcal{M}_{C2}}, T_I^{\mathcal{M}_{C3}}, T_I^{\mathcal{M}'_{C3}}$, respectively, and let the corresponding exponential counterparts be $T_e^{\mathcal{M}_{C2}}, T_e^{\mathcal{M}_{C3}}, T_e^{\mathcal{M}'_{C3}}$. Then, from the results in Section 3 and (11), we have

$$T_I^{\mathcal{M}_{C2}} \geq_{cv} T_e^{\mathcal{M}_{C2}}, \quad (12)$$

$$T_I^{\mathcal{M}_{C3}} \leq_{cv} T_e^{\mathcal{M}_{C3}} \quad \text{and} \quad T_I^{\mathcal{M}'_{C3}} \leq_{cv} T_e^{\mathcal{M}'_{C3}}. \quad (13)$$

Further, as seen in Section 3.4 and following the similar line as above, we have

$$T_I^{(RWP)} \leq_{cv} T_e^{(RWP)} \quad \text{and} \quad T_I^{(RW)} \geq_{cv} T_e^{(RW)}. \quad (14)$$

REMARK 6. [5] shows a convex ordering relationship indexed by different degrees of correlation in the class of RW type mobility models, in which the inter-contact time has always DFR according to our study here. In contrast, we provide much more general classes of mobility patterns with various aging properties and a way to compare the non-exponential inter-contact times with different aging properties with the corresponding exponentially distributed inter-contact time with the same mean.

How does the convex ordering result in (12)–(14) impact system performance? Consider the following simple scenario: a set of *i.i.d.* mobile nodes following some given stochastic mobility pattern with certain aging property. Let T be the inter-contact time between a pair of these mobile nodes. As has been done frequently, suppose that the inter-contact time is now *assumed* to be exponentially distributed with the same mean $\mathbb{E}\{T\}$, and let T_e denote this exponential random variable. Then, if all nodes are initially located independently according to their stationary distribution, the time to forward/copy a packet from a node A to any other node B upon encounter (link delay) becomes the residual lifetime of the inter-contact time T between A and B at equilibrium [8, 23] (denoted by T^F), i.e.,

$$\mathbb{P}\{T^F > t\} \triangleq \frac{1}{\mathbb{E}\{T\}} \int_t^\infty \mathbb{P}\{T > s\} ds = \frac{\mathbb{E}\{[T - t]^+\}}{\mathbb{E}\{T\}}.$$

Note that for the exponential random variable T_e, we have

$$\mathbb{P}\{T_e^F > t\} \triangleq \frac{1}{\mathbb{E}\{T_e\}} \int_t^\infty \mathbb{P}\{T_e > s\} ds = \mathbb{P}\{T_e > t\}.$$

Thus, from the convexity of $\psi(x) = [x - t]^+$ in x for any $t \geq 0$ and from (12)–(14), we have, for each $t \geq 0$

$$\mathbb{P}\{T^F > t\} \geq \mathbb{P}\{T_e^F > t\} = \mathbb{P}\{T_e > t\}, \qquad (15)$$

for all mobility patterns with DFR or NWU. Or, equivalently, for any non-decreasing function $\varphi(\cdot)$, we have $\mathbb{E}\{\varphi(T^F)\} \geq \mathbb{E}\{\varphi(T_e)\}$. In other words, in this case, the analysis assuming exponentially distributed inter-contact time results in *under-estimation* of the real system performance (in the sense of stochastic ordering in the link delay) when the real underlying mobility pattern is any of $\mathcal{M}_{C2}$ or RW models. Similarly, in case of IFR or NBU (e.g., $\mathcal{M}_{C3}, \mathcal{M}'_{C3}$ or RWP models), the inequality in (15) is reversed and the analysis assuming exponentially distributed inter-contact time results in *over-estimation* of the real system performance (in the sense of stochastic ordering in the link delay). We expect that this stochastic comparison result forms an analytic foundation toward the design and comparison of forwarding/routing protocols over mobile nodes with general non-exponential inter-contact time by analytically comparing its performance with that of the much simpler case – 'exponentialized' inter-contact time.

