On the Forwarding Performance under Heterogeneous Contact Dynamics in Mobile Opportunistic Networks

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Abstract—In this paper we focus on how the heterogeneous contact dynamics of mobile nodes impact the performance of forwarding algorithms in mobile opportunistic networks. To this end, we consider two representative heterogeneous network models, each of which captures heterogeneity among node pairs (individual) and heterogeneity in underlying environment (spatial), respectively, and examine the full extent of difference in delay performance they cause on forwarding algorithms through formal stochastic comparisons. We first show that these heterogeneous models correctly capture non-Poisson contact dynamics observed in real traces. We then rigorously establish *stochastic/convex ordering relationships* on the delay performance of direct forwarding and multicopy two-hop relay protocol under these heterogeneous models and the corresponding homogeneous model, all of which have the same average inter-contact time of a random pair of nodes. In particular, we demonstrate that the heterogeneous models predict an entirely *opposite* ordering relationship in delay performance depending on which of the two heterogeneity structures is captured. We also provide simulation results including the delay performance of epidemic routing protocol to support the analytical findings. Our results thus suggest that the heterogeneity in mobile nodes' contact dynamics should be properly taken into account for the performance evaluation of forwarding algorithms. Our results will also be useful for better design of forwarding algorithms correctly exploiting the heterogeneity structure.

Index Terms—Mobile opportunistic networks, heterogeneous contact dynamics, non-Poisson contact dynamics, forwarding performance, stochastic/convex ordering relationships

1 Introduction

In mobile opportunistic networks (MONs) a.k.a. delay/disruption tolerant networks (DTNs), frequent disruptions in end-to-end connectivity arise due to many factors such as node mobility, power limitations, etc. To overcome the intermittent connectivity, mobile nodes relay or copy messages to other mobile nodes upon encounter by the so-called *'store-carry-and-forward'* principle, which ensures that the messages eventually reach their destinations. The performance of message delivery depends on how to relay or copy messages to mobile nodes. Thus, many forwarding algorithms such as epidemic routing [2], two-hop relay [3], [4], spray and wait [5], to name a few, have been proposed and commonly analyzed based upon a 'homogeneous' network model in which contacts between *any* pair of nodes occur according to a Poisson process [5], [4], [6], [7], [8]. This homogeneous model is typically supported by observing that the inter-contact time[1] between two successive contacts for *any* node pair follows an exponential distribution via numerical simulations under synthetic mobility models [4], [7]. Other analytical works also resort to the homogeneous model for their investigation on the cost-delay tradeoff [11], [12], the design of forwarding policy [13], and content distribution [14].

However, many measurement studies [15], [16], [17], [18], [19], [20] point out the existence of heterogeneity in a wide range of mobile networking scenarios, while recent empirical and/or analytical observations [9], [21] show that the inter-contact time distribution is no longer pure exponential; rather, it is a mixture of power-law and exponential distributions. In particular, from real mobility traces and survey data, [17], [18] observe the characteristics of heterogeneity in mobile nodes' contact dynamics, and [15], [16], [18], [19], [20] uncover spatially and/or socially formatted community structures in nodes' mobility, which all make contact dynamics deviate from a pure Poisson process. The observed heterogeneity structures have been actively used for the development of new mobility models [15], [22], [20] and empirically exploited to design new forwarding algorithms [16], [23], [18].

In the literature, there are two widely used heterogeneous network models [17], [24], [25], [26], [27], [19], [28], each of which captures *individual (social)* or *spatial* heterogeneity observed in real traces. In the individually heterogeneous network model [17], [24], [25], [26], [27], the heterogeneity is characterized by allowing different contact rates for different node pairs, while the inter-contact time distribution of each pair is still exponential (but with different rates). On the other hand, in

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A preliminary version of this paper has appeared in the proceedings of WiOpt 2009 [1].

1. The inter-contact time of two mobile nodes is defined as the time interval from when their communication becomes unavailable to the time when the communication resumes. See [9], [10] for its formal definition.

the spatially heterogeneous network model [19], [28], the heterogeneity arises on each spatial cluster (site) in which mobile nodes reside, while they can move to the other spatial clusters. (See Section 2 for more details.) However, none of these works analytically investigates how the heterogeneity structure impacts the performance of forwarding algorithms, not to mention whether the considered heterogeneity improves or deteriorates the performance.

1.1 Summary of Contributions

In this paper, we examine how the forwarding performance under the two heterogeneous network models deviates from that under the aforementioned homogeneous model. We first show that *each* of the heterogeneous models correctly captures the non-Poisson contact dynamics (i.e., non-exponential inter-contact time distribution of a random pair of nodes) as observed in real traces. Then, we rigorously establish stochastic/convex ordering relationships among the delay performance of direct forwarding and multicopy two-hop relay protocol [4], [7], [29], [30] under the two heterogeneous models and the corresponding homogeneous model, all of which are *indistinguishable* from the viewpoint of the average inter-contact time of a random node pair.

Specifically, we prove that the message delivery delays of direct forwarding and multicopy two-hop relay protocol under the spatially heterogeneous model are *stochastically larger* than those under the corresponding homogeneous model, respectively. We also prove that the delay of direct forwarding under the individually heterogeneous model is *larger* than that under the corresponding homogeneous model *in convex ordering* (see Section 4 for its formal definition), while the average delay of multicopy two-hop relay protocol under the individually heterogeneous model is *smaller* than that under the corresponding homogeneous model. As a special case of the above results, we show that the heterogeneity structure in the spatially heterogeneous model *deteriorates* the average delay performance of multicopy two-hop relay protocol, whereas the other heterogeneity structure in the individually heterogeneous model *improves* its average delay performance when compared with that under the corresponding homogeneous model. This implies that each of the two heterogeneous models predicts an entirely opposite average delay performance. We also observe this opposite performance result for epidemic routing protocol[2] [2], [4], [6], [7], [30], [24] via numerical simulations.

We further demonstrate that the delay performance of direct forwarding and multicopy two-hop relay protocol under the spatially heterogeneous model is *worse* than that under the individually heterogeneous model, even when the *entire distributions* of inter-contact time of a random node pair under both heterogeneous models are precisely matched. Our results collectively suggest that merely capturing non-Poisson contact dynamics from the viewpoint of a random node pair is not enough and that one should carefully evaluate the performance of forwarding algorithms under a properly chosen heterogeneous network setting. Our results will also be useful in correctly exploiting the underlying heterogeneity structure so as to achieve better forwarding performance.

2. The forwarding algorithms considered in this paper are 'oblivious' to the underlying network structures, which enables an unbiased evaluation on the forwarding performance under heterogeneous contact behaviors of mobile nodes.

1.2 Outline of Paper

The rest of this paper is organized as follows. Section 2 gives preliminaries on the formal description of two representative (individually and spatially) heterogeneous network models. Section 3 presents the characteristics of inter-contact time under each of the two heterogeneous models. Sections 4 and 5 provide our theoretical results on the stochastic comparison of message delivery delays for direct forwarding and multicopy two-hop relay protocol under each of the heterogeneous models and its corresponding homogeneous model, respectively. We provide simulation results in Section 6 and conclude in Section 7.

2 Preliminaries

In this section, we present the details of two heterogeneous network models to be used for our paper. In general, mobile nodes typically belong to different societal groups, with different preferred sites following different mobility patterns. For example, in Fig. 1, there exist several popular places (e.g., library, dormitory, or dining hall) in a campus and students may form spatially separate clusters around the popular places, while occasionally move to other clusters according to their own daily schedules. Further, in each spatial cluster, students from different groups (e.g., ECE/CS departments or undergraduate/graduate) typically mix together, but making more frequent contacts with others from the same group than from different groups. These social (or individual) and spatial heterogeneity structures can be captured under the following two heterogeneous network models [17], [24], [25], [26], [27], [19], [28], each of which directly characterizes the heterogeneity in mobile nodes' contact dynamics in a different manner, rather than defining detailed mobile trajectories inside a small domain or group (or social 'clique').

2.1 Individually Heterogeneous Network Model

An individually heterogeneous network model (simply, an individual model) was introduced in [17] and described as follows. This individual model has been also used in [25], [24], [26], [27].

Consider a set of mobile nodes $\mathcal{N}$ in the network. The pairwise inter-contact time between nodes i and j follows an independent exponential distribution with

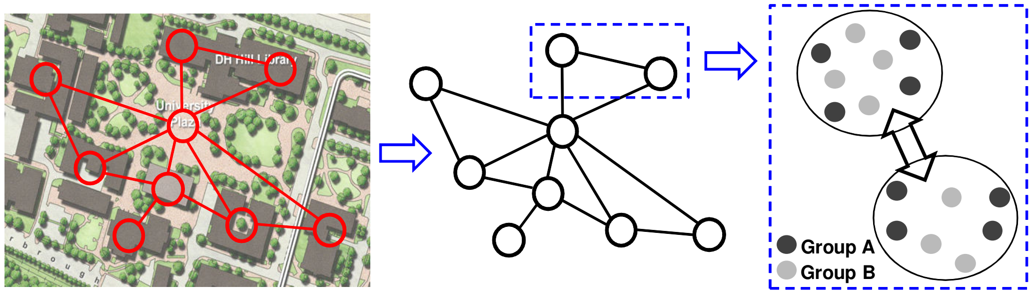
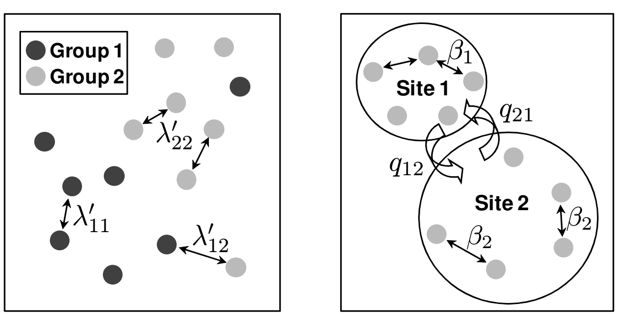


Fig. 1. An example for spatial and social (individual) heterogeneity in an opportunistic campus mobile network.

rate λ_{ij}, i.e., contacts between nodes i and j occur according to a Poisson process with rate parameter λ_{ij}, where $i, j \in N$ and $i \neq j$ and $\lambda_{ji} = \lambda_{ij}$. The pairwise inter-contact times between any two node pairs are also mutually independent. In this model, the heterogeneity in mobile nodes' contact dynamics is captured by different contact rates λ_{ij}.

