Exploiting Heterogeneity to Prolong the Lifetime of Large-Scale Wireless Sensor Networks

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***Abstract*—In wireless sensor networks (WSNs), sensor nodes are typically power-constrained with limited lifetime, and thus it is necessary to know how long the network sustains its networking operations. We consider the network lifetime as the time until that a majority of functional nodes remains connected of one another, forming a giant component, in the network. We then analytically examine such network lifetime of a large-scale WSN via the theory of site percolation on a random graph model with a given degree distribution. In particular, we develop an analytical framework to quantify the network lifetime if the node lifetime can be controlled based on its degree, and show in theory and simulation that, by properly exploiting the heterogeneity over the node degrees, we can always increase the network lifetime when compared with that under the comparable degree-independent node lifetime.**

I. Introduction

Wireless sensor networks (WSNs) consist of sensor nodes equipped with their own battery having limited lifetime, which makes the operations of network available only within a limited amount of time. It is crucial to examine and estimate how long the network is properly functioning, or *network lifetime*, after which the deployment of additional sensor nodes is inevitable so as to ensure the network connectivity and maintain the networking operations of interest, but requires lots of cost and effort. Hence, in the literature, there has been much work on analyzing the network lifetime under a certain consideration and devising an algorithm or network protocol to prolong the network lifetime.

The network lifetime has been typically defined as the time until any first sensor node dies or runs out of its battery power [4], [7]. Or, it can be the time after which isolated sensor nodes (a blind spot) appear [9], [5]. However, these definitions are too stringent or conservative to declare that the network no longer functions. Even if a single node runs out of its battery power or isolated nodes appear, other sensor nodes can ensure the network connectivity, still enabling timely information delivery over the network. Since WSNs are typically composed of low-cost, low-power sensor devices, it is mostly likely that a few sensor nodes stop functioning right after network deployment. To be fault-tolerant, sensor devices are densely deployed and node redundancy is normally granted [4], which relaxes the requirement of full connectivity.

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In this paper, we focus on the lifetime of a large-scale WSN, which is defined as the time before a giant component, in which a majority of functional nodes are connected with each other, remains to form in the network or after which the network becomes fully fragmented. (See its definition in Section II.) The presence of a giant component can ensure the network to properly operate, even if it loses a connection with a static base station, through leveraging multiple and/or mobile sinks (e.g., data mules). This version of network lifetime was studied via a percolation theory in [15] where the lifetime of every node is $i.i.d.$. The percolation theory has been also used in [9], [5] to analyze the related devolution process of a large-scale WSN with $i.i.d.$ node lifetime.

While we also consult the percolation theory, in contrast, our focus is to examine the network lifetime in the presence of *degree-dependent* node lifetime or how the underlying heterogeneity over the node degrees can be exploited to prolong the network lifetime. Specifically, we provide an analytical framework to evaluate the network lifetime induced from degree-dependent node lifetime (including the degree-independent or $i.i.d.$ node lifetime as a special case) by extending the theory of site percolation on a random graph model with an arbitrarily given degree distribution, which has been popular in statistical physics literature [2], [13], [12]. We then recover the main results obtained in [15] on the network lifetime, determined by a priori given degree-independent node lifetime distributions, as an evidence to demonstrate the effectiveness of our framework. Our main result then show that if the lifetime of each sensor node can be controlled based on its own degree under a certain condition, then its resulting network lifetime can be longer than that under its comparable degree-independence node lifetime. To the best of our knowledge, this work is the first attempt to analyze the network lifetime, or a critical time before which the network keeps a giant component, in the presence of degree-dependent node lifetime via a percolation theory.

II. Preliminaries

A. Network Model

We consider n sensor nodes placed on a graph (or network) $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, n\}$ is a set of sensor nodes and $\mathcal{E}$ is a set of edges. If a sensor node $i \in \mathcal{N}$ can reliably communicate with other sensor node $j \in \mathcal{N}$, then an edge between sensor nodes i, j exists, i.e., $(i,j) \in \mathcal{E}$ $(i \neq j)$. The degree of a node i is the number of its edges, i.e. $|\{j \in \mathcal{N} \mid (i,j) \in \mathcal{E}\}|$. We also assume that G is undirected, i.e., each communication link is symmetric.