Lastly, we want to point out that although (14) shows that the inter-contact time T_I under RWP model is smaller than its exponential counterpart in convex ordering, the distribution of T_I is still quite close to an exponential distribution. For example, the survival function under RWP model in Figure 4(a) is almost invariant with respect to τ for large τ. Also, Figure 4(b) shows that compared to the case of RW model, $r(t)$ under RWP model varies over much smaller range (order of 10^{-4} compared to 10^{-3} in RW case over the same time interval). Thus, we expect that the resulting performance under RWP models will be better than the case of pure exponential inter-contact time, but quite close. However, (14) also applies to any other mobility patterns (e.g., vehicles' movement on routes, visitors' tour in a park, etc.) under the class of $\mathcal{M}_{C3}$ and $\mathcal{M}'_{C3}$, in which the distribution of inter-contact time can be much different from exponential distribution, leading to bigger 'gap' in the aforementioned stochastic ordering.[5]

5. APPLICATION OF AGING PROPERTY IN FORWARDING/ROUTING

In Section 4, we show how the aging property in mobility pattern leads to performance comparison between approaches based on exponential and non-exponential inter-contact time. In this section, we discuss how the aging property can be *actively exploited* toward better design of forwarding/routing algorithms over mobile nodes.

For simplicity of exposition, here we denote by $Age(A, B)$ the time elapsed since the latest encounter between nodes A and B and by $Res(A, B)$ the remaining time until nodes A and B meet again (residual lifetime). This age information $Age(A, B)$ between two mobile nodes has been utilized in wireless sensor networks [15, 31, 19] to estimate the change in the relative positions of nodes A and B and thus change in the overall network topology caused by nodes' mobility. Similarly, this age information has been partially exploited in MANET forwarding/routing algorithm design [10, 34]. For example, [10] proposes an efficient route discovery strategy from the source node $S = R_0$, through searching for a series of intermediate nodes $R_1, R_2, \ldots, R_n$ to the destination node D with the property that for any $i \in [0, n-1]$, $Age(R_{i+1}, D) \leq Age(R_i, D)$. [10] then shows that as $Age(A, B)$ increases, the empirical conditional mean of the distance between A and B also increases and finally converges to a constant after certain time $T_{\mathcal{M}}$, which depends on the underlying mobility pattern $\mathcal{M}$ of mobile nodes A and B. [34] present mobility assisted routing algorithms, under which only a small number of copies of a packet are allowed in the network and each copy, if currently resides in node A, can be further relayed to other node B upon encounter only when $Age(B, D) \leq Age(A, D) - t_0$ with some constant parameter $t_0 > 0$ of their choice. This algorithm is shown to perform well under RW mobility models.

Compared to the existing works showing that the smaller $Age(A, B)$ indicates the smaller mean distance between A and B before certain time $T_{\mathcal{M}}$ (from now on, the time scale of our discussion is between time 0 and $T_{\mathcal{M}}$), our results shows that smaller $Age(A, B)$ can lead to either stochastically smaller or larger residual lifetime $Res(A, B)$ when the inter-contact time has DFR or IFR. For instance, under DFR mobility patterns, we have

$$Age(A,D) \leq Age(B,D) \Rightarrow Res(A,D) \leq_{st} Res(B,D), \quad (16)$$

and in this case a mobile node 'closer' in distance to the destination D indeed will meet it in shorter time. However, under IFR mobility patterns such as RWP model or $\mathcal{M}_{C3}$ in Section 3.3, we have

$$Age(A,D) \leq Age(B,D) \Rightarrow Res(A,D) \geq_{st} Res(B,D), \quad (17)$$

which implies that a mobile node that is closer to D in distance will now spend stochastically longer time to meet the destination.[6] In other words, while indeed successively finding 'better' nodes (in terms of smaller residual lifetime with D) under DFR type mobility patterns, the same algorithm based on age information as above may choose 'worse' set of nodes for IFR type mobility patterns. In fact, the simulation results in [34] have shown that while their forwarding algorithm improves the system performance in terms of packet delay under RW mobility model, the same algorithm then leads to worse performance under RWP mobility model. The authors in [34] discussed that the inferior performance under RWP mobility model might be caused by the 'high mobility', still assuming that the age information should be utilized in the same way as it was for RW mobility models, and they regarded the age information not so much helpful (due to 'high mobility') for RWP mobility models. From our results, however, the true reason for this performance discrepancy under RWP and RW models should be that RWP mobility model has IFR property, while RW mobility model has DFR property, as shown in Figure 4. In this regard, our results suggest that existing forwarding algorithms need to be modified to adapt to different aging properties of inter-contact times, so as to correctly predict the future dynamics (residual lifetime) based on the past

[5] A simple example is the periodic walk in Figure 3(a), under which the inter-contact time ccdf is a step function.