If $\lambda_{ij} = \lambda$ for all i, j, then the individual model reduces to the homogeneous network model (a.k.a. Poisson contact model) in which contacts between *any pair* of mobile nodes occur according to a Poisson process with same rate parameter λ. This heterogeneous model also captures a social community structure [26] as a special case. Suppose that there are K different social groups G_i ($i = 1, \dots, K$) forming a partition of $\mathcal{N}$, i.e., $\mathcal{N} = \bigcup_{i=1}^{K} G_i$. Let λ'_{lk} be common contact rate between any member of G_l and another member of G_k for $l, k = 1, \dots, K$. That is, $\lambda_{ij} = \lambda'_{lk}$ for all $i \in G_l$ and $j \in G_k$ where $l, k = 1, \dots, K$. Fig. 2(a) shows an example with $K = 2$.

The individual model has been validated in [17], [26]. They independently show through statistical methods that empirical pairwise inter-contact time distributions, obtained in real mobility traces, for a large portion of node pairs can be well fitted by exponential distributions but with different rates.



(a) A case of two social groups (b) A case of two spatial clusters

Fig. 2. Examples of the individual and spatial models.

2.2 Spatially Heterogeneous Network Model

A spatially heterogeneous network model (simply, a spatial model) was introduced in [19] and also similarly used in [28]. This spatial model is formally described below.

Consider a set of mobile nodes $\mathcal{N}$ in the network. There are M different spatial clusters (or preferred sites). Let S_i be a spatial cluster (site) i, where $i = 1, \dots, M$. Then, each mobile node independently moves between sites and encounters with others within each site as follows:

(i) Each mobile node in site S_i moves to site S_j with rate q_{ij} at any time t.
(ii) Any pair of mobile nodes in site S_i has Poisson contacts with rate β_i, i.e. the inter-contact time for a node pair in site S_i is independently and exponentially distributed with mean $1/\beta_i$. The pairwise inter-contact times between any two node pairs in site S_i are also mutually independent. Fig. 2(b) depicts an example with $M = 2$.

Let $X(t) \in \{S_1, \dots, S_M\} \triangleq \Omega$ be the site that a mobile node belongs to at time t. From the condition (i), $\{X(t)\}_{t \geq 0}$ is a continuous time Markov chain with transition rate matrix (or infinitesimal generator) $\mathbf{Q} = \{q_{ij}\}$. We assume $\{X(t)\}$ is irreducible, i.e., any mobile node can reach everywhere in finite time with positive probability. For analytical simplicity, we also assume that $q_{ij} = q_{ji}$, i.e., the transition rates of mobile nodes between sites S_i and S_j are the same.

The spatial model was justified in [19]. In particular, the condition (ii) is supported in [19] by empirically observing that 90% of all the inter-contacts gathered in a confined area, a subset of whole network domain, approximately follows an exponential distribution but with different rates over different subsets. The Poisson contacts over a small confined area has been also theoretically justified in [10], regardless of the mobility pattern of each mobile node inside that small confined area. In this spatial model, the heterogeneity arises by allowing different contact rates β_i over different sites.

3 Inter-contact Time under Heterogeneous Network Models

In this section, we show that each heterogeneous network model can capture non-Poisson contact dynamics as observed in real traces. For notational simplicity, we enumerate each of node pairs and define an index set for the node pair as $\mathcal{I} = \{1, 2, \ldots, |\mathcal{N}|(|\mathcal{N}| - 1)/2\}$. We also define by I a random variable to indicate a random node pair, which is uniformly distributed over $\mathcal{I}$. Further, we define by T_I and T_i the *aggregate* inter-contact time over all node pairs and *pairwise* inter-contact time for a given node pair $i \in \mathcal{I}$, respectively. Here, the aggregate inter-contact time distribution can be obtained by randomizing the pairwise inter-contact time distributions over all node pairs, i.e.,

$$\mathbb{P}\{T_I > t\} = \mathbb{E}\{\mathbb{P}\{T_I > t | I\}\} = \sum_{i \in \mathcal{I}} \mathbb{P}\{T_i > t\} \frac{1}{|\mathcal{I}|}.$$

We will use different superscripts 'IN','SP', and 'HO' to distinguish T_I and T_i for the individual, spatial, and homogeneous models, respectively.

From the definition of the individual model, it follows that

$$\mathbb{P}\{T_i^{\mathsf{IN}} > t\} = e^{-\lambda_i t}, \text{ and } \mathbb{P}\{T_I^{\mathsf{IN}} > t\} = \sum_{i \in \mathcal{I}} e^{-\lambda_i t} \frac{1}{|\mathcal{I}|},$$

where λ_i is the contact rate of a given node pair $i \in \mathcal{I}$. We can rewrite this as

$$\mathbb{P}\{T_I^{\mathsf{IN}} > t\} = \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\}, \tag{1}$$

where X_{IN} is a discrete random variable taking values $\frac{1}{\lambda_i}$ with probability $\frac{1}{|\mathcal{I}|}$. Note that the actual distribution of X_{IN} can be quite general by suitably setting λ_i.[3]

For the spatial model, we have the following results:

Proposition 1: For the spatial heterogeneous model as defined earlier, we have for any $i \in \mathcal{I}$,

$$\mathbb{P}\{T_i^{\mathsf{SP}} > t\} = \mathbb{P}\{T_I^{\mathsf{SP}} > t\} = \mathbb{E}\{e^{-t/X_{\mathsf{SP}}}\}, \tag{2}$$

for some positive random variable X_{SP}. □

Proof: See Section 8.1. □

Proposition 1 says that the inter-contact time for spatial model follows a hyper-exponential distribution. Here, the random variable X_{SP} depends on $\mathbf{Q}$ and β_i. We refer to the proof of Proposition 1 for more details. From (1)–(2) and by noting that $\mathbb{E}\{T\} = \int_0^\infty \mathbb{P}\{T > t\}dt$, we have

$$\mathbb{E}\{T_I^{\mathsf{IN}}\} = \mathbb{E}\{X_{\mathsf{IN}}\} = \sum_{i \in \mathcal{I}} \frac{1}{\lambda_i} \frac{1}{|\mathcal{I}|}, \text{ and } \mathbb{E}\{T_I^{\mathsf{SP}}\} = \mathbb{E}\{X_{\mathsf{SP}}\}.$$

It was addressed in [31] how to approximate a power-law (heavy-tail) distribution in the regions of *primary interest* by a mixture of exponentials while the approximated distribution still has an exponential tail. This implies that the observed 'dichotomic' inter-contact time distribution with power-law and exponential mixture [9] can be approximated by hyper-exponential distributions within any desired degree of accuracy. Note that the 'dichotomic' inter-contact time distribution is mainly obtained from aggregate inter-contact time samples. Since the aggregate inter-contact time distribution under both the individual and spatial models is a form of hyper-exponential distributions as shown in (1)–(2), both models can capture this non-exponential inter-contact time behavior.

Throughout the rest of this paper, we focus on the stochastic comparison of message delivery delays for direct forwarding and multicopy two-hop relay protocol under the individual, spatial, and corresponding homogeneous models. By the homogeneous model, we hereafter mean that for all $i \in \mathcal{I}$,

$$\mathbb{P}\{T_i^{\mathsf{HO}} > t\} = \mathbb{P}\{T_I^{\mathsf{HO}} > t\} = e^{-t/\tau}, \tag{3}$$

where $\tau = \mathbb{E}\{X_{\mathsf{IN}}\} = \mathbb{E}\{X_{\mathsf{SP}}\}$, unless otherwise specified. Thus, under the constructed homogeneous model, the inter-contact time of any pair of nodes is exponentially distributed (thus giving Poisson contacts) with the same *average* aggregate inter-contact time as the other heterogeneous models.

Throughout the rest of this paper, we assume the followings as in other analytical works [4], [7], [29], [30], [25], [19]. First, the network is sparse and network traffic is light. We also assume that each node has no resource constraint, i.e., it has infinite bandwidth and buffer. Hence, the interference and contention incurred during message transfers are not primary factors that govern the forwarding performance. In addition, we assume that a message transfer between any two nodes at their contact instant takes negligible time with respect to their inter-contact time. Finally, we consider the delay performance of a source and destination pair which is uniformly chosen over $\mathcal{I}$ unless specified.

4 Delay Performance of Direct Forwarding

In this section, we first stochastically compare the delay performance of direct forwarding (i.e., a source node waits until it meets a destination node to deliver a message) under the three models. Although the direct forwarding is very simple and there certainly exist other algorithms with better performance, its performance can serve as a basis for performance evaluation or prediction of two-hop or multi-hop forwarding algorithms. Let $D_{\mathsf{IN}}^{[1]}$, $D_{\mathsf{SP}}^{[1]}$, and $D_{\mathsf{HO}}^{[1]}$ be the message delivery delay of direct forwarding under the individual, spatial, and homogeneous models, respectively.

To proceed, we need the following definitions for the stochastic and convex orderings between two random variables Y and Z, denoted by $Y \geq_{(\cdot)} Z$, if $\mathbb{E}\{\phi(Y)\} \geq \mathbb{E}\{\phi(Z)\}$ for a class of functions ϕ for which the expectation exists.

Definition 1: [32] Y is said to be larger than Z *in the usual stochastic order* (denoted by $Y \geq_{st} Z$) if $\mathbb{E}\{\phi(Y)\} \geq \mathbb{E}\{\phi(Z)\}$ holds for any increasing function ϕ, or equivalently if $\mathbb{P}\{Y > u\} \geq \mathbb{P}\{Z > u\}$ for all $u \in \mathbb{R}$. □

3. For example, setting $\lambda_1 = \lambda_2 \neq \lambda_i$ for $i \geq 3$ will give non-uniform distribution while setting $\lambda_i = \lambda$ for all i makes $X_{\mathsf{IN}} = 1/\lambda$, for which the aggregate inter-contact time follows a pure exponential distribution.

