# Crossing Over the Bounded Domain: From Exponential To Power-law Inter-meeting time in MANET

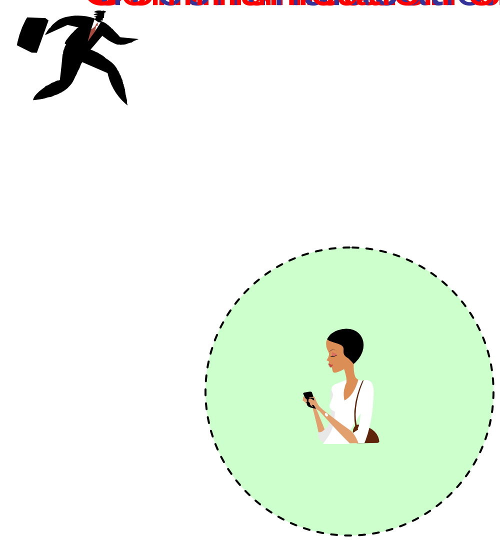
**Han Cai, Do Young Eun**

Department of Electrical and Computer Engineering

North Carolina State University

# Motivation – inter-meeting time

Communication begin!
Communication end!



- Significance of Inter-meeting time
  - One of contact metrics (especially important for DTN)

# Motivation – exp. inter-meeting

- Assumed for tractable analysis [1, 2]
- Supported by numerical simulations based on mobility model (RWP) [3, 4]
- Theoretical result to upper bound first and second moment [5] using BM model on a sphere


- [1] Grossglauser, M., and Tse, D. N. C. Mobility increases the capacity of Ad Hoc wireless networks. *IEEE/ACM Transactions on Networking, 2002.*
- [2] Sharma, G., and Mazumdar, R. On achievable delay/capacity trade-offs in Mobile Ad Hoc Networks. *WIOPT, 2004.*
- [3] Sharma, G., and Mazumdar, R. Scaling Laws for Capacity and Delay in Wireless Ad Hoc Networks with Random Mobility. In *ICC, 2004.*
- [4] Groenevelt, R., Nain, P., and Koole, G. Message delay in MANET. In *Proceedings of ACM SIGMETRICS* (New York, NY, June 2004).
- [5] Sharma, G., Mazumdar, R., and Shroff, N. B. Delay and Capacity Trade-offs in Mobile Ad Hoc Networks: A Global Perspective. In *Infocom 2006.*

# Motivation – power-law inter-meeting (1)

- Recently discovered: power-law [6, 7]

<u>Effect of power-law on system performance</u> [6]

"If $\alpha < 1$, none of these algorithms, including flooding, can achieve a transmission delay with a finite expectation."


- [6] Chaintreau, A., Hui, P., Crowcroft, J., Diot, C., Gass, R., and Scott, J. Impact of human mobility on the design of opportunistic forwarding algorithms. In *Proceedings of IEEE INFOCOM* (Barcelona, Catalunya, SPAIN, 2006).
- [7] Hui, P., Chaintreau, A., Scott, J., Gass, R., Crowcroft, J., and Diot, C. Pocket switched networks and the consequences of human mobility in conference environments. In *Proceedings of ACM SIGCOMM (WDTN-05).*

# Motivation – power-law inter-meeting (2)

<u>Effect of infrastructure and multi-hop transmission [8]</u>

"... A consequence of this is that there is a need for good and efficient forwarding algorithms that are able to make use of these communication opportunities effectively."


- [8] Lindgren, A., Diot, C., and Scott, J. Impact of communication infrastructure on forwarding in pocket switched networks. In *Proceedings of the 2006 SIGCOMM workshop on Challenged networks* (Pisa, Italy, September 2006).

# Motivation – power-law inter-meeting (3)

- Recent study on power-law (selected)
  - Call for new mobility model [6]
    - Use 1-D random walk model to produce power-law inter-meeting time [9]
  - Call for new forwarding algorithm [8]


- [9] Boudec, J. L., and Vojnovic, M. Random Trip Tutorial. In *ACM Mobicom* (Sep. 2006).

# Our work

- What's the fundamental reason for exponential & power-law behavior?
- In this paper, we
  - Identify what causes the observed exponential and power-law behavior
  - Mathematically prove that most current synthetic mobility models necessarily lead to exponential tail of the inter-meeting time distribution
  - Suggest a way to observe power-law inter-meeting time
  - Illustrate the practical meaning of the theoretical results

# Content

- **Inter-meeting time with exponential tail**
- From exponential to power-law inter-meeting time
- Scaling the size of the space
- Simulation