For analytical treatment, we consider a generalized random graph model with any given degree distribution (a.k.a. configuration model), which is widely used in statistical physics and computer science (e.g., [2], [13], [12], [14]). In this model, the degree of any randomly chosen node follows the given degree distribution in the limit of large graph size ($n \to \infty$). Also, nodes in G are inter-connected at random with their degrees so that the degrees of any two adjacent nodes remain independent. Thus, the nodes in G have degrees that are independently and identically distributed (*i.i.d.*) of one another in the large-n limit. Since our focus is to investigate the behavior of network lifetime in such limit (or large-scale network) as studied in [9], [5], [15], it is enough to measure all relevant quantities from the viewpoint of a *randomly chosen node*. This model is simple yet versatile. First, one can assign any arbitrary degree distribution to properly capture or represent the level of heterogeneity over the node degrees, allowing us to examine the impact of degree-dependent node lifetime upon the network lifetime, which is defined below, under a wide range of scenarios. Also, under this model, we can recover the results for the network lifetime recently obtained in [15] where node lifetimes are assumed to be *i.i.d.* as a special case.

B. Definitions

We use the following definitions throughout the paper. First, a *component* in G is a subset of nodes in G where each node is reachable from others, i.e., there exists at least one path from a node to any other in the component. Each component can be disconnected from other components. If the size of the largest component in G (or the expected number of nodes in the component), say $\mathbb{E}\{C\}$, scales with n ($\mathbb{E}\{C\} \to \infty$) as $n \to \infty$, we call it a *giant component* in G. There exists a critical point at which the size of the largest component in G undergoes a sudden change, or *phase transition*, from a constant size, to being proportional to n in the large-n limit [11], [12]. In other words, a giant component forms in G above the critical point of phase transition.

Each sensor node in G is equipped with its own battery, and it only functions for a limited amount of time, or *node lifetime*. Here, the node lifetime can vary over the nodes due to the different battery drain rates. The size of the largest component consisting of *alive* nodes that correctly function in G, say $\mathbb{E}\{C(t)\}$, is now a function of time t. Assuming that there exists a giant component in G at time $t=0$, as will be shown later, there exists a *critical time* T_c before which such giant component remains to form, i.e., $\mathbb{E}\{C(t)\} \to \infty$ as $n \to \infty$ for $t < T_c$, while the existence of a giant component is not ensured or the network may be fully partitioned for $t > T_c$. In this paper, we define the critical time T_c as the *network lifetime* before which a majority of nodes in G remain to work properly, and our main focus is to investigate the network lifetime when the lifetime of each node varies depending on its degree.

III. A Critical Point of Site Percolation

To analyze the network lifetime in the presence of degree-dependent node lifetime, we consult the theory of site percolation [2], [12]. When some fraction of nodes (or sites) are removed from a graph (or network) in a certain way, from the percolation theory one can find a critical point of *percolation transition* or percolation threshold above which a giant component of remaining nodes forms or below which such giant component no longer exists.

Consider a random graph G with a given degree distribution whose mean is finite. If a node is correctly functioning, we say that the node is *occupied* (or alive). Then, each node is occupied with some occupation probability, which is a function of its own degree. In this set-up, it was shown in [2], [12] that there exists a critical point of percolation transition at which a giant component consisting of occupied nodes first forms. Since the percolation threshold will be a basis for our subsequent analysis, we here provide a brief review on it.

Suppose that G is below the critical point of percolation transition in the large-n limit. Let C be the number of occupied nodes in a component to which a *randomly chosen node*, say $i \in \mathcal{N}$, belongs. In order to find the percolation threshold, we first need to know how many occupied neighbors node i has and then how many alive neighbors those occupied neighbors also have, *other than* node i. Let D be a nonnegative integer-valued random variable to denote the degree of randomly chosen node i having the given degree distribution. We also define by q_d the occupation probability that a node is occupied given that it has degree d. Then, the joint probability that node i has degree d and is also occupied, is $f_o(d) \triangleq \mathbb{P}\{D=d\} \cdot q_d$. Hence, the probability that node i is occupied is $\sum_{d=0}^{\infty} f_o(d)$.