Definition 2: [32] We define a *convex* (resp. *concave*) order, written $Y \geq_{cx} Z$ (resp. $Y \geq_{cv} Z$), if $\mathbb{E}\{\phi(Y)\} \geq \mathbb{E}\{\phi(Z)\}$ holds for any convex (resp. concave) function ϕ. Similarly, we also define an *increasing convex* (resp. *increasing concave*) order, written $Y \geq_{icx} Z$ (resp. $Y \geq_{icv} Z$), if $\mathbb{E}\{\phi(Y)\} \geq \mathbb{E}\{\phi(Z)\}$ holds for any increasing[4] convex (resp. increasing concave) function ϕ. □

From Definitions 1–2, one can easily establish the following implications. If $Y \geq_{st} Z$, then $Y \geq_{icx} Z$ and $Y \geq_{icv} Z$. Similarly, if $Y \geq_{cx} Z$ (resp. $Y \geq_{cv} Z$), then $Y \geq_{icx} Z$ (resp. $Y \geq_{icv} Z$). Also, by noting that ϕ is concave if $-\phi$ is convex, from Definition 2, $Y \geq_{cx} Z$ implies $Y \leq_{cv} Z$. Moreover, by Definition 1, if $Y \geq_{st} Z$, then $\mathbb{E}\{Y\} \geq \mathbb{E}\{Z\}$, while from Definition 2, if $Y \geq_{cx} Z$, then $\mathbb{E}\{Y\} = \mathbb{E}\{Z\}$ and $\mathrm{Var}\{Y\} \geq \mathrm{Var}\{Z\}$ by taking $\phi(\cdot) = (\cdot)^2$.

The message delivery delay for a given source and destination pair is nothing but their residual (or remaining) inter-contact time after the message is generated at the source node. First, for the homogeneous model, we have

$$\mathbb{P}\{D^{[1]}_{\mathsf{HO}} > t\} = \mathbb{P}\{T^{\mathsf{HO}}_I > t\} = e^{-t/\tau}$$

due to the memoryless property of the exponential inter-contact time distribution with mean τ for any pair of nodes. Similarly, for the individual model,

$$\mathbb{P}\{D^{[1]}_{\mathsf{IN}} > t | I = i\} = \mathbb{P}\{T^{\mathsf{IN}}_I > t | I = i\} = e^{-\lambda_i t}$$

for a given pair $i \in \mathcal{I}$, thus from (1), we have

$$\begin{aligned}\mathbb{P}\{D^{[1]}_{\mathsf{IN}} > t\} &= \mathbb{E}\{\mathbb{P}\{D^{[1]}_{\mathsf{IN}} > t | I\}\} = \mathbb{P}\{T^{\mathsf{IN}}_I > t\} \\ &= \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\}.\end{aligned}$$

However, for the spatial model, the inter-contact time of a given pair $i \in \mathcal{I}$ is no longer memoryless but of hyper-exponential form as in (2). Under stationary regime, note that the residual inter-contact time R_i of a pair $i \in \mathcal{I}$ follows the equilibrium distribution of T_i [33], [29], [21], i.e., $\mathbb{P}\{R_i > t\} = \frac{1}{\mathbb{E}\{T_i\}} \int_t^\infty \mathbb{P}\{T_i > u\} du$. Then, from (2), we can write for any $i \in \mathcal{I}$,

$$\begin{aligned}\mathbb{P}\{R_i > t\} &= \frac{1}{\mathbb{E}\{T_i\}} \int_t^\infty \mathbb{E}\{e^{-u/X}\} du \\ &= \frac{1}{\mathbb{E}\{T_i\}} \mathbb{E}\left\{\int_t^\infty e^{-u/X} du\right\} = \frac{1}{\mathbb{E}\{X\}} \mathbb{E}\{X e^{-t/X}\}, \quad (4)\end{aligned}$$

where T_i and X here represent T^{SP}_i and X_{SP} for the spatial model, respectively. Since (4) holds for any $i \in \mathcal{I}$, we have

$$\mathbb{P}\{D^{[1]}_{\mathsf{SP}} > t\} = \mathbb{P}\{R_i > t\} = \frac{1}{\mathbb{E}\{X_{\mathsf{SP}}\}} \mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\}.$$

Now, we present our results for stochastic comparison on the delay performance of direct forwarding under the individual, spatial, and homogeneous models.

Proposition 2: Let $X_{\mathsf{IN}1}$, $X_{\mathsf{IN}2}$ be random variables in (1) for two different scenarios under the individual model, and $D^{[1]}_{\mathsf{IN}1}$, $D^{[1]}_{\mathsf{IN}2}$ be the corresponding message delivery delays of direct forwarding. Then, if $X_{\mathsf{IN}1} \geq_{cx} X_{\mathsf{IN}2}$, we have $D^{[1]}_{\mathsf{IN}1} \geq_{cx} D^{[1]}_{\mathsf{IN}2}$. □

Proof: By noting that $\mathbb{P}\{D^{[1]}_{\mathsf{IN}} > t\} = \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\}$ and $\mathbb{E}\{X_{\mathsf{IN}1}\} = \mathbb{E}\{X_{\mathsf{IN}2}\}$, we have $\mathbb{E}\{D^{[1]}_{\mathsf{IN}1}\} = \mathbb{E}\{D^{[1]}_{\mathsf{IN}2}\}$. Thus, in order to prove $D^{[1]}_{\mathsf{IN}1} \geq_{cx} D^{[1]}_{\mathsf{IN}2}$, it is enough to show that $\int_a^\infty \mathbb{P}\{D^{[1]}_{\mathsf{IN}1} > t\} dt \geq \int_a^\infty \mathbb{P}\{D^{[1]}_{\mathsf{IN}2} > t\} dt$ for all $a > 0$ [32]. It is equivalent to showing that

$$\mathbb{E}\{X_{\mathsf{IN}1} \cdot e^{-a/X_{\mathsf{IN}1}}\} \geq \mathbb{E}\{X_{\mathsf{IN}2} \cdot e^{-a/X_{\mathsf{IN}2}}\}, \quad (5)$$

for all $a > 0$.

Let $g(x) \triangleq x e^{-a/x}$. It is easy to see that $g(x)$ is a convex function of $x > 0$ for all $a > 0$. Thus, from $X_{\mathsf{IN}1} \geq_{cx} X_{\mathsf{IN}2}$ and Definition 2, the above inequality (5) holds by taking $\phi(x) = x e^{-a/x}$. This completes the proof. □

Proposition 2 says the message delivery delay gets larger in the sense of convex order, as the underlying individual model becomes 'more heterogeneous' (in larger convex ordering of X). In particular, if $\mathbb{E}\{T^{\mathsf{IN}}_I\} = \mathbb{E}\{T^{\mathsf{HO}}_I\} = \tau$ (the same average aggregated inter-contact time under the individual and homogeneous models), we have

$$D^{[1]}_{\mathsf{IN}} \geq_{cx} D^{[1]}_{\mathsf{HO}},$$

since $X_{\mathsf{IN}} \geq_{cx} \mathbb{E}\{X_{\mathsf{IN}}\} = \mathbb{E}\{T^{\mathsf{IN}}_I\} = \tau$. This means that the message delivery delay of direct forwarding under the individual model is *more variable* than that under the homogeneous model, while the average delays under both models are the same.

Proposition 3: If $\mathbb{E}\{T^{\mathsf{SP}}_I\} = \mathbb{E}\{T^{\mathsf{HO}}_I\}$, then $D^{[1]}_{\mathsf{SP}} \geq_{st} D^{[1]}_{\mathsf{HO}}$. □

Proof: Recall that $\mathbb{P}\{D^{[1]}_{\mathsf{HO}} > t\} = e^{-t/\tau}$ and

$$\mathbb{P}\{D^{[1]}_{\mathsf{SP}} > t\} = \frac{1}{\mathbb{E}\{X_{\mathsf{SP}}\}} \mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\}.$$

Note also that $X_{\mathsf{SP}} \geq_{cx} \mathbb{E}\{X_{\mathsf{SP}}\} = \mathbb{E}\{T^{\mathsf{SP}}_I\} = \tau$. Let X_{HO} be a random variable that takes the value $\tau = \mathbb{E}\{X_{\mathsf{SP}}\}$ with probability 1. Then, we can write

$$\mathbb{P}\{D^{[1]}_{\mathsf{HO}} > t\} = \frac{1}{\mathbb{E}\{X_{\mathsf{HO}}\}} \mathbb{E}\{X_{\mathsf{HO}} \cdot e^{-t/X_{\mathsf{HO}}}\}.$$

Hence, since $X_{\mathsf{SP}} \geq_{cx} X_{\mathsf{HO}}$ and $x e^{-t/x}$ is a convex function of $x > 0$ for all $t > 0$, from Definition 2, we have

$$\mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\} \geq \mathbb{E}\{X_{\mathsf{HO}} \cdot e^{-t/X_{\mathsf{HO}}}\}$$

for all $t > 0$. Then, by noting that $\mathbb{E}\{X_{\mathsf{SP}}\} = \mathbb{E}\{X_{\mathsf{HO}}\}$, it follows that $\mathbb{P}\{D^{[1]}_{\mathsf{SP}} > t\} \geq \mathbb{P}\{D^{[1]}_{\mathsf{HO}} > t\}$ for all $t > 0$. From Definition 1, the result follows. □

Proposition 3 says that the message delivery delay of direct forwarding under the spatial model is stochastically larger than that under the homogeneous model, when the average inter-contact time under both models are matched. From Propositions 2 and 3, we see that the delay performance of direct forwarding under each

4. Here, 'increasing' means non-decreasing.

heterogeneous model deviates from that under the homogeneous model in a different manner, though three models are the same in the average aggregate inter-contact time point of view.

Next, we compare the delay performance of direct forwarding under the spatial and individual models, when their *entire distributions* of the aggregate inter-contact time remain identical. This can be achieved by setting $X_{\mathsf{SP}} \overset{d}{=} X_{\mathsf{IN}}$ in (1)–(2). Still, our next result tells us that the delay of direct forwarding under the spatial model is always stochastically larger than that under the individual model.