# Basic assumptions and definitions

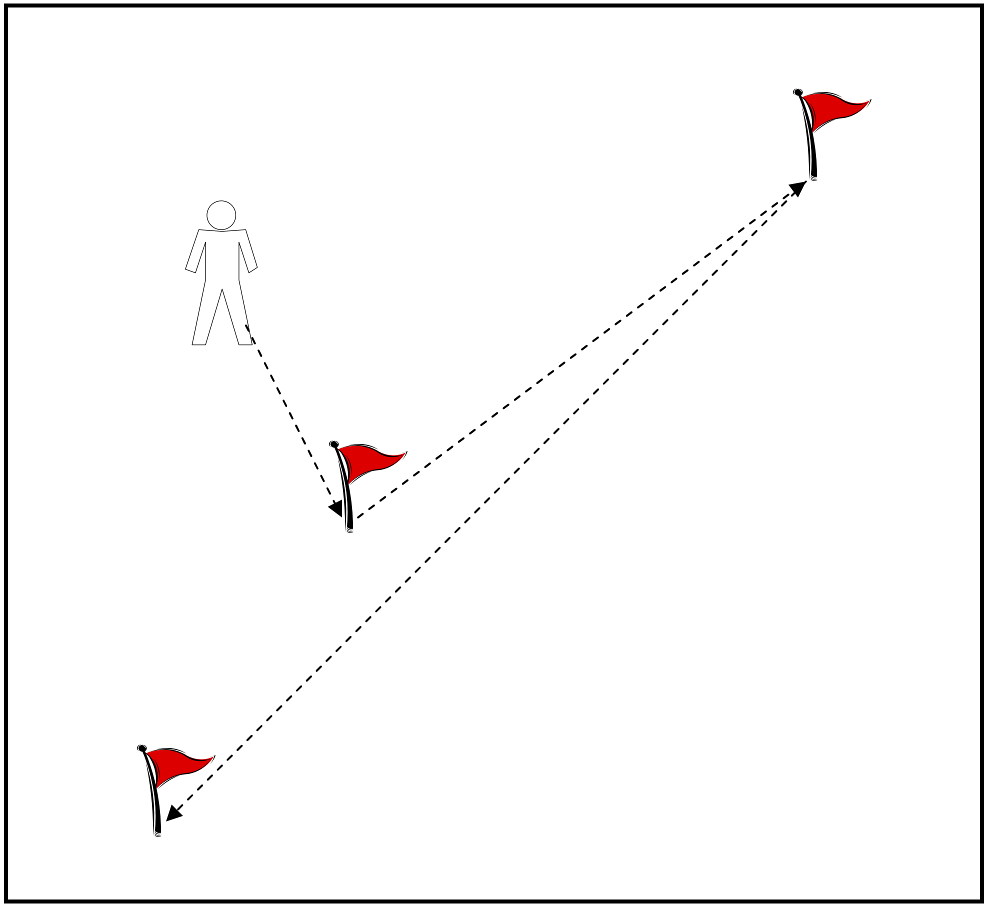
- The inter-meeting time $T_I$ of nodes A and B is defined as

$$T_I \triangleq \inf_{t>0} \{t : \|A(t) - B(t)\| \leq d\}$$

given that $\|A(0) - B(0)\| = d$ and $\|A(0^+) - B(0^+)\| > d.$

- Two nodes under study are independent, unless otherwise specified

# Random Waypoint Model



- We consider
  - Zero pause time
  - Random pause time (light-tail)

# RWP with zero pause time

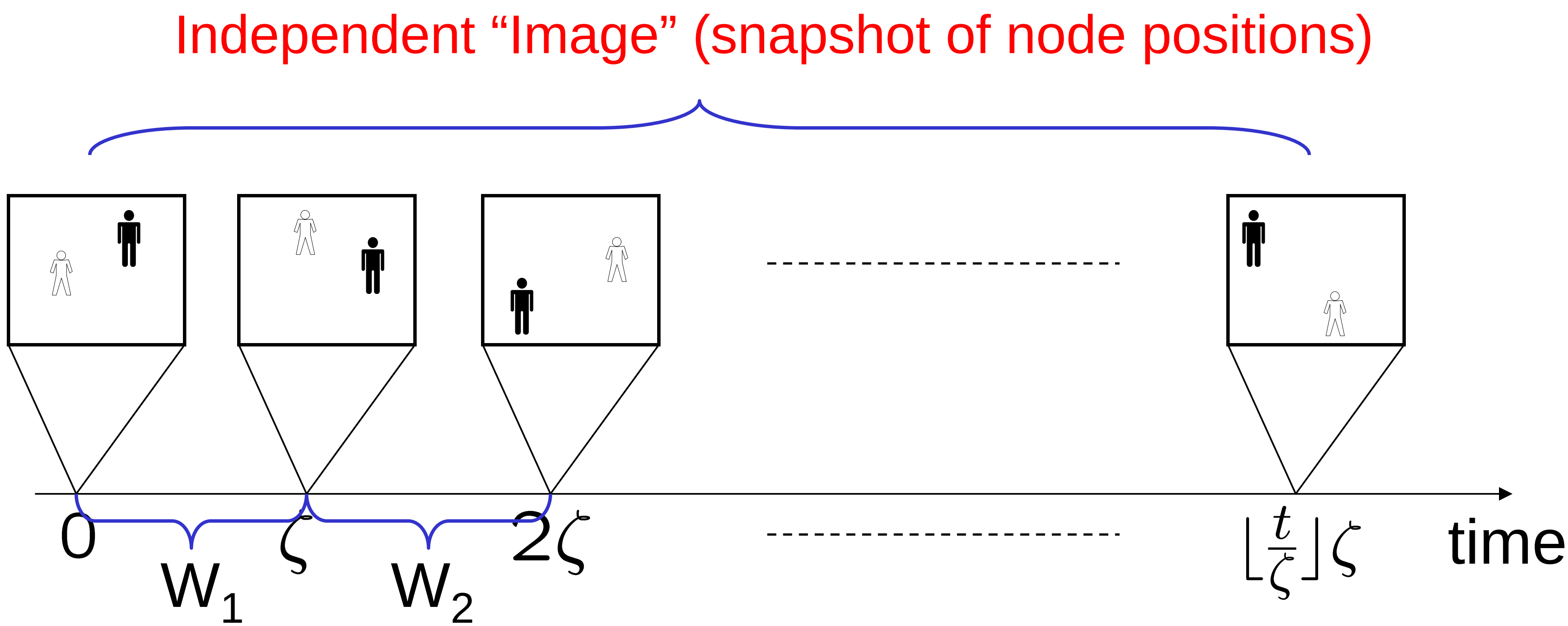
**Proposition 1:** Under zero pause time, there exists constant $c > 0$ such that

$$P\{T_I > t\} < e^{-ct},$$

for all sufficiently large t.