In addition, let D_1 be the number of edges of a neighbor of node i other than the edge connecting node i and the neighbor. It is just one less than the total number of edges of the neighbor. Then, by noting that a node with higher degree has higher chance to be connected to node i, one can see that D_1 is proportional to $d \cdot \mathbb{P}\{D=d\}$, and after a correct normalization, we have

$$\mathbb{P}\{D_1 = d\} = \frac{(d+1)\mathbb{P}\{D=d+1\}}{\sum_{d=0}^{\infty}(d+1)\mathbb{P}\{D=d+1\}} = \frac{(d+1)\mathbb{P}\{D=d+1\}}{\mathbb{E}\{D\}}. \quad (1)$$

Then, the joint probability that a neighbor of node i has degree $d+1$ and is also occupied, is $g_o(d) \triangleq \mathbb{P}\{D_1 = d\} \cdot q_{d+1}$. Thus, the probability that a neighbor of node i is occupied is $\sum_{d=0}^{\infty} g_o(d)$. Note that $f_o(d) \neq g_o(d)$ in general.

Observe that if node i is *not occupied* with probability $1 - \sum_{d=0}^{\infty} f_o(d)$, then $C = 0$. Also, occupied nodes in a component to which node i belongs, if node i has *degree* d and is *occupied* with probability $f_o(d)$, are composed of node i itself and other occupied nodes in d different components originating from each of the d neighbors of node i. Note that node i is not included in any one of such d components. Let $S_1, \ldots, S_d$ be the size of each of such d different components. Since degrees of node i and its neighbors are $i.i.d.$, $S_1, \ldots, S_d$ are also $i.i.d.$ with common distribution F.* Hence, for each $d \geq 0$, it follows that

$$C \ \stackrel{d}{=} \ 1 + \sum_{k=1}^{d} S_k \text{ with probability } f_o(d), \qquad (2)$$

where $\stackrel{d}{=}$ means equal in distribution. Subsequently,

$$\mathbb{E}\{C\} = \sum_{d=0}^{\infty} f_o(d) + \mathbb{E}\{S_1\} \sum_{d=0}^{\infty} d \cdot f_o(d). \qquad (3)$$

We can compute $\mathbb{E}\{S_1\}$ in terms of $g_o(d)$ (or $\mathbb{P}\{D=d\}$ and q_d) in a similar way. If *a neighbor* of node i is not occupied with probability $1-\sum_{d=0}^{\infty} g_o(d)$, then $S_1=0$. Each component originating from the neighbor of node i, if the neighbor has *degree* $d+1$ and is *occupied* with probability $g_o(d)$, consists of the neighbor itself and other occupied nodes in d different components initiated from each of its d neighbors. Note that one of $d+1$ edges of the neighbor connects itself and node i. Also, these d components do not include the neighbor of i. Thus, if we let $\tilde{S}_k$ $(k = 1, 2, \ldots, d)$ be $i.i.d.$ copy of S_1, after repeating the same argument above, S_1 must satisfy the following self-consistent relationship

$$S_1 \ \stackrel{d}{=} \ 1 + \sum_{k=1}^{d} \tilde{S}_k \text{ with probability } g_o(d) \qquad (4)$$

for each $d \geq 0$, and so we have

$$\mathbb{E}\{S_1\} = \sum_{d=0}^{\infty} g_o(d) + \mathbb{E}\{S_1\} \sum_{d=0}^{\infty} d \cdot g_o(d). \qquad (5)$$

From (3) and (5), we finally have

$$\mathbb{E}\{C\} = \sum_{d=0}^{\infty} f_o(d) + \frac{\sum_{d=0}^{\infty} g_o(d)}{1 - \sum_{d=0}^{\infty} d \cdot g_o(d)} \sum_{d=0}^{\infty} d \cdot f_o(d). \quad (6)$$