Proposition 4: If $T_I^{\mathsf{SP}} \overset{d}{=} T_I^{\mathsf{IN}}$, then $D_{\mathsf{SP}}^{[1]} \geq_{st} D_{\mathsf{IN}}^{[1]}$. □

Proof: Recall that $\mathbb{P}\{D_{\mathsf{IN}}^{[1]} > t\} = \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\}$ and

$$\mathbb{P}\{D_{\mathsf{SP}}^{[1]} > t\} = \frac{1}{\mathbb{E}\{X_{\mathsf{SP}}\}}\mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\}.$$

Since $e^{-t/x}$ is increasing in $x > 0$ for any given $t > 0$, we have

$$\mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\} \geq \mathbb{E}\{X_{\mathsf{SP}}\} \cdot \mathbb{E}\{e^{-t/X_{\mathsf{SP}}}\}. \qquad (6)$$

Then, from the assumption that

$$\mathbb{P}\{T_I^{\mathsf{SP}} > t\} = \mathbb{E}\{e^{-t/X_{\mathsf{SP}}}\} = \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\} = \mathbb{P}\{T_I^{\mathsf{IN}} > t\}$$

for any given $t > 0$, and from (6) we have

$$\begin{aligned}\mathbb{P}\{D_{\mathsf{SP}}^{[1]} > t\} &= \frac{1}{\mathbb{E}\{X_{\mathsf{SP}}\}}\mathbb{E}\{X_{\mathsf{SP}} \cdot e^{-t/X_{\mathsf{SP}}}\} \\ &\geq \mathbb{E}\{e^{-t/X_{\mathsf{SP}}}\} = \mathbb{E}\{e^{-t/X_{\mathsf{IN}}}\} = \mathbb{P}\{D_{\mathsf{IN}}^{[1]} > t\},\end{aligned}$$

for all $t > 0$. From Definition 1, the result follows. □

To sum up, from Propositions 2–4, we observe that the performance of direct forwarding varies depending on which of the two heterogeneous models is chosen, i.e., how the non-Poisson contact dynamics observed in the real traces are modeled. In addition, the aggregated inter-contact time statistics (the whole distribution) are still insufficient to correctly predict the forwarding performance, even though many existing works [33], [9], [19] have relied on the aggregated inter-contact time samples to uncover the characteristics of mobile nodes' contact patterns and justify their modeling choices.

5 Delay Performance of Multicopy Two-hop Relay Protocol

We now turn our attention to multicopy two-hop relay protocol [4], [7], [29], [30] as a test case for a further investigation of the impact of the heterogeneity structure on the forwarding performance. In this protocol, only source node can replicate a message and forward its copy to any relay node that does not have the message copy upon encounter.

Consider the delivery of a single message in the network with $|\mathcal{N}| = n + 2$. Given a pair of source s and destination d which is uniformly chosen over $\mathcal{I}$, there are n possible relay nodes $(r_1, r_2, \ldots, r_n)$. We hereafter use T_{ij}, instead of T_l ($l \in \mathcal{I}$), to represent the pairwise inter-contact time of nodes i and j if needed to specify nodes (i, j) of each node pair, where $i, j \in \{s, r_1, \ldots, r_n, d\}$ and $i \neq j$. Similarly, R_{ij} stands for the residual inter-contact time of nodes i and j. Then, as shown in [29], [30], the message delivery delay of the multicopy two-hop relay protocol (denoted by D) – the time interval from the time when the message is generated at a source node to the time when any copy of the message first reaches its destination, is given by

$$D \overset{d}{=} \min\{R_{sd}, R_{sr_1} + R_{r_1 d}, \ldots, R_{sr_n} + R_{r_n d}\}. \qquad (7)$$

As before, $D_{\mathsf{IN}}^{[2]}$, $D_{\mathsf{SP}}^{[2]}$, and $D_{\mathsf{HO}}^{[2]}$ denote the message delivery delay of multicopy two-hop relay protocol under individual, spatial, and homogeneous models, respectively. Here, we use superscript $D^{[2]}$ to indicate the multicopy two-hop relay protocol, whereby $D^{[1]}$ was used for the direct forwarding (single-hop) protocol in Section 4.

We first show that the stochastic ordering relationship in Proposition 3 still holds for the message delay delays of multicopy two-hop relay protocol under the spatial and homogeneous models.

Proposition 5: If $\mathbb{E}\{T_I^{\mathsf{SP}}\} = \mathbb{E}\{T_I^{\mathsf{HO}}\}$, then $D_{\mathsf{SP}}^{[2]} \geq_{st} D_{\mathsf{HO}}^{[2]}$. □

Proof: Let R_{ij}^{SP} and R_{ij}^{HO} be the residual inter-contact time of a given node pair (i, j) under the spatial and homogeneous models, respectively. From Proposition 3, we have $R_{ij}^{\mathsf{SP}} \geq_{st} R_{ij}^{\mathsf{HO}}$. From the independence of R_{ij}^{SP} and R_{ij}^{HO} over different node (i, j) pairs, the stochastic order is also closed under convolutions [32]. Thus, $R_{sd}^{\mathsf{SP}} \geq_{st} R_{sd}^{\mathsf{HO}}$ and $R_{sr_i}^{\mathsf{SP}} + R_{r_i d}^{\mathsf{SP}} \geq_{st} R_{sr_i}^{\mathsf{HO}} + R_{r_i d}^{\mathsf{HO}}$ ($i = 1, 2, \ldots, n$). Then, it easily follows that these stochastic ordering relationships still hold for their first order statistic, i.e.,

$$\begin{aligned}&\min\{R_{sd}^{\mathsf{SP}}, R_{sr_1}^{\mathsf{SP}} + R_{r_1 d}^{\mathsf{SP}}, \ldots, R_{sr_n}^{\mathsf{SP}} + R_{r_n d}^{\mathsf{SP}}\} \\ &\quad \geq_{st} \min\{R_{sd}^{\mathsf{HO}}, R_{sr_1}^{\mathsf{HO}} + R_{r_1 d}^{\mathsf{HO}}, \ldots, R_{sr_n}^{\mathsf{HO}} + R_{r_n d}^{\mathsf{HO}}\}.\end{aligned}$$

That is, $D_{\mathsf{SP}}^{[2]} \geq_{st} D_{\mathsf{HO}}^{[2]}$, which completes the proof. □

Proposition 5 also implies that the hyper-exponential inter-contact time yields stochastically larger delay than the exponential inter-contact time for the mulicopy two-hop relay protocol when their average inter-contact times are matched.

Next, we show the stochastic comparison for the delays of multicopy two-hop relay protocol under the individual and homogeneous models. Specifically, we first compare the delay performance for *a given source and destination pair* under the individual and homogeneous models, as the pairwise inter-contact times are statistically different for different node pairs under the individual model unlike to the spatial and homogeneous models. Later on, we will continue our stochastic comparison on the delay performance for *a uniformly and randomly chosen source and destination pair* under both models. In this stochastic comparison, we assume that each message reaches its destination via relay nodes only and the direct path from source to destination is not considered. This may be the case with a moderate to large number of mobile nodes (e.g., campus-wide MONs), i.e., the 'best' of n relay nodes is likely to reach the destination earlier than the source node does.

Let $D^{[2]}_{\mathsf{IN}(s,d)}$ be the message delivery delay for a given source and destination (s,d) pair under the individual model. Here, for a proper comparison between $D^{[2]}_{\mathsf{IN}(s,d)}$ and $D^{[2]}_{\mathsf{HO}}$, we set the average inter-contact time for the corresponding homogeneous model as

$$\tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right]. \tag{8}$$

That is, the average inter-contact time for any node pair under the constructed homogeneous model is simply the arithmetic mean of the average inter-contact times over all node pairs in n two-hop relay paths under the individual model. Let T^{IN}_{ij} and T^{HO}_{ij} be the inter-contact time of each node pair (i,j) under the individual and homogeneous models, respectively. Then, due to the memoryless property of exponential pairwise inter-contact time distributions under the individual and homogeneous models, $D^{[2]}_{\mathsf{IN}(s,d)}$ and $D^{[2]}_{\mathsf{HO}}$ are given by

$$D^{[2]}_{\mathsf{IN}(s,d)} = \min\{T^{\mathsf{IN}}_{sr_1} + T^{\mathsf{IN}}_{r_1d}, \dots, T^{\mathsf{IN}}_{sr_n} + T^{\mathsf{IN}}_{r_nd}\}, \tag{9}$$

$$D^{[2]}_{\mathsf{HO}} = \min\{T^{\mathsf{HO}}_{sr_1} + T^{\mathsf{HO}}_{r_1d}, \dots, T^{\mathsf{HO}}_{sr_n} + T^{\mathsf{HO}}_{r_nd}\}. \tag{10}$$

Instead of directly comparing $D^{[2]}_{\mathsf{IN}(s,d)}$ with $D^{[2]}_{\mathsf{HO}}$, we compare *each* of these message delivery delays with that under a *partially homogeneous* setting (a special case of the individual model). Fig. 3(b) shows this partially homogeneous setting in which the delay over each relay path is now a sum of two *i.i.d.* exponential random variables with mean $\frac{1}{2}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$ (homogeneous for a given path, but heterogeneous over different paths). Let S_{sr_i} and S_{r_id} be *i.i.d.* exponential random variables with mean

$$\frac{1}{\mu_i} \triangleq \frac{1}{2}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right], \tag{11}$$

where $i = 1, \dots, n$. Then, the message delivery delay in this partially homogeneous model, $\tilde{D}^{[2]}_{\mathsf{IN}(s,d)}$, is given by

$$\tilde{D}^{[2]}_{\mathsf{IN}(s,d)} = \min\{S_{sr_1} + S_{r_1d}, \dots, S_{sr_n} + S_{r_nd}\}. \tag{12}$$

Fig. 3 depicts the aforementioned three different settings of n two-hop relay paths with varying degrees of heterogeneity over the average inter-contact times in the network.

To proceed, we collect several definitions on majorization [34] ordering. This is a partial order over vectors of real numbers and is useful in capturing the degree of heterogeneity in vector components.