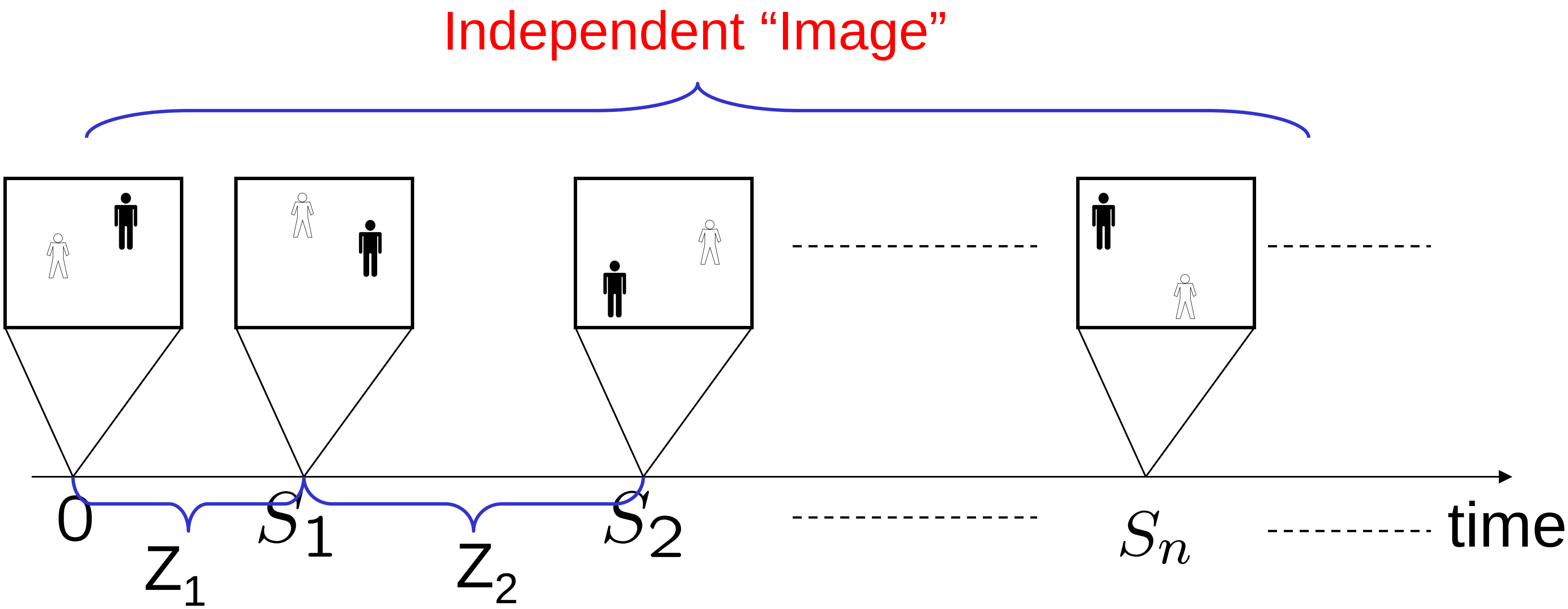
- Proposition 1 is also true for "bounded" pause time case.

# Proof sketch for Proposition 1




- $W_1 = W_2 = \cdots = \zeta$
- # of independent "image" = $O(t)$
- Each "image": P {not meeting} < c < 1

# Random pause time: the difficulty




- $Z_1 = Z_2 = \cdots = \zeta$ ✗
- # of independent “image” = O(t) ?

# RWP with random pause time

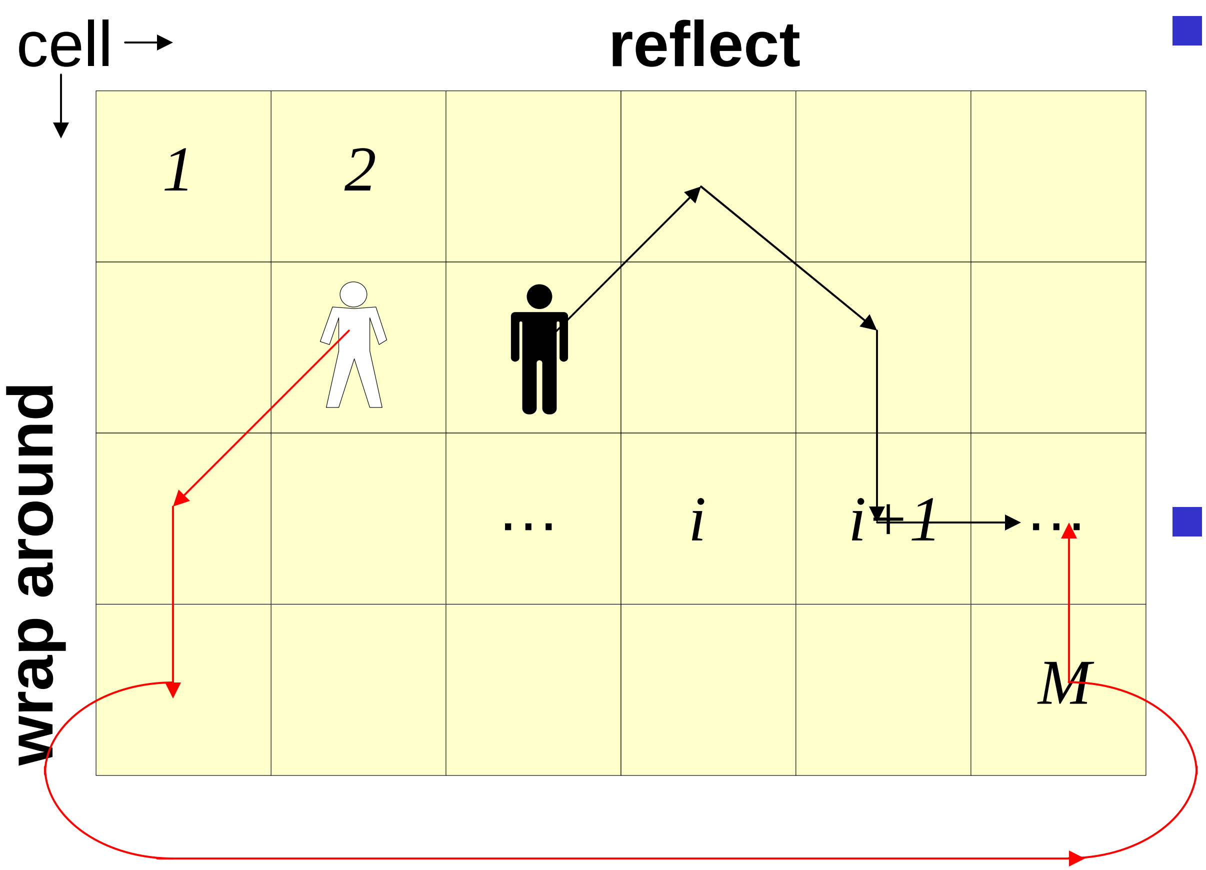
**Theorem 1:** Under random pause time, there exists constant $c > 0$ such that

$$P\{T_I > t\} < e^{-ct},$$

for all sufficiently large t.