One can now observe that if $\sum_{d=0}^{\infty} d \cdot g_o(d) = 1$, then $\mathbb{E}\{C\} \to \infty$, i.e., a giant component composed of occupied nodes first appears in G or the occupied nodes in G start to be percolated. By recalling $g_0(d) = \mathbb{P}\{D_1 = d\} \cdot q_{d+1}$ and from (1), the critical point of percolation transition, $\sum_{d=0}^{\infty} d \cdot g_o(d) = 1$, can be written as

$$\frac{\sum_{d=1}^{\infty} d(d-1)\mathbb{P}\{D = d\} q_d}{\mathbb{E}\{D\}} = 1. \qquad (7)$$

This is exactly the same as the percolation threshold obtained in [12, pp.609–614] via a different method. See also [2].† Since the sum in the LHS of (7) monotonically increases as edges are added to the graph G for given q_d, it follows that a giant component consisting of occupied nodes exists, if and only if the LHS of (7) is larger than one, i.e.,

$$\frac{\sum_{d=1}^{\infty} d(d-1)\mathbb{P}\{D = d\} q_d}{\mathbb{E}\{D\}} > 1. \qquad (8)$$

In what follows, we demonstrate how these percolation threshold and condition can be interpreted so as to obtain the network lifetime in the presence of degree-dependent node lifetime.

IV. From Site Percolation To Network Lifetime

We first show there exists a critical time T_c, or the network lifetime, after which a giant component composed of active nodes starts to disintegrate in a random graph G with a given degree distribution, provided that the existence of such giant component (or network connectivity) is initially ensured at $t = 0$. Then, we analytically examine how the degree-dependent node lifetime impacts the network lifetime through the comparison with the network lifetime driven from its comparable degree-independent node lifetime.

Let $L(d)$ be a random variable denoting the lifetime of any node with degree d (degree-dependent node lifetime). Then, the lifetime of a randomly chosen node i becomes $L(D)$, a function of random variable D. If node i has degree d, then the probability that it is alive or properly functioning at time $t>0$ is given by

$$\mathbb{P}\{L(D)>t|\ D=d\} = \mathbb{P}\{L(d)>t\}, \qquad (9)$$

where the equality is from the independence of $L(d)$ and D. In other words, the probability that node i has degree d and is alive at time t, is $\mathbb{P}\{D=d\} \cdot \mathbb{P}\{L(d)>t\}$.

Now, we fix $t \geq 0$ and set $q_d = \mathbb{P}\{L(d)>t\}$, the probability that a node with degree d is alive (or occupied) at time t. We define a function $\mathcal{L}(t)$ as

$$\mathcal{L}(t) \triangleq \frac{\sum_{d=1}^{\infty} d(d-1)\mathbb{P}\{D=d\}\mathbb{P}\{L(d)>t\}}{\mathbb{E}\{D\}}. \qquad (10)$$

It then follows from (8) that, for a given time t, a giant component of active nodes exists if and only if $\mathcal{L}(t) > 1$. Observe that $\mathbb{P}\{L(d)>t\}$ is decreasing in $t \geq 0$ for any given d, $\mathcal{L}(t)$ is also decreasing in t. Hence, from (7), (8) and (10), the critical time T_c, or the network lifetime, can be obtained as

$$T_c = \inf\{t \geq 0 : \mathcal{L}(t) = 1\}. \qquad (11)$$

It implies that $\mathcal{L}(t) > 1$ for $t < T_c$, or a giant component composed of functional (active) nodes remains to form in G for $t < T_c$, while the existence of such giant component is not guaranteed and network may be fully partitioned for $t > T_c$.

To avoid triviality, we assume that all the nodes are initially functioning (active) and there exists a giant component of active nodes at $t = 0$, i.e., $\mathcal{L}(0) > 1$. Since $\mathbb{P}\{L(d)>0\} = 1$, from (10), the condition $\mathcal{L}(0) > 1$ amounts to $\mathbb{E}\{D^2\}/\mathbb{E}\{D\} > 2$, which is the condition for a critical point of phase transition as mentioned in Section II for a *static* random graph with given degree distribution [11], [12]. Note that we are here dealing with a dynamic (time-varying) random graph in which each node with degree d will stop functioning and thus be removed from the network after its (degree-dependent) lifetime $L(d)$.