Definition 3: [34] For $\vec{y}, \vec{z} \in \mathbb{R}^n$, $\vec{y}$ is said to be *majorized* by $\vec{z}$, or $\vec{z}$ majorizes $\vec{y}$, (written $\vec{y} \prec \vec{z}$), if $\sum_{i=1}^{m} y_{[i]} \le \sum_{i=1}^{m} z_{[i]}$, $(m = 1, 2, \dots, n-1)$, and $\sum_{i=1}^{n} y_{[i]} = \sum_{i=1}^{n} z_{[i]}$, where $y_{[1]} \ge y_{[2]} \ge \dots \ge y_{[n]}$ $(z_{[1]} \ge z_{[2]} \ge \dots \ge z_{[n]})$ denote the components of $\vec{y}$ (resp. $\vec{z}$) in decreasing order. □

From (11) and Definition 3, we have

$$\left(\frac{1}{\lambda_{sr_i}}, \frac{1}{\lambda_{r_id}}\right) \succ \left(\frac{1}{\mu_i}, \frac{1}{\mu_i}\right) \tag{13}$$

for any $\lambda_{sr_i}, \lambda_{r_id} > 0$, and $(1/\mu_i, 1/\mu_i)$ is the smallest in the sense of majorization ordering. Further, note that from (8) and (11),

$$\tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right] = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mu_i}.$$

This implies that

$$\left(\frac{1}{\mu_1}, \dots, \frac{1}{\mu_n}\right) \succ (\tau, \dots, \tau) \tag{14}$$

for any $\mu_i > 0$.

Definition 4: [34] For $\vec{y}, \vec{z} \in \mathbb{R}^n$, a real-valued function ψ defined on $\mathbb{R}^n$ is said to be *Schur-convex*, if $\vec{y} \prec \vec{z}$ implies $\psi(\vec{y}) \le \psi(\vec{z})$. Similarly, ψ is said to be *Schur-concave*, if $\vec{y} \prec \vec{z}$ implies $\psi(\vec{y}) \ge \psi(\vec{z})$. □

We also need the following result on the preservation of the increasing concave ordering.

Proposition 6: [35, Proposition 9.5.4] If $Y_1, \dots, Y_n$ are independent random variables and $Z_1, \dots, Z_n$ are independent random variables, and $Y_i \ge_{icv} Z_i$ for each $i = 1, \dots, n$, then $f(Y_1, \dots, Y_n) \ge_{icv} f(Z_1, \dots, Z_n)$ for all increasing and componentwise concave function f. □

Now we present our main result on the stochastic comparison among $D^{[2]}_{\mathsf{IN}(s,d)}$, $\tilde{D}^{[2]}_{\mathsf{IN}(s,d)}$, $D^{[2]}_{\mathsf{HO}}$ – the message delivery delay of multicopy two-hop relay protocol over the network setting in Fig. 3(a), (b), (c), respectively.

Theorem 1: If $\mathbb{E}\{T^{\mathsf{HO}}_{I}\} = \tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right]$, then $D^{[2]}_{\mathsf{IN}(s,d)} \le_{icv} \tilde{D}^{[2]}_{\mathsf{IN}(s,d)} \le_{st} D^{[2]}_{\mathsf{HO}}$. □

Proof: **(A) Proof of $D^{[2]}_{\mathsf{IN}(s,d)} \le_{icv} \tilde{D}^{[2]}_{\mathsf{IN}(s,d)}$:** Let U_1 and U_2 be *i.i.d.* exponential random variables with rate one. Then, observe that

$$T^{\mathsf{IN}}_{sr_i} \overset{d}{=} \frac{1}{\lambda_{sr_i}}U_1, \text{ and } T^{\mathsf{IN}}_{r_id} \overset{d}{=} \frac{1}{\lambda_{r_id}}U_2.$$

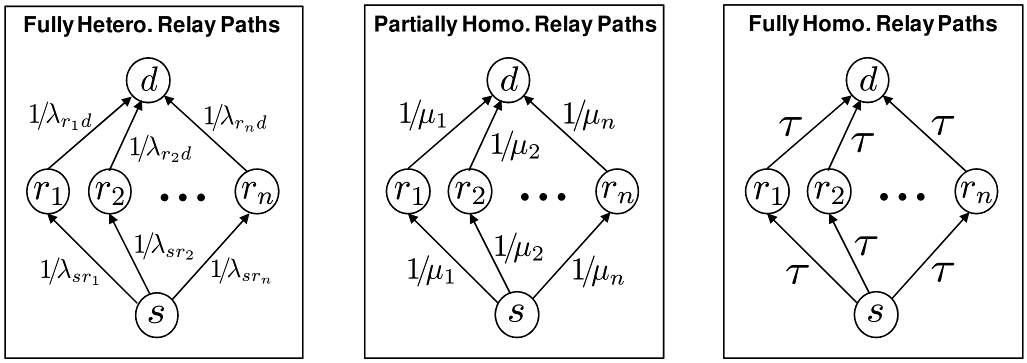
Similarly, we have

$$S_{sr_i} \overset{d}{=} \frac{1}{\mu_i}U_1, \text{ and } S_{r_id} \overset{d}{=} \frac{1}{\mu_i}U_2.$$

Thus, from the independence of $T^{\mathsf{IN}}_{sr_i}$ and $T^{\mathsf{IN}}_{r_id}$ and the independence of S_{sr_i} and S_{r_id}, we have

$$\begin{aligned} T^{\mathsf{IN}}_{sr_i} + T^{\mathsf{IN}}_{r_id} &\overset{d}{=} \frac{1}{\lambda_{sr_i}}U_1 + \frac{1}{\lambda_{r_id}}U_2, \\ S_{sr_i} + S_{r_id} &\overset{d}{=} \frac{1}{\mu_i}U_1 + \frac{1}{\mu_i}U_2. \end{aligned} \tag{15}$$

Note that if $Y_1, \dots, Y_n$ are exchangeable random variables, then $\psi(\vec{a}) = \mathbb{E}\{f(\sum a_iY_i)\}$ is Schur-convex on $\mathbb{R}^n$



(a) (b) (c)

Fig. 3. Three different settings of n two-hop relay paths with varying degrees of heterogeneity: (a) a fully heterogeneous setting, (b) a partially homogeneous setting where each relay path is homogeneous. (i.e., two-hop components in each relay path have the same average inter-contact time as $1/\mu_i = \frac{1}{2}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$), and (c) a fully homogeneous setting where the average inter-contact time for any node pair is $\tau = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$.

for any convex function f [34, p.287, Proposition B.2]. Thus, from (13), (15), and Definition 4, we have,

$$\begin{aligned}\mathbb{E}\{f(T^{\mathsf{IN}}_{sr_i} + T^{\mathsf{IN}}_{r_id})\} &= \mathbb{E}\left\{f\left(\frac{1}{\lambda_{sr_i}}U_1 + \frac{1}{\lambda_{r_id}}U_2\right)\right\} \\ &\geq \mathbb{E}\left\{f\left(\frac{1}{\mu_i}U_1 + \frac{1}{\mu_i}U_2\right)\right\} \\ &= \mathbb{E}\{f(S_{sr_i} + S_{r_id})\},\end{aligned}$$

for any convex function f. Equivalently, from Definition 2, we have $T^{\mathsf{IN}}_{sr_i} + T^{\mathsf{IN}}_{r_id} \geq_{cx} S_{sr_i} + S_{r_id}$ for each $i = 1, \ldots, n$. As mentioned in Section 4, it follows that

$$\begin{aligned}T^{\mathsf{IN}}_{sr_i}+T^{\mathsf{IN}}_{r_id} \geq_{cx} S_{sr_i}+S_{r_id} &\Rightarrow T^{\mathsf{IN}}_{sr_i}+T^{\mathsf{IN}}_{r_id} \leq_{cv} S_{sr_i}+S_{r_id} \\ &\Rightarrow T^{\mathsf{IN}}_{sr_i}+T^{\mathsf{IN}}_{r_id} \leq_{icv} S_{sr_i}+S_{r_id},\end{aligned}$$

for any i. Then, since $\min\{x_1, \ldots, x_n\}$ is increasing on $\mathbb{R}^n$ and concave in each argument x_i, from (9), (12), and Proposition 6, we have

$$D^{[2]}_{\mathsf{IN}(s,d)} \leq_{icv} \tilde{D}^{[2]}_{\mathsf{IN}(s,d)}. \tag{16}$$

(B) Proof of $\tilde{D}^{[2]}_{\mathsf{IN}(s,d)} \leq_{st} D^{[2]}_{\mathsf{HO}}$: Let $\nu_i \triangleq \frac{1}{\mu_i}$ and

$$g(\nu_i) \triangleq \mathbb{P}\{S_{sr_i} + S_{r_id} > t\} = \left(1 + \frac{t}{\nu_i}\right)e^{-t/\nu_i}$$

for any given $t > 0$. We also define another function by

$$h(\vec{\nu}) \triangleq \mathbb{P}\left\{\tilde{D}^{[2]}_{\mathsf{IN}(s,d)} > t\right\} = \prod_{i=1}^{n} g(\nu_i),$$

where $\vec{\nu} \triangleq (\nu_1, \nu_2, \ldots, \nu_n)$. It is straightforward to check $\log g(\nu_i)$ is concave in $\nu_i > 0$ for all $t > 0$. Then, since $g(\nu_i)$ is log-concave, $h(\vec{\nu}) = \prod g(\nu_i)$ is Schur-concave on $(0,\infty)^n$ [34, p.73, Proposition E.1]. Thus, from (10), (12), (14) and Definition 4, we have for any $\vec{\nu} \in (0,\infty)^n$,

$$\begin{aligned}\mathbb{P}\left\{\tilde{D}^{[2]}_{\mathsf{IN}(s,d)} > t\right\} &= h(\nu_1, \ldots, \nu_n) \\ &\leq h(\tau, \ldots, \tau) = \mathbb{P}\left\{D^{[2]}_{\mathsf{HO}} > t\right\},\end{aligned}$$

for any given $t > 0$. In other words, by Definition 1,

$$\tilde{D}^{[2]}_{\mathsf{IN}(s,d)} \leq_{st} D^{[2]}_{\mathsf{HO}}. \tag{17}$$

From (16) and (17), we are done. □

By noting that $\leq_{st} \Rightarrow \leq_{icv}$, Theorem 1 implies that if $\mathbb{E}\{T^{\mathsf{HO}}_I\} = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$, then

$$D^{[2]}_{\mathsf{IN}(s,d)} \leq_{icv} D^{[2]}_{\mathsf{HO}}.$$

Since $\phi(x) = x$ is increasing and concave, from Definition 2, it further implies

$$\mathbb{E}\{D^{[2]}_{\mathsf{IN}(s,d)}\} \leq \mathbb{E}\{D^{[2]}_{\mathsf{HO}}\}. \tag{18}$$

We now move on to the stochastic comparison on message delivery delays for a *uniform* source and destination pair. Note that the average message delivery delay of a uniform pair is nothing but the arithmetic mean of the average message delivery delays over all possible $|\mathcal{N}|(|\mathcal{N}| - 1)/2$ source and destination (s, d) pairs. Also, as in (18), for each (s, d) pair, if $\mathbb{E}\{T^{\mathsf{HO}}_I\} = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i}+1/\lambda_{r_id}]$, then $\mathbb{E}\{D^{[2]}_{\mathsf{HO}}\}$ becomes an upper bound of $\mathbb{E}\{D^{[2]}_{\mathsf{IN}(s,d)}\}$. Hence, after computing the arithmetic mean of the upper bounds of $\mathbb{E}\{D^{[2]}_{\mathsf{IN}(s,d)}\}$ over all (s, d) pairs, we obtain the following corollary.