- Proposition 1 is extended to random pause time case, i.e., the pause time may be infinite.

# Random Walk Models (MC)




- Markov Chain RWM: transition matrix

$$P = \{ p_{ij} \},$$

prob. of jumping from cell i to j

- Boundary behavior
  - Reflect
  - Wrap around

- Two node meet if and only if they are in the same cell
- General version of discrete isotropic RWM

# Assumptions on RWM

- After deleting any single state from the MC model, the resulting state space is still a communicating class.
  - The failure of any one cell will not disconnect the mobility area – if an obstacle is present, the moving object (people, bus, etc.) will simply bypass it, rather than stuck on it

- For any possible trajectory of node B, node A eventually meets node B with positive probability (No conspiracy).

# RWM: exponential inter-meeting

**Theorem 2:** Suppose that node A moves according to the RWM and satisfies assumptions on RWM. Then, there exists constant $\gamma > 0$ such that

$$P\{T_I > t\} \leq e^{-\gamma t},$$

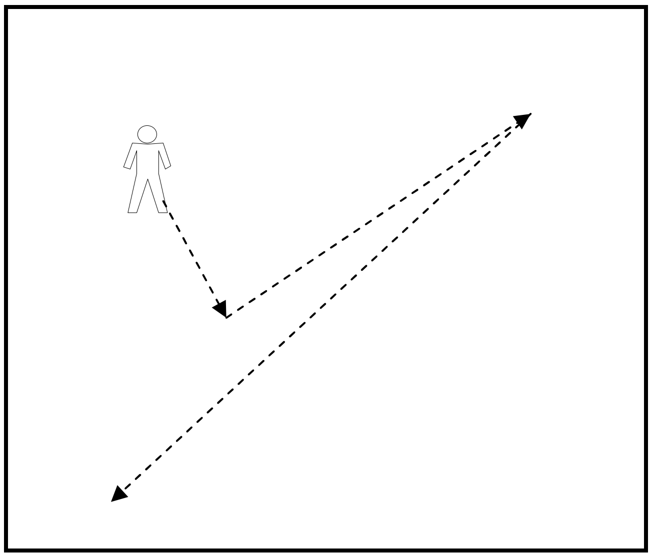
for all sufficiently large t.

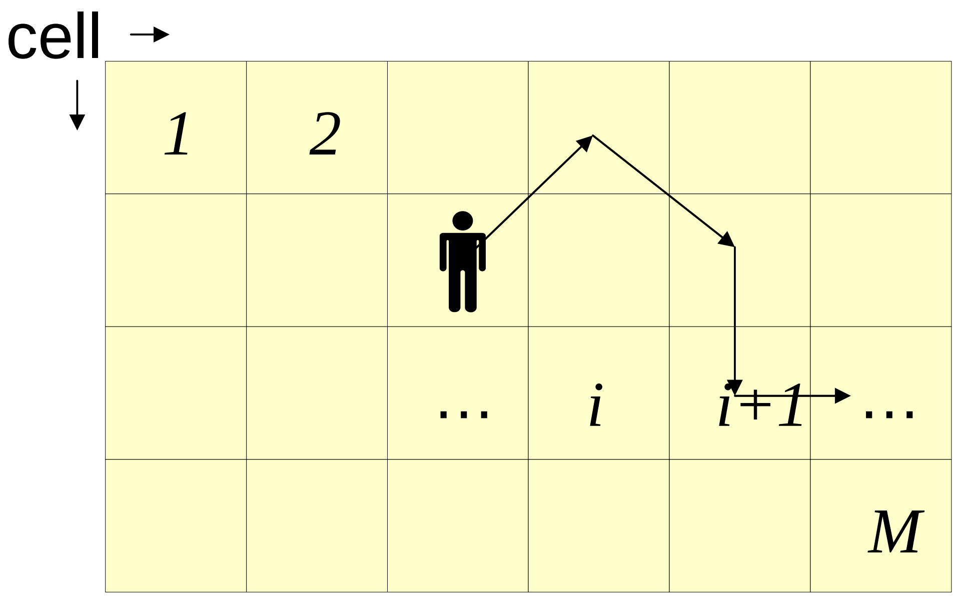
- Only one node is required to move as RWM.
  - Theorem 2 applies to inter-meeting time of two nodes moving as: RWM+RWM, RWM+RWP, RWM+RD, RWM+BM, etc.
- Effect of spatial constraints (e.g., obstacles) is also reflected (by assigning $p_{ij}$ ).

# Content

- Inter-meeting time with exponential tail
- **From exponential to power-law inter-meeting time**
- Scaling the size of the space
- Simulation

# Common factor leads to exponential tail?





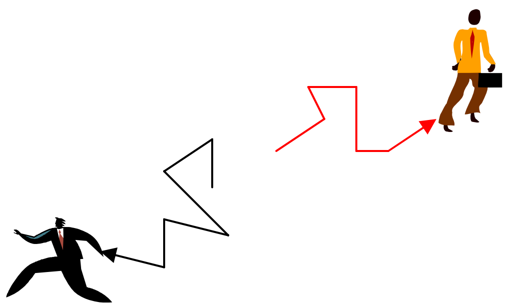

- What is common in all these models?