[6] An intuitive explanation is that under IFR mobility patterns, a mobile node A with small $Age(A, D)$ ('closer' to D) is most likely taking the direction that is away from D due to its tour (or traffic circle) type mobility pattern and it will take longer to come back and meet D again.

events (age information) in an attempt to exploit the non-exponential (memory) structure. Further, we expect that not only the ordering in (16) and (17) but also the 'range' over which the failure rate function $r(t)$ fluctuates, can be utilized to predict how much improvements the forwarding with aging information will bring.

6. CONCLUSION

In this paper, by exploring the *memory* structure of contact dynamics, we showed that recently discovered non-exponential inter-contact time can bring tremendous *opportunities* to the study in MANET if correctly exploited. Under four classes of stochastic mobility patterns, we mathematically proved that they produce inter-contact times with constant/decreasing/increasing failure rate as well as new-better-than-used property. We then presented convex ordering relationships between inter-contact times with different aging properties, based on which we compared two approaches using non-exponential and exponential inter-contact time. We also discussed the implication of our results on forwarding/routing algorithm design. We expect that our work in this paper will form a rigorous foundation toward analysis and design of MANET protocols over non-exponential contacts via stochastic ordering and thus bridge the gap between two parallel bodies of approaches focusing on realism or mathematical simplicity.

APPENDIX

Proof of Theorem 1: From Section 2.2, we only need to show that $\bar{F}_t(\tau)$ in (2) does not depend on t. Since $\bar{F}_t(\tau) = \bar{F}_t(1)\bar{F}_{t+1}(1)\cdots\bar{F}_{t+\tau-1}(1)$, it suffices to show that $\bar{F}_t(1) = \mathbb{P}\{T_I \geq t+1 | T_I \geq t\}$ does not depend on t. Observe that

$$\begin{aligned}
&\mathbb{P}\{T_I \geq t+1|T_I \geq t\} = 1 - \mathbb{P}\{T_I = t|T_I \geq t\}\\
&= 1 - \mathbb{P}\{C(t) = 0|T_I \geq t\}\\
&= 1 - \sum\nolimits_{i=1}^{N} \mathbb{P}\{C(t-1) = i|T_I \geq t\} \cdot p_{i0}. \qquad (18)
\end{aligned}$$

Under the 'in-home' condition in Definition 7, $p_{i0} = \gamma$ ($\gamma \in (0,1)$). Hence, from (18), $\mathbb{P}\{T_I \geq t+1|T_I \geq t\} = 1-\gamma$ does not depend on t.

As for the 'out-home' condition in Definition 7, first note that when $\mathcal{G}$ is 2-connected, P^* is irreducible. Since all elements of P^* are non-negative, by Perron–Frobenius Theorem [7], $\vec{p}_0^{\,out} > 0$ in (6) exists. Now, define a matrix Q by replacing the first row of matrix P with $[1, 0, \ldots, 0]$ and consider another mobile node B whose mobility pattern is defined by the transition matrix Q and position at time t by $B(t)$. Note that 0 is an absorbing state for node B, hence the probability that B has never visited site 0 during time instances $1, 2, \ldots, t-1$ is simply $\mathbb{P}\{B(t-1) \neq 0\}$.

If $C(0) = 0$, then at time $t = 1$, C leaves site 0 ($\sum_{k=1}^{N} p_{0k} = 1$) and arrives at site $i \in [1, N]$ with probability p_{0i}. Suppose at time $t = 1$, node B is located at site i (with probability p_{0i}). Then, the distributions of $B(t)$ and $C(t)$ from $t \geq 1$ until they visit site 0 again are identical. Thus, we have

$$\begin{aligned}
\mathbb{P}\{C(t-1) = i|T_I \geq t\} &= \mathbb{P}\{B(t-1) = i|B(t-1) \neq 0\}\\
&= q_i(t-1)/\big[1 - q_0(t-1)\big], \qquad (19)
\end{aligned}$$

where $q_j(t) = \mathbb{P}\{B(t) = j\}$. Finally, note that $\vec{p}_0^{\,\text{out}}$ in (6) is quasi-stationary distribution [24] in that when $\vec{p}_0^{\,\text{out}}$ is node B's initial distribution, $\frac{q_i(t)}{1-q_0(t)}$ is invariant with respect to t. Hence, from (18) and (19), $\mathbb{P}\{T_I \geq t+1|T_I \geq t\}$ does not depend on t and this completes the proof.