Corollary 1: If $\mathbb{E}\{T^{\mathsf{HO}}_I\} = \mathbb{E}\{T^{\mathsf{IN}}_I\} = \sum_{i\in\mathcal{I}} \frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}$, then $\mathbb{E}\{D^{[2]}_{\mathsf{IN}}\} \leq \mathbb{E}\{D^{[2]}_{\mathsf{HO}}\}$. □

Proof: See Section 8.2. □

As shown in Theorem 1 and Corollary 1, the path diversity (heterogeneity) over n relay paths under the individual model results in better delay performance of multicopy two-hop relay protocol. It also turns out that Proposition 5 still holds under the same scenario (i.e., no direct path is used) considered in Corollary 1. Thus, under this scenario, if $\mathbb{E}\{T^{\mathsf{SP}}_I\} = \mathbb{E}\{T^{\mathsf{IN}}_I\} = \mathbb{E}\{T^{\mathsf{HO}}_I\}$, we have

$$\mathbb{E}\{D^{[2]}_{\mathsf{IN}}\} \leq \mathbb{E}\{D^{[2]}_{\mathsf{HO}}\} \leq \mathbb{E}\{D^{[2]}_{\mathsf{SP}}\}. \tag{19}$$

This means that the heterogeneity structure in the spatial model makes the average delay performance of multicopy two-hop relay protocol worse, whereas the other heterogeneity structure in the individual model is beneficial to its average delay performance when compared with that under the corresponding homogeneous model. In addition, even if the whole aggregate inter-contact time distribution under both the spatial and individual models remains the same, i.e., $T_I^{\mathsf{SP}} \overset{d}{=} T_I^{\mathsf{IN}}$, since $\mathbb{E}\{T_I^{\mathsf{SP}}\} = \mathbb{E}\{T_I^{\mathsf{IN}}\}$, it follows from (19) that

$$\mathbb{E}\{D_{\mathsf{IN}}^{[2]}\} \leq \mathbb{E}\{D_{\mathsf{SP}}^{[2]}\}.$$

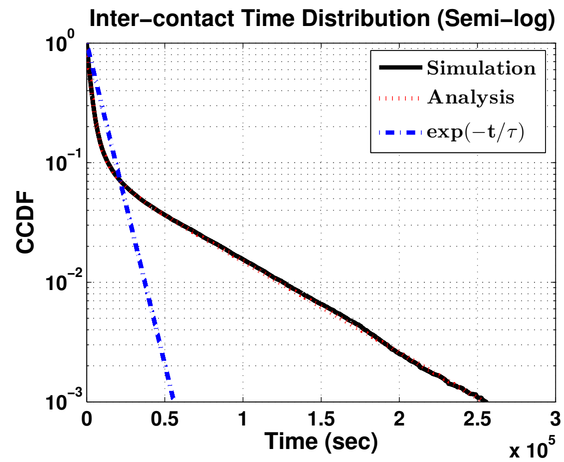
Along with Proposition 4, it clearly shows that the aggregate inter-contact time statistics are still not sufficient in accurately estimating the forwarding performance.

From our theoretical results, we expect that the delay performance of other two-hop or multi-hop forwarding protocols under each heterogeneous model differs considerably from that under the homogeneous model and there exists a significant performance gap between the two heterogeneous models as we observed, even when the entire aggregate inter-contact time distributions are precisely matched.

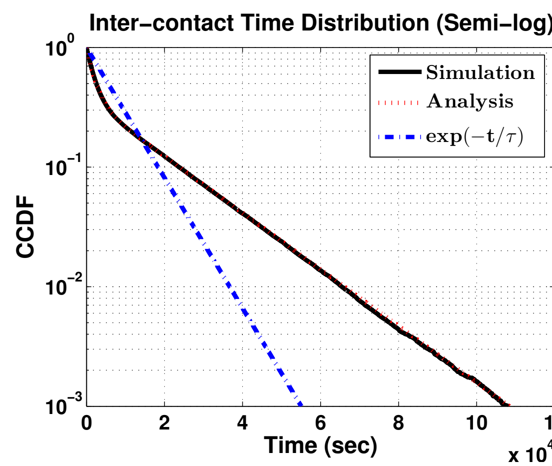
6 Simulation Results

In this section, we present simulation results on the average delay performance of multicopy two-hop relay protocol and epidemic routing protocol for a uniform source and destination pair under the individual, spatial, and homogeneous models, all of which are the same in the average inter-contact time of a random pair of nodes (the same average aggregate inter-contact time) to support our analytical findings. In the epidemic routing protocol [2], [4], [6], [7], [30], [24], a commonly used reference forwarding algorithm for MONs, every node can copy a message and forward its copy ('infect') to any other node that does not have the message already upon encounter. We use a custom event-driven simulator implemented using C++ to conduct numerical simulations.

Specifically, for the spatial model, we consider a two-states spatial model (a special case) where random contact events of each node pair occur according to a Poisson process with rate β_i only when two nodes of the pair reside in the same state (site) $i \in \{1, 2\}$, while every node can move between two states with transition rates q_{12}, q_{21}. In addition, we consider the following scenario for the individual model: all node pairs (total $|\mathcal{N}|(|\mathcal{N}|-1)/2$ pairs) are equally divided into 5 groups, in which the contact rate of any node pair in each group is the same, but different from that of the other group, though the inter-contact time distribution of each pair is still exponential. For the homogeneous model, by its definition, contact events of *any* node pair happen according to a Poisson process with same rate parameter. We below explain parameter settings which ensure the same average aggregate inter-contact time for all the three models.



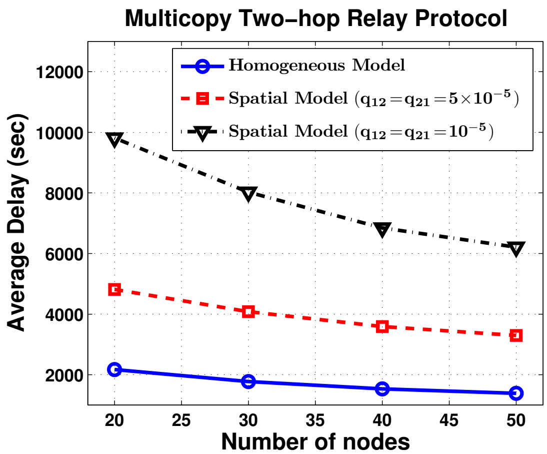
(a) $q_{12} = q_{21} = 10^{-5}$



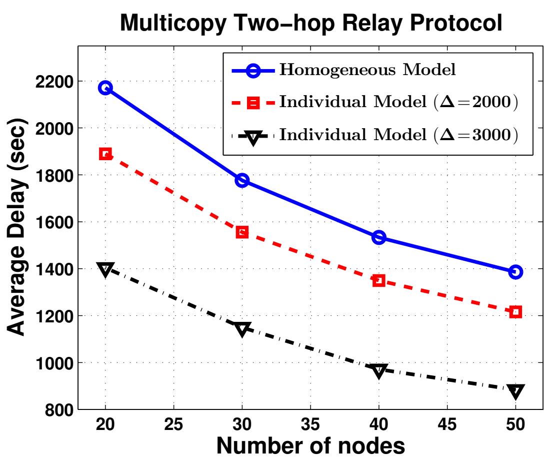
(b) $q_{12} = q_{21} = 5 \times 10^{-5}$

Fig. 4. Pairwise inter-contact time distribution under the two-states spatial model with varying q_{12} $(= q_{21})$. $\tau = 8000$ for both cases.