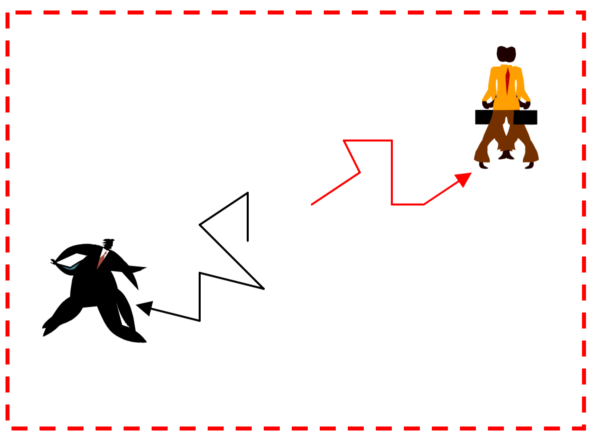
# Common factor leads to exponential tail?

## Finite Boundary!!!

- "Boundary" is incorporated in definition
  - RWM: wrapping or reflecting boundary behavior
  - RWP: boundary concept inherited in model definition (destination for each jump is uniformly chosen from a bounded area)

# Finite boundary: exponential tail





- Two nodes not meet for a long time

  → most likely move towards different directions

  → prolonged inter-meeting time

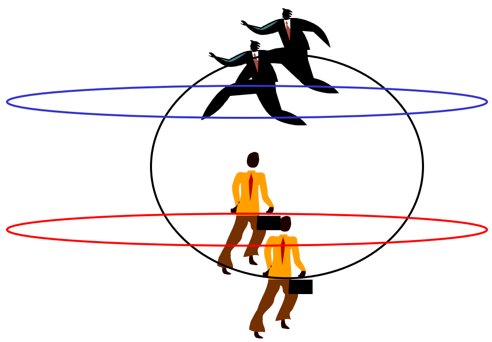
  <**strong memory**>

- Finite boundary erase this memory <**memoryless**>

# Other factors than boundary?

- For most current synthetic models, finite boundary critically affects tail behavior of inter-meeting time

- Other possible factors
  - Dependency between mobile nodes
  - Heavy-tailed pause time (with infinite mean)
  - Correlation in the trajectory of mobile nodes



- Our study focuses on:
  - Independence case
  - Weak-dependence case

# Removing the boundary ...

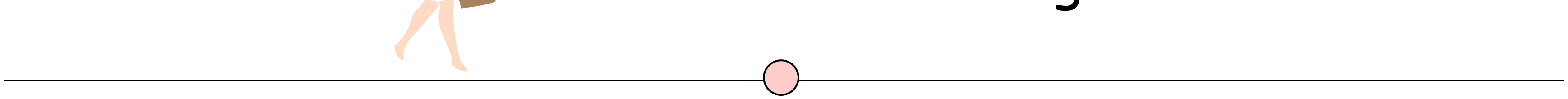
- Isotropic random walk in $R^2$
  - Choose a random direction uniformly from $[0, 2\pi)$
  - Travel for a random length in $(0, \infty)$
  - Repeat the above process

**Theorem 4:** Two independent nodes A, B move according to the 2-D isotropic random walk model described above. Then, there exists constant $C > 0$ such that the inter-meeting time $T_I$ satisfies:

$$P\{T_I > t\} \geq Ct^{-1/2},$$ for all sufficiently large t.

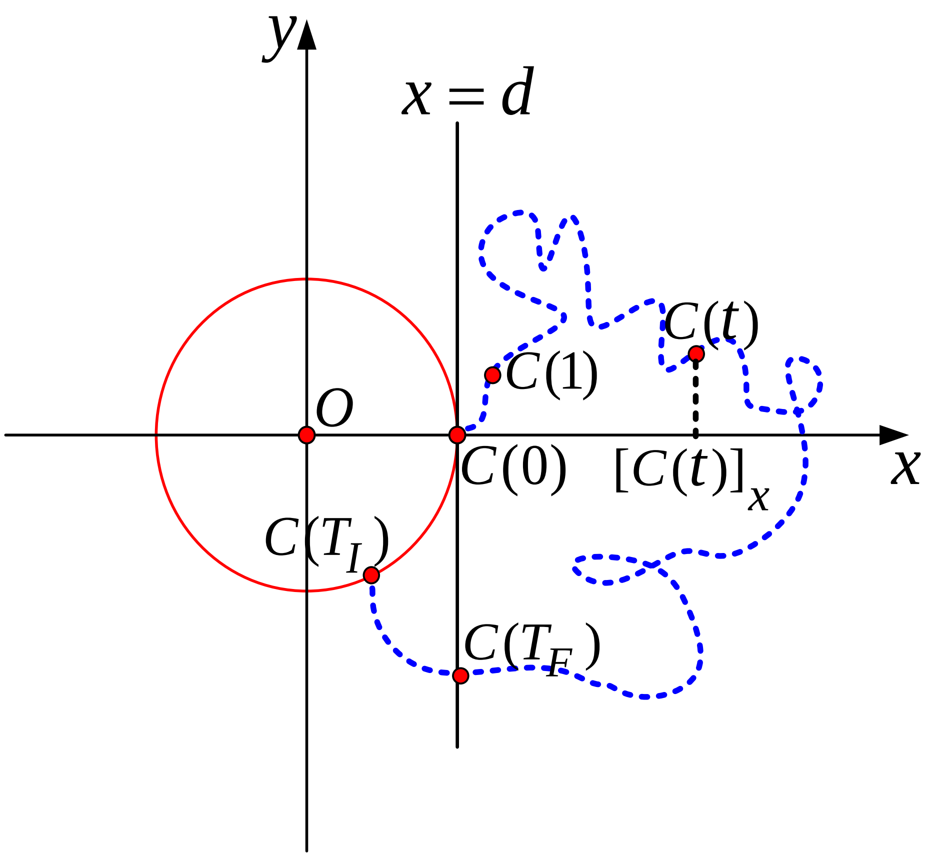
# Proof sketch for Theorem 4 (1)

- 1-D isotropic random walk
  - P {jump left over L} = P {jump right over L}
  - First passage time: starting from a non-origin $x_0$, minimum time to return to the origin




- **<u>Sparre-Andersen Theorem</u>**: For any one-dimensional discrete time random walk process starting at non-origin $x_0$ with each step chosen from a continuous, symmetric but otherwise arbitrary distribution, the First Passage Time Density (FPTD) to the origin asymptotically decays as $t^{-1.5}$.