Proof of Theorem 2: We will first show that the traffic-circle condition in Definition 9 implies the total-positivity condition, i.e. $Q = \{q_{ij}\} \in \text{TP}_2$. From Definition 8, we only need to show that

$$q_{ij}q_{i'j'} - q_{ij'}q_{i'j} \geq 0, \qquad (20)$$

for all $i, j, i', j' \in [0, N]$ satisfying $i < i'$ and $j < j'$. Since $q_{00} = 1$ and $q_{0j} = 0$, (20) trivially holds when $i = 0$. When $i > 0$, if $j' \geq i$ or $j \geq i'$, then at least one of $q_{ij'} = 0$ and $q_{i'j} = 0$ is true, thus (20) automatically holds. Therefore, we only need to consider the case when $j < j' < i < i'$. However, from the traffic-circle condition, we have $q_{ij} = q_{ij'} = 1/i$ (note that $i > 0$), and $q_{i'j'} = q_{i'j} = 1/i'$. In other words, $q_{ij}q_{i'j'} - q_{ij'}q_{i'j} = 0$ and (20) still holds. In consequence, we have shown that $Q \in TP_2$ under traffic-circle condition.

Now, similar to the proof of Theorem 1, consider another mobile node B whose mobility pattern is defined by transition matrix Q. If $C(0) = 0$, then $C(1) = N$ from $p_{0N} = 1$ in Definition 9. Suppose that at time $t = 1$ node B is also at site N. Then, as before, distributions of nodes B and C's trajectories until they visit site 0 again are identical, and thus T_I has the same distribution as that of the first hitting time T_F of mobile node B from site N to site 0. From Theorem 3.21 in [26], T_F has IFR. This completes the proof.

Proof of Theorem 3: We will first show that under the progressive-route condition in Definition 12, the $(N+1) \times (N+1)$ matrix Q is stochastically monotone, where $Q = \{q_{ij}\}$ is from replacing the first row of transition matrix P by $[1, 0, \ldots, 0]$. Let the row vectors of matrix Q be

$$\vec{q}_i = [q_{i0}, q_{i1}, \ldots, q_{iN}], \quad i = 0, 1, \ldots, N.$$

From Definition 11, we only need to shows that

$$\vec{q}_0 \leq_{st} \vec{q}_1 \leq_{st} \ldots \leq_{st} \vec{q}_N. \qquad (21)$$

Since $\vec{q}_0 = [1, 0, \ldots, 0]$, clearly $\vec{q}_0 \leq_{st} \vec{q}_1$ (note that the sum of all entries in $\vec{q}_1$ is 1). For any $i \in [1, N]$, from the progressive-route condition in Definition 12, we have

$$\sum\nolimits_{j=1}^{k} q_{ij} = \begin{cases} (k+1)\big(1-(N-i+1)p_i\big)/i & \text{if } k \in [0, i-1], \\ 1-(N-k)p_i & \text{if } k \in [i, N]. \end{cases}$$

Therefore, for any $i \in [1, N]$, when $k \in [0, i-1]$,

$$\begin{aligned}
&\sum\nolimits_{j=1}^{k} q_{ij} = (k+1)\big(1-(N-i+1)p_i\big)/i\\
&\geq (k+1)\big(1-(N-(i+1)+1)p_{i+1}\big)/i \qquad (22)\\
&\geq (k+1)\big(1-(N-(i+1)+1)p_{i+1}\big)/(i+1) = \sum\nolimits_{j=1}^{k} q_{(i+1)j},
\end{aligned}$$

where (22) is from (10), i.e., $(N-i+1)p_i \leq (N-(i+1)+1)p_{i+1}$. When $k = i$,

$$\sum\nolimits_{j=1}^{k} q_{ij} = 1-(N-i)p_i \geq 1-(N-i)p_{i+1} = \sum\nolimits_{j=1}^{k} q_{(i+1)j}.$$

Similarly, when $k \in [i, N]$, $\sum_{j=1}^{k} q_{ij} \geq \sum_{j=1}^{k} q_{(i+1)j}$ from $p_i \leq p_{i+1}$. Hence, (21) holds and the matrix Q is stochastically monotone.

Now, again, similar to the proof of Theorem 2, we can define another mobile node B whose mobility pattern is defined by transition matrix Q, and T_I has the same distribution as that of the first hitting time T_F of mobile node B from site N to the site 0. From Theorem 3.20 in [26], $T_F \in$ NBU and this completes the proof.

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