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*More precisely, in the regime below the percolation threshold we assumed at the beginning, the size of any existing component in G is finite. Since the degrees of any two adjacent nodes are $i.i.d.$, in the large-n limit, clustering coefficient – the probability that two neighbors of a node are also neighbors of one another, tends to zero, and thus all finite components do not contain loops and are rather tree-like, which in turn makes the sizes of such components be $i.i.d.$ [2], [13], [12].

†A set of equations, obtained via a formalism of generating functions, to find the percolation threshold was first given in [2]. The percolation threshold is not clearly stated there, but can be obtained from Equations (1)–(4) therein.

A. Degree-independent Node Lifetime

We first discuss the network lifetime for the case of degree-independent node lifetime, i.e., the lifetime of every node is *i.i.d.* with common distribution, regardless of its degree, i.e. $L \stackrel{d}{=} L(d)$ for all d. Thus (10) becomes

$$\mathcal{L}(t) = \frac{\mathbb{E}\{D^2\} - \mathbb{E}\{D\}}{\mathbb{E}\{D\}} \mathbb{P}\{L > t\}. \tag{12}$$

Here, from (11)–(12), one can easily see that for any given distribution of L, the network tends to live longer (T_c gets larger) as the underlying network topology becomes more heterogeneous in the sense of larger $\mathbb{E}\{D^2\}$ while $\mathbb{E}\{D\}$ is kept the same. When D is Poisson distributed with $\mathbb{E}\{D\} = \mu > 1$ (to ensure the initial connectivity of the network, i.e. $\mathcal{L}(0) > 1$) in the large-n limit, we have the following.

Proposition 1: If L is exponentially distributed with mean $1/\alpha$, i.e., $\mathbb{P}\{L > t\} = e^{-\alpha t}$, then $T_c = \frac{1}{\alpha}\log(\mu)$. In addition, if L follows a Pareto (or power-law) distribution, i.e., $\mathbb{P}\{L > t\} = \left(\frac{t}{\eta}\right)^{-\rho}$ for $\rho > 1$, then $T_c = \eta(\mu)^{1/\rho}$. □

Proof: Since D is Poisson distributed with $\mathbb{E}\{D\} = \mu$, we have $\mathbb{E}\{D^2\} = \mu^2 + \mu$ and thus $\mathcal{L}(t) = \mu\mathbb{P}\{L > t\}$ from (12). Then, from (11), the results follow. ■

Remark 1: The Poisson degree distribution often arises in the modelling of wireless ad-hoc or sensor networks (e.g., [8], [15]). Specifically, n (sensor) nodes are uniformly distributed on a square area of size A and two nodes are connected if they are within distance of r, which is called a random geometric graph. Then, as $n \to \infty$ and $A \to \infty$ while the node density $\nu = n/A$ is kept constant, the node degree approximately follows a Poisson distribution with mean $\mu = \nu\pi r^2$ [3], [8]. □

Remark 2: It was shown in [15] that the last time until the network possesses a giant component composed of surviving nodes (i.e., network lifetime T_c in our definition) is $\Theta(\log(\log n))$ and $\Theta((\log n)^{1/\rho})$ for the exponential and Pareto node lifetime, respectively, under a random geometric graph where the node density ν is fixed as in Remark 1, but $\mu = \nu\pi r^2 = \Theta(\log n)$ (or $r^2 = \Theta(\log n)$) in the large-n limit.[‡] In this set-up, we can recover these results from Proposition 1, which clearly demonstrates the effectiveness of our approach based on a random graph model with a given degree distribution. □

B. Degree-dependent Node Lifetime

We turn our attention to the network lifetime in the presence of degree-dependent node lifetime. Specifically, we examine how the network lifetime can be prolonged if one can control the lifetime of each node as a function of its degree. To this end, we consider the following scenario of WSNs with random duty-cycling which is adopted for energy/power conservation: each node wakes up according to a Poisson process with some wake-up rate to communicate with its awake neighbors.[§]

[‡]As the average node degree is on the order of $\log n$ while node density remains fixed, the network lifetime scales with the number of nodes n.