In the simulations of the two-states spatial model, we use $\beta_1 = 10^{-4}$ and $\beta_2 = 4 \times 10^{-4}$ for the contact rates associated with states S_1 and S_2, respectively, and consider two different cases of the transition rates q_{12} and q_{21}, i.e., $q_{12} = q_{21} = 10^{-5}$ and $q_{12} = q_{21} = 5 \times 10^{-5}$. We here present the complementary cumulative distribution function (CCDF) of the inter-contact time of a node pair on semi-log scale under the above parameter settings in Fig. 4. The graphs labeled 'simulation' are plotted based on inter-contact time samples of a node pair from numerical simulations, while the other graphs labeled 'analysis' are obtained from the derivation of pairwise inter-contact time distribution shown in the proof of Proposition 1 (i.e., numerical computation of (22) in Section 8.1). From Fig. 4, we can identify that the simulation results show a good agreement with the analysis of the pairwise inter-contact time distribution. In Fig. 4, an exponential distribution with mean τ, which is set to be the same average inter-contact time observed in each simulation, is also drawn to explicitly show that the

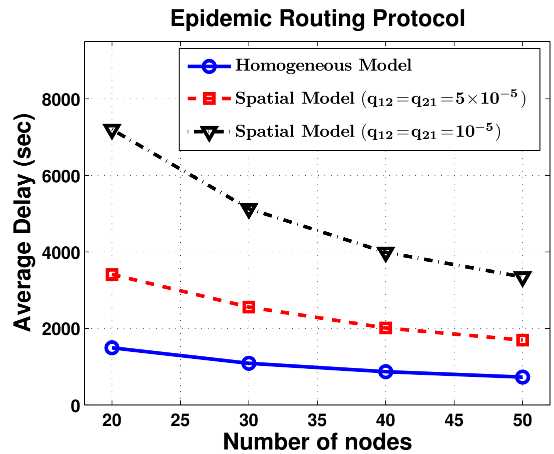


(a) Spatial model vs. homogeneous model

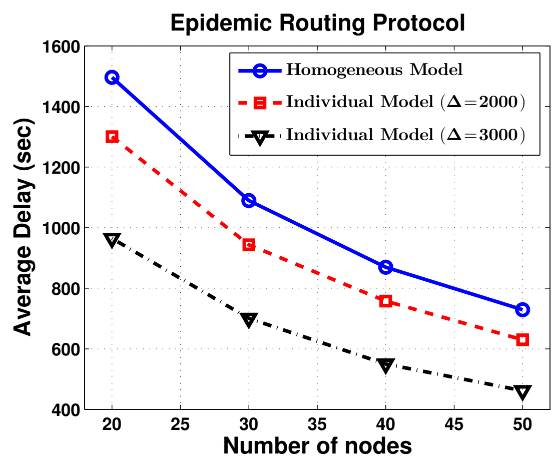


(b) Individual model vs. homogeneous model

Fig. 5. The average delay of multicopy two-hop relay protocol under the spatial, individual, and homogeneous models, all of which have the same average aggregate inter-contact time.



(a) Spatial model vs. homogeneous model



(b) Individual model vs. homogeneous model

Fig. 6. The average delay of epidemic protocol under the spatial, individual, and homogeneous models, all of which remain the same in the average inter-contact time of a random pair of nodes.

pairwise inter-contact time distribution under the two-states spatial model deviates much from that under the homogeneous model, a pure exponential distribution. The average inter-contact time observed under each of both simulations is almost the same as 8000 seconds (i.e., $\tau = 8000$). This value of τ is also used for the average inter-contact time of any node pair under the homogeneous model in the subsequent simulations. In addition, for the simulations of the individual model, the average inter-contact time for each of 5 groups of node pairs is given by $\tau - 2\Delta, \tau - \Delta, \tau, \tau + \Delta, \tau + 2\Delta$, respectively, where $\tau = 8000$ and $\Delta = 2000, 3000$. In this way, the average inter-contact time of a randomly chosen pair of nodes remains the same for the three models.

We then conduct numerical simulations to measure the average delay performance of multicopy two-hop relay protocol and epidemic routing protocol under each of the three models with the above parameter settings. In each simulation, a message is independently generated at a random time for a uniformly and randomly chosen source and destination pair and total 10^5 messages are generated during the simulation. All simulation results are obtained based upon the message delivery of 10^5 messages. We also change the number of nodes $|\mathcal{N}|$ from 20 to 50 in the simulations.

Fig. 5 shows the average delays of multicopy two-hop relay protocol under the spatial, individual, and homogeneous models. First, as shown in Fig. 5(a), the spatial model with a different set of transition rates yields worse performance than the homogeneous model in terms of the average delay, which is in good agreement with our analytical findings in Section 5 (see Proposition 5 and (19)). This performance degradation results from that pairwise inter-contact time under the spatial model is no longer memoryless and more variable, which tends to have longer inter-contact time sample with higher probability, as seen from Fig. 4. We can further observe that for the spatial model, the average delay performance under the case of $q_{12}=q_{21}=10^{-5}$ is worse than the other. This is also due to higher variability of inter-contact time under the former case.

In addition, as can be seen from Fig. 5(b), the individual model (with $\Delta=2000, 3000$) results in better average delay performance than the homogeneous model. This also confirms our findings in Theorem 1 and Corollary 1. As mentioned in Section 5, the main reason of the performance improvement is the *path diversity* in the individual model. In other words, since the message delivery delay is the *minimum* path delay over direct and two-hop relay paths, the diversity in path delays under the individual model can improve the delay performance. We also observe that the more variable scenario of the individual model in terms of the average inter-contact time for each node pair ($\Delta=3000$), the smaller average delay of the multicopy two-hop relay protocol. This improvement comes from higher path diversity for the case of $\Delta=3000$.

We also present the average delays of epidemic routing protocol under the spatial, individual, and homogeneous models in Fig. 6. We can see the same trend in the average delay performance of epidemic routing protocol as observed above for the multicopy two-hop relay protocol. Hence, all the simulation results collectively exhibit an opposite prediction on the forwarding performance from each of the two heterogeneous models and also show the presence of a performance gap between the performance predictions, which confirms our analytical results.

7 Conclusion

In this paper we have mainly focused on how the underlying heterogeneity structure in mobile nodes' contact dynamics impacts the performance of forwarding algorithms in MONs. Based upon two representative heterogeneous network models, we have investigated their non-Poisson contact dynamics and stochastically compared their delay performance of direct forwarding and multicopy two-hop relay protocol with those under the homogeneous model. In particular, our findings show that each heterogeneous model predicts an entirely opposite delay performance when compared with that under the homogeneous model. Simulation results including the delay performance of epidemic routing protocol are also provided to support these findings. Our results call for much more careful studies on the forwarding performance under non-Poisson contacts, and perhaps more importantly, under properly chosen heterogeneous models.

8 Proofs

8.1 Proof of Proposition 1

We here prove that the inter-contact time of any node pair which is uniformly chosen in $\mathcal{I}$ has a hyper-exponential distribution under the spatial model. By the definition of the spatial model, the distribution of inter-contact time of a node pair is identical to the others, it is enough to show the inter-contact time distribution of a given node pair $i \in \mathcal{I}$.

Let T_{AB} be inter-contact time between randomly chosen nodes A and B. Without loss of generality, we assume that a contact between nodes A and B occurs at time 0. Let $A(t), B(t) \in \Omega$ be the sites that nodes A and B belong to at time t, respectively. By the definition of the spatial model, we know $\{A(t)\}_{t\geq 0}$ and $\{B(t)\}_{t\geq 0}$ are continuous time Markov chains with state space Ω. We hereafter use state i, instead of state S_i, for simplicity ($\Omega = \{1, 2, \ldots, M\}$). Since each of the Markov chains is irreducible and its state space is finite, it is ergodic and thus there exists a unique stationary distribution $\vec{\pi} = [\pi_i, i \in \Omega]$ such that $\vec{\pi}\mathbf{Q} = \vec{0}$ [35], [36]. We assume that the system is in the steady-state with its stationary distribution $\vec{\pi}$. Recall that the transition rate matrix $\mathbf{Q}$ of the Markov chains is given by

$$\mathbf{Q} = \begin{bmatrix} -q_1 & q_{12} & \cdots & q_{1M} \\ q_{21} & -q_2 & \cdots & q_{2M} \\ \vdots & \vdots & \vdots & \ddots \\ q_{M1} & q_{M2} & \cdots & -q_M \end{bmatrix},$$

where $q_i = \sum_{k\neq i} q_{ik}$. We also define a matrix $\mathbf{B}$ by $\mathbf{B} = \text{diag}\{\beta_1, \beta_2, \ldots, \beta_M\}$. From the definition of the spatial model, we know that a contact process based on $\mathbf{B}$ between nodes A and B is modulated by $\{A(t)\}$ and $\{B(t)\}$. In other words, a contact between nodes A and B happens according to a Poisson process with rate β_i, only when two nodes reside in the same state $i \in \Omega$. One can expect the similarity as a point process between the contact process under the spatial model and an arrival process governed by the Markov Modulated Poisson Process (MMPP) [37] widely used in teletraffic engineering. In the MMPP, packet arrivals occur according to a Poisson process with a different rate which is modulated by an irreducible continuous time Markov chain.

Let $C(t) \triangleq (A(t), B(t)) \in \Omega^2$ to represent a pair of states that nodes A and B belong at time t. Then, $\{C(t)\}_{t\geq 0}$ is a continuous time Markov chain with state space Ω^2 and its transition rate matrix $\mathbf{Q}' = \{q'_{\vec{u},\vec{v}}\}_{\vec{u},\vec{v}\in\Omega^2}$ is also written as

$$\mathbf{Q}' = \begin{bmatrix} -q'_1 & q_{12} & q_{13} & \cdots & 0 \\ q_{21} & -q'_2 & q_{23} & \cdots & 0 \\ q_{31} & q_{32} & -q'_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -q'_{M^2} \end{bmatrix}.$$

Here, the entries of the rate matrix $\mathbf{Q}'$ are ordered lexicographically, i.e., $(1,1), (1,2), \ldots, (1,M), (2,1), (2,2), \ldots, (M,M)$, and $q'_l = \sum_{k\neq i} q_{ik} + \sum_{k\neq j} q_{jk}$ for each $l = M(i-1)+j$, where $i, j \in \Omega$. Also, the stationary distribution $\vec{\pi}'$ of $\{C(t)\}$ is now given by $\vec{\pi}' = [\pi_1^2 \ \pi_1\pi_2 \ \cdots \ \pi_1\pi_M \ \pi_2\pi_1 \ \cdots \ \pi_M^2]$.

For notational convenience, we define another $M^2 \times M^2$ matrix $\mathbf{B}'$ from the $M \times M$ matrix $\mathbf{B}$, as the rate matrix $\mathbf{Q}'$ is a $M^2 \times M^2$ matrix. The $\mathbf{B}'$ is a diagonal matrix with $\mathbf{B}'_{jj} = \beta_i$ if $j = M(i-1)+i$, otherwise zero, where $j \in \{1, 2, \ldots, M^2\}$ and $i \in \{1, 2, \ldots, M\}$. Thus, the contact process based on $\mathbf{B}'$ between nodes A and B is modulated by $\{C(t)\}$. That is, when the Markov chain $\{C(t)\}$ is in state (i, i), contacts occur according to the Poisson process of β_i. Therefore, it has exactly the same structure of MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$).