# Proof sketch for Theorem 4 (2)




- Difference walk
- Find lower bound
  - $T_F \leq T_I$
- Map to 1-D
  - $C(t) \rightarrow [C(t)]_x$
- Apply S-A Theorem

# Content

- Inter-meeting time with exponential tail
- From exponential to power-law inter-meeting time
- **Scaling the size of the space**
- Simulation

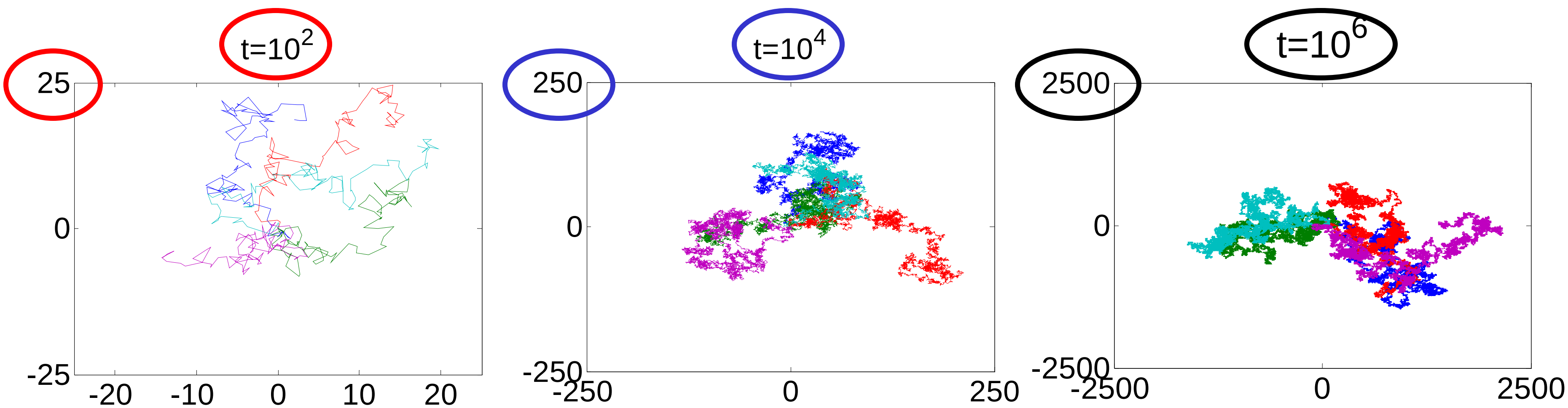
# Questions

- About the boundary
  - In reality, all domain under study is bounded
  - In what sense does "infinite domain" exist?

- About exponential/power-law behavior
  - Where does the transition from exponential to power-law happen?

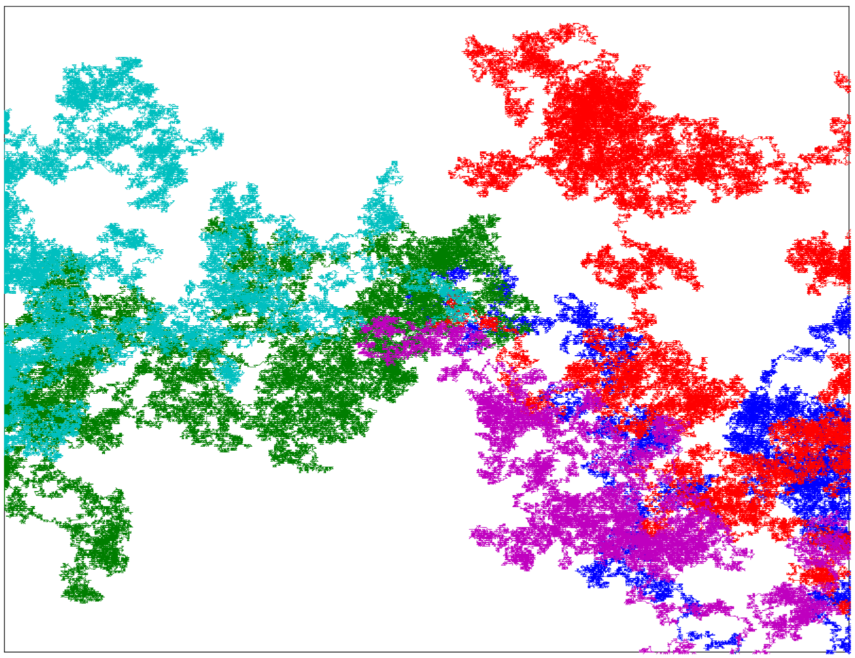
# Time/space scaling

- The **interaction** between the **timescale** under discussion and the **size** of the boundary
  - Position of node A (following 2-D isotropic random walk) at time t: A(t), satisfies $\mathbb{E}\{\|A(t)\|^2\} = t$
  - "Average amount of displacement": standard deviation of A(t), scales as $O(\sqrt{t})$
  - Standard BM: position scale as $O(\sqrt{t})$