[§]This setting has been used in [8], [7], [10], while the underlying packet-forwarding algorithm and the target in each work are different from others.

Clearly, the lifetime of each node and the induced network lifetime both depend on the wake-up rate of each node since each node consumes certain amount of its batter power whenever it wakes up and stays on. We consider a class of wake-up rate controls in which the wake-up rate of each node with degree d is given by

$$\lambda(d) = \lambda_0 d^{-\beta}, \tag{13}$$

where λ_0 is an initial wake-up rate (constant) and β is our control parameter. This set of wake-up rate controls was considered in our previous work [10], which addressed how to choose β to improve the delay performance of an opportunistic forwarding, but disregarding the lifetime aspect of the network. In contrast, our focus here is on which values of β lead to longer network lifetime when compared to that under its comparable degree-independent node lifetime as will be specified shortly.

We assume that the lifetime of a node $L(d)$ with degree d is given by

$$L(d) = \frac{B}{a\lambda(d)} = \frac{B}{a\lambda_0} d^{\beta}, \tag{14}$$

where B is an initial battery power and a is a common unit of battery consumption made whenever a node wakes up. This power consumption model and its resulting constant node lifetime were similarly used in [7]. This choice allows tractable analysis for the network lifetimes under various scenarios and their quantitative comparison while still capturing the inversely-proportional relationship between the node lifetime and its wake-up rate. We however note that more sophisticated power consumption models and their resulting random node lifetimes can be still considered in our framework at the cost of more complicated analysis. As a special case, if $\lambda(d) = \lambda$ (the same wake-up rate for every node), then the node lifetime in (14) becomes degree-independent, i.e. $L = L(d)$ for all d.

To find the values of β in (13)–(14) leading to longer network lifetime, we compare each resultant network lifetime with the network lifetime $\bar{T}_c$ obtained under the degree-independent node lifetime. Since a randomly chosen node has a degree D, it follows that

$$\bar{T}_c = \mathbb{E}\{L(D)\} = \frac{B}{a\lambda_0}\mathbb{E}\{D^{\beta}\}, \tag{15}$$

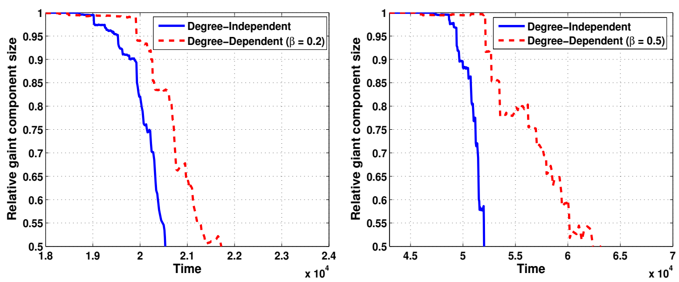
where the expectation is with respect to D. It means that every node will be no longer functional at the same time $\bar{T}_c$. We use $\bar{T}_c$ for the network lifetime under this degree-independent node lifetime to be distinguished from the network lifetime T_c under the more general degree-dependent case.

Now, we can obtain the desirable value of β in (13)–(14) such that $T_c \geq \bar{T}_c$ by showing $\mathcal{L}(\bar{T}_c) \geq 1$. Observe that

$$\mathbb{P}\{L(d) > \bar{T}_c\} = \mathbf{1}_{\{L(d) > \bar{T}_c\}} = \mathbf{1}_{\{d > \mathbb{E}\{D^{\beta}\}^{1/\beta}\}}, \tag{16}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. Since $\mathbb{E}\{D^{\beta}\}^{1/\beta}$ is increasing in $\beta > 0$ [1], from (10) and (16), if one finds β^* such that

$$\mathcal{L}(\bar{T}_c) = \frac{1}{\mathbb{E}\{D\}}\mathbb{E}\left\{D(D-1)\mathbf{1}_{\{D > \mathbb{E}\{D^{\beta^*}\}^{1/\beta^*}\}}\right\} \geq 1, \tag{17}$$



(a) $\beta = 0.2$ (b) $\beta = 0.5$

Fig. 1. The comparison between the network lifetime induced from the degree-dependent node lifetime with different β and its corresponding network lifetime under the degree-independent node lifetime.

then $T_c \geq \bar{T}_c$ for any given $\beta \in (0, \beta^*]$. By using this property, for a random graph G with a Poisson degree distribution with mean $\mu > 1$, we show the following.