Consider the epochs of successive contacts in the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) to obtain the pairwise inter-contact time distribution between nodes A and B, i.e., $\mathbb{P}\{T_{AB} > t\}$. As mentioned above, the contact process between nodes A and B starts at an "arbitrary contact epoch", i.e., $t = 0$ is a contact epoch. It is called *i*nterval-stationary process in the MMPP [37]. We denote J_n, $n \geq 0$, to be the state of the Markov chain $\{C(t)\}$ associated with the n^{th} contact (J_0 is the state at $t = 0$). We also denote X_n, $n \geq 1$, to be the inter-contact time between the $(n-1)^{st}$ and the n^{th} contacts with $X_0 = 0$. Then, the sequence $\{(J_n, X_n), n \geq 0\}$ is a Markov renewal sequence with transition probability matrix [37], [38]

$$\begin{aligned}\mathbf{F}(\mathbf{t}) &= \int_0^t e^{(\mathbf{Q}'-\mathbf{B}')u} du \mathbf{B}' = \left[\mathbf{I} - e^{(\mathbf{Q}'-\mathbf{B}')t}\right](\mathbf{B}'-\mathbf{Q}')^{-1}\mathbf{B}' \\ &= \left[\mathbf{I} - e^{(\mathbf{Q}'-\mathbf{B}')t}\right]\mathbf{F}(\infty), \end{aligned} \tag{20}$$

where the element $F_{ij}(t)$ of $\mathbf{F}(\mathbf{t})$ is the conditional probability $\{J_n = j, X_n \leq t \mid J_{n-1} = i\}$ for any $n \geq 1$, and $\mathbf{I}$ is a $M^2 \times M^2$ identity matrix.

The matrix $\mathbf{F}(\infty) = (\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}'$ is stochastic and its stationary vector $\vec{p}$ is given by [37], [38]

$$\vec{p} = \vec{p}\ (\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}' = \frac{1}{\vec{\pi}'\vec{\beta}'}\vec{\pi}'\mathbf{B}',$$

where $\vec{\beta}'_{M^2\times 1} \triangleq [\beta'_1\ \beta'_2\ \cdots\ \beta'_{M^2}]^T$. Here, $\beta'_j = \beta_i$ if $j = M(i-1)+i$, otherwise zero, where $j \in \{1, \ldots, M^2\}$ and $i \in \{1, \ldots, M\}$. Thus, an element of $\vec{p}$ is $\frac{\pi_i^2\beta_i}{\sum_{k=1}^{M}\pi_k^2\beta_k}$ if it corresponds to that of state (i, i), otherwise 0. Here, since the contact process between nodes A and B governed by the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) is interval-stationary, the initial probability vector $\{J_0\}$ of the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) is chosen to be $\vec{p}$. Thus, since $\mathbb{P}\{T_{AB} > t\} = \mathbb{P}\{X_n > t\}$ for any $n \geq 1$, from (20) with $\vec{p}$, we have

$$\begin{aligned}\mathbb{P}\{T_{AB} > t\} &= \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}(\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}'\vec{e} \\ &= \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}\vec{e}, \end{aligned} \tag{21}$$

where $\vec{e}_{M^2\times 1} = [1\ 1\ \cdots\ 1]^T$. The second equality is from the fact that the matrix $(\mathbf{B}'-\mathbf{Q}')^{-1}\mathbf{B}'$ is stochastic. Note that (21) is the marginal distribution of an inter-contact time between two successive contact epochs.

Recall that $q_{ij} = q_{ji}$ in the rate matrix $\mathbf{Q}$ of each of $\{A(t)\}$ and $\{B(t)\}$, where $i, j \in \Omega$. Hence, it is easy to see that the matrix $\mathbf{Q}' - \mathbf{B}'$ is symmetric, and thus its eigenvalues and eigenvectors are real. By the spectral theorem [39], the matrix $\mathbf{Q}'$–$\mathbf{B}'$ can be diagonalized by an orthogonal matrix. In other words, $\mathbf{Q}' - \mathbf{B}' = \mathbf{M}\mathbf{U}\mathbf{M}^{-1}$, where $\mathbf{M}$ is a $M^2 \times M^2$ orthogonal matrix containing orthonormal eigenvectors of $\mathbf{Q}'$–$\mathbf{B}'$, and $\mathbf{U}$ is a $M^2 \times M^2$ diagonal matrix in which each diagonal element is its an eigenvalue. Thus, (21) becomes

$$\mathbb{P}\{T_{AB} > t\} = \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}\vec{e} = \vec{p}\ \mathbf{M}e^{\mathbf{U}t}\mathbf{M}^{-1}\vec{e}. \tag{22}$$

Here, since all the eigenvalues of $\mathbf{Q}' - \mathbf{B}'$ are real and $e^{(\mathbf{Q}'-\mathbf{B}')t} \to \mathbf{0}$ as $t \to \infty$ in (20), all the eigenvalues (in $\mathbf{U}$) should be negative [39]. Therefore, (22) becomes a weighted sum of exponentials (i.e., hyper-exponential). This completes the proof.

8.2 Proof of Corollary 1

We here show that if $\mathbb{E}\{T_I^{\mathsf{HO}}\} = \mathbb{E}\{T_I^{\mathsf{IN}}\} = \sum_{i\in\mathcal{I}}\frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}$, then $\mathbb{E}\{D_{\mathsf{IN}}^{[2]}\} \leq \mathbb{E}\{D_{\mathsf{HO}}^{[2]}\}$. First, observe that from the independence of T_{ij}^{HO} over different node (i, j) pairs, for any source and destination pair, we have

$$\begin{aligned}\mathbb{E}\{D_{\mathsf{HO}}^{[2]}\} &= \int_0^\infty \mathbb{P}\{\min\{T_{sr_1}^{\mathsf{HO}}+T_{r_1d}^{\mathsf{HO}}, \ldots, T_{sr_n}^{\mathsf{HO}}+T_{r_nd}^{\mathsf{HO}}\} > t\}dt \\ &= \int_0^\infty \prod_{i=1}^{n}\mathbb{P}\{T_{sr_i}^{\mathsf{HO}}+T_{r_id}^{\mathsf{HO}} > t\}dt \\ &= \int_0^\infty (1 + t/\tau)^n e^{-nt/\tau}dt \\ &= \tau\sum_{i=0}^{n}\frac{n!}{(n-i)!n^{i+1}} = \tau f(n), \end{aligned} \tag{23}$$

where $\tau = \mathbb{E}\{T_I^{\mathsf{HO}}\}$ and $f(n) \triangleq \sum_{i=0}^{n}\frac{n!}{(n-i)!n^{i+1}}$. Also, from Theorem 1, we know that if $\tau = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$, then

$$\mathbb{E}\{D_{\mathsf{IN}(s,d)}^{[2]}\} \leq \mathbb{E}\{D_{\mathsf{HO}}^{[2]}\} = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right]f(n), \tag{24}$$

where the equality is from (23).

As mentioned earlier, the average delay for a uniform source and destination pair is the arithmetic mean of the average delays for all $|\mathcal{N}|(|\mathcal{N}|-1)/2$ source and destination (s, d) pairs. We hereafter use (i, j), instead of (s, d), to clearly distinguish each source and destination pair, where $i, j \in \mathcal{N} \triangleq \{1, 2, \ldots, n+2\}$. Then, the average delay for a uniform source and destination pair is given by

$$\begin{aligned}\mathbb{E}\{D_{\mathsf{IN}}^{[2]}\} &= \frac{2}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j>i}^{n+2}\mathbb{E}\{D_{\mathsf{IN}(i,j)}^{[2]}\} \\ &= \frac{1}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\mathbb{E}\{D_{\mathsf{IN}(i,j)}^{[2]}\} \\ &\leq \frac{1}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{1}{2n}\sum_{k\neq i,j}^{n+2}\left(\frac{1}{\lambda_{ik}} + \frac{1}{\lambda_{kj}}\right)f(n) \\ &= \frac{f(n)}{(n+2)(n+1)2n}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\left(\frac{1}{\lambda_{ik}} + \frac{1}{\lambda_{kj}}\right), \end{aligned} \tag{25}$$

where the first equality is from the symmetry of the contact process of each node pair ($\lambda_{ij} = \lambda_{ji}$) under the individual model, and the inequality is from (24). Further, the summation terms in (25) can be simplified as follows. Observe that

$$\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{ik}} = \sum_{i=1}^{n+2}\left(\sum_{k\neq i}^{n+2}\frac{n+1}{\lambda_{ik}} - \sum_{j\neq i}^{n+2}\frac{1}{\lambda_{ij}}\right) = \sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{n}{\lambda_{ij}},$$

and

$$\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{kj}} = \sum_{i=1}^{n+2}\sum_{j=1}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{kj}} - \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{1}{\lambda_{kj}}$$
$$= \sum_{j=1}^{n+2}\left(\sum_{k\neq j}^{n+2}\frac{n+2}{\lambda_{kj}} - \sum_{h\neq j}^{n+2}\frac{1}{\lambda_{hj}}\right) - \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{1}{\lambda_{kj}}$$
$$= \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{n}{\lambda_{kj}} = \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{n}{\lambda_{jk}}.$$

Thus, (25) can be rewritten as

$$\mathbb{E}\{D^{[2]}_{\mathsf{IN}}\} \leq \frac{f(n)}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{1}{\lambda_{ij}} = \left(\frac{2}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j>i}^{n+2}\frac{1}{\lambda_{ij}}\right)f(n) = \left(\sum_{i\in\mathcal{I}}\frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}\right)f(n) = \mathbb{E}\{T_I^{\mathsf{IN}}\}f(n), \quad (26)$$

where the equalities are from the definition of the individual model. Then, from the assumption that $\mathbb{E}\{T_I^{\mathsf{HO}}\} = \mathbb{E}\{T_I^{\mathsf{IN}}\} = \sum_{i\in\mathcal{I}}\frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}$ and from (23) and (26), we have

$$\mathbb{E}\{D^{[2]}_{\mathsf{IN}}\} \leq \mathbb{E}\{T_I^{\mathsf{IN}}\}f(n) = \tau f(n) = \mathbb{E}\{D^{[2]}_{\mathsf{HO}}\}.$$

This completes the proof.

Acknowledgments

This work was supported in part by National Science Foundation under grants CNS-0831825, CCF-0830680, and CAREER Award CNS-0545893.

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