# BM: time/space scaling

t=1000000

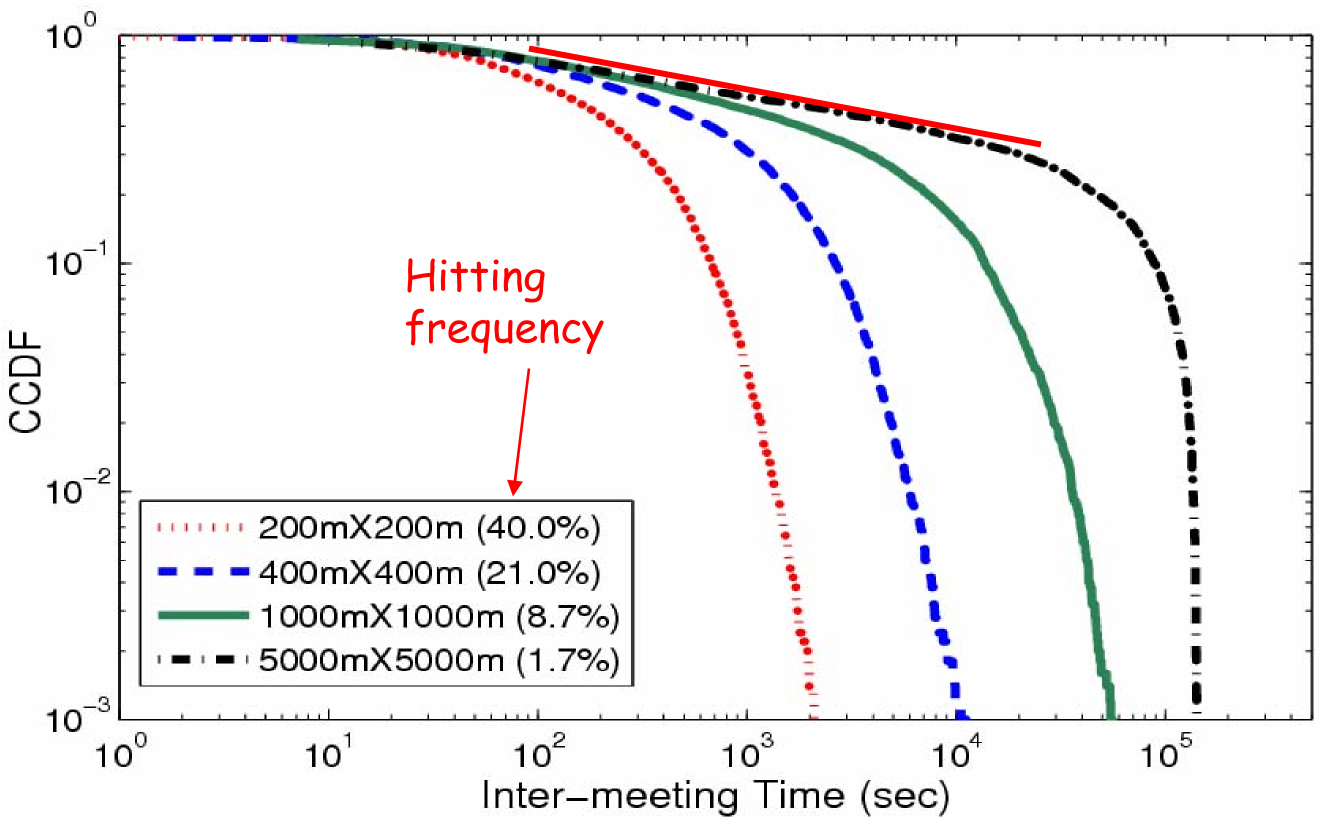


- Area: 800X800 $m^2$
- Is 200X200 domain bounded?
  - Unbounded over time scale [0,100]
  - Bounded over time scale [0,1000000]
- KEY: whether the boundary effectively "erases" the memory of node movement

# Content

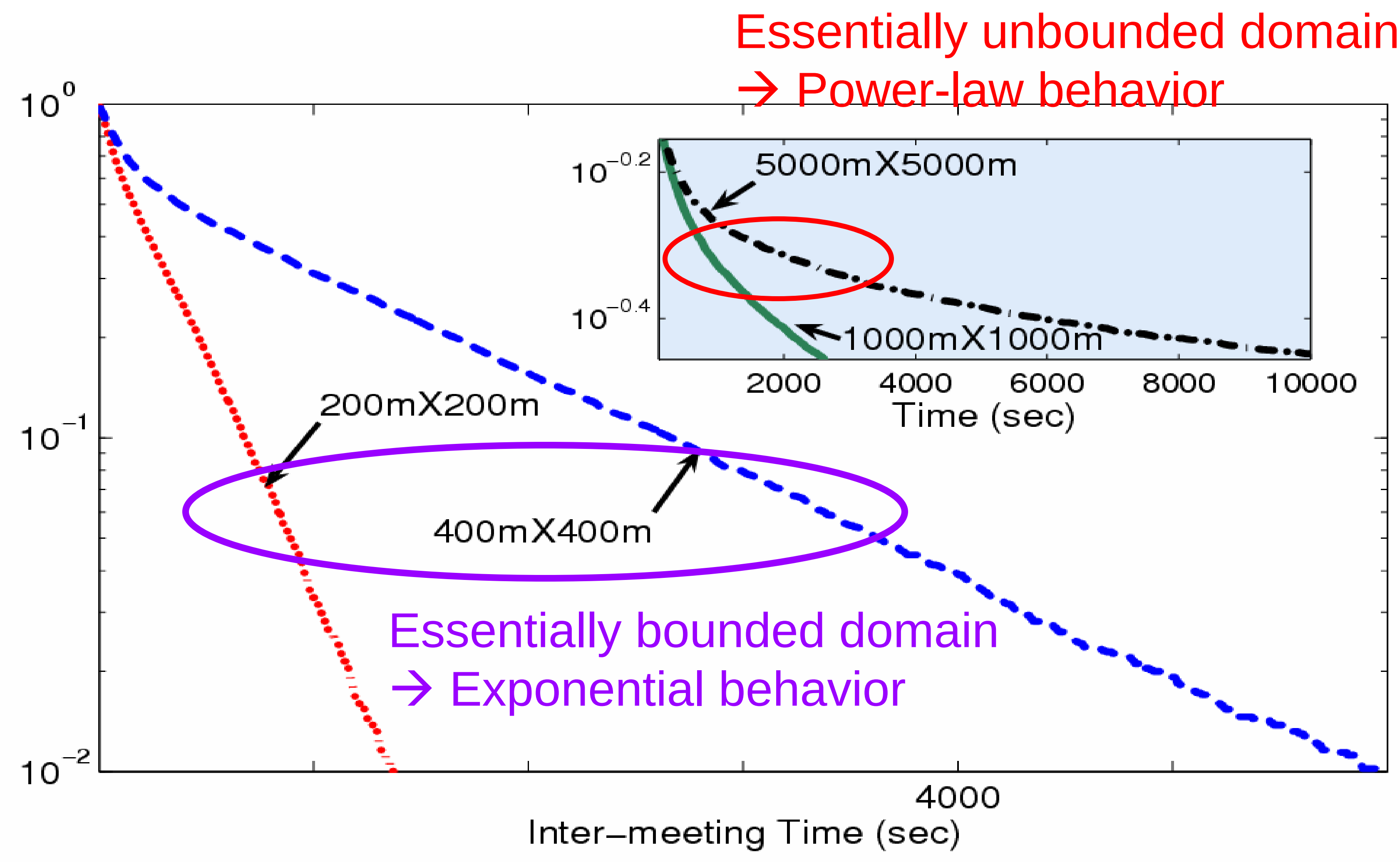
- Inter-meeting time with exponential tail
- From exponential to power-law inter-meeting time
- Scaling the size of the space
- **Simulation**