Theorem 1: For any given $\beta \in (0, 1]$, we have $T_c \geq \bar{T}_c$. □

Proof: We set $\beta^* = 1$. After little algebraic computation, (17) becomes

$$\mu \sum_{d=\lfloor\mu\rfloor-1}^{\infty} \frac{\mu^d}{d!} e^{-\mu} \geq 1, \tag{18}$$

which is equivalent to

$$\mathbb{P}\{D \leq \lfloor\mu\rfloor - 2\} \;\leq\; 1 - \frac{1}{\mu}, \tag{19}$$

where $\lfloor\mu\rfloor$ is the largest integer n with $n \leq \mu$. For $1 < \mu < 3$, this trivially holds. To show that (19) holds for $\mu \geq 3$, we recall from [6, Theorem 1] that for $m \geq 2$ and $\mu \geq 2$,

$$\mathbb{P}\{D \leq \lfloor\mu\rfloor - m\} \leq e^{\frac{1}{8\mu}} \left(1 + \frac{1}{\mu}\right) Q\left(\frac{m - 3/2}{\sqrt{\mu}}\right), \tag{20}$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy$. Since $Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}}$ for all $x \geq 0$, from (20), we have

$$\mathbb{P}\{D \leq \lfloor\mu\rfloor - 2\} \leq \frac{1}{2}\left(1 + \frac{1}{\mu}\right) \leq 1 - \frac{1}{\mu}, \tag{21}$$

where the second inequality holds for $\mu \geq 3$, and this completes the proof. ■

To support our analysis, we also conduct simulations to numerically measure the network lifetime. To this end, whenever any sensor dies out (runs out of battery), we compute a relative giant component size – defined by the ratio of the number of surviving nodes in the largest component to the total number of surviving nodes at each observation time instant as used in [15], and plot this value over time. We use a sample topology of random geometric graph (defined in Remark 1) generated as follows: $n = 400$ nodes are uniformly distributed over a square area $[0, \sqrt{\frac{n}{\nu}}]^2$ with node density $\nu = 2$, and any two nodes are connected if they are within $r = 2$. For the degree-dependent node lifetime, we use $\frac{B}{a} = 100$ and $\lambda_0 = 0.01$ with $\beta = 0.2$ or 0.5 in (14). The actual lifetime of each node with its wake-up rate $\lambda(d)$ in (13) is now the sum of 100 exponential inter-wake-up durations (from Poisson wake-up), whose mean is $L(d)$. For fair comparison, as was done in (15), the empirical average of the mean lifetime over $n = 400$ nodes under degree-dependent case is used as the mean lifetime of every node for the corresponding degree-independent case for each choice of β, while the lifetime of each node (with common wake-up rate) is still the sum of 100 exponential inter-wake-up durations. All simulation results are obtained by averaging over 10 independent trials. As shown in Fig. 1, we can observe that the network lifetime driven from the degree-dependent node lifetime is prolonged for both $\beta = 0.2, 0.5$ (a majority of surviving nodes remains connected of one another for a longer period of time), which coincides with our analysis.

V. Conclusion

We have studied the network lifetime of a large-scale WSN in the presence of degree-dependent node lifetime via our analytical framework developed based upon the percolation theory on a random graph model with an arbitrarily given degree distribution. In particular, we demonstrated that a proper control of node lifetime exploiting the heterogeneity over the node degrees leads to longer network life, saving huge cost and effort for network maintenance. We expect that our framework can be easily extended to address the temporal characteristics of networks beyond WSNs with general degree distributions.

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