# RWM: $P\{T_I > t\}$ (log-log)

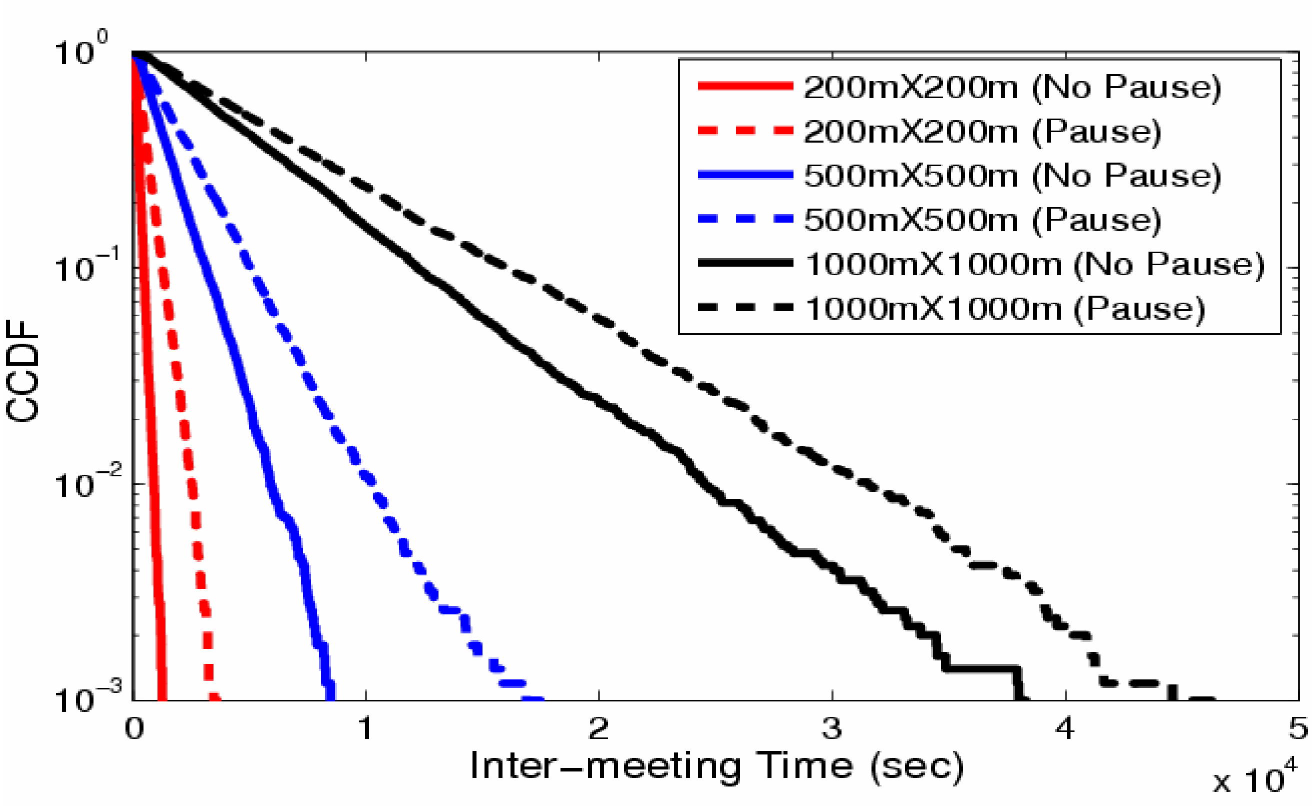



- RWM: change direction uniformly every 50 seconds
- Speed: U(1.00, 1.68)
- Simulation period T: 40 hours
- Avg. amount of displacement: 500m

# RWM: $P\{T_I > t\}$ (linear-log)



# RWP: $P\{T_I > t\}$ (linear-log)




- Irrespective of the domain size, the tail of inter-meeting exhibits an exponential behavior
- For either zero pause or random pause cases, the slope of the CCDF decreases as domain size increases

# Conclusion

- "Finite boundary" is a decisive factor for the tail behavior of inter-meeting time, we prove
  - The exponential tailed inter-meeting time based on RWP, RWM model
  - The power-law tailed inter-meeting time after removing the boundary

- Time/space scaling, i.e., the interaction between domain size and time scale under discussion is the key to understand the effect of boundary

# Thank You!

Questions ?