Toward Stochastic Anatomy of Inter-meeting Time Distribution under General Mobility Models

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ABSTRACT

Recent discovery of the mixture (power-law and exponential) behavior of inter-meeting time distribution of mobile nodes presents new challenge to the problem of mobility modeling and its effect on the network performance. Existing studies on this problem via the *average* inter-meeting time become insufficient when the inter-meeting time distribution starts to deviate from exponential one. This insufficiency necessarily leads to the increasing difficulty in the performance analysis of forwarding algorithms in mobile ad-hoc networks (MANET). In this paper, we analyze the effect of mobility patterns on the inter-meeting time *distribution*. We first identify the critical timescale in the inter-meeting distribution, at which the transition from power-law to exponential takes place, in terms of the domain size and the statistics of the mobility pattern. We then prove that *stronger correlations* in mobility patterns lead to *heavier* (non-exponential) 'head' of the inter-meeting time distribution. We also prove that there exists an *invariance* property for several contact-based metrics such as inter-meeting, contact, inter-any-contact time under both distance-based (Boolean) and physical interference (SINR) based models, in that the averages of those contact-based metrics do not depend on the degree of correlations in the mobility patterns. Our results collectively suggest a *convex ordering* relationship among inter-meeting times of various mobility models indexed by their degrees of correlation, which is in good agreement with the ordering of network performance under a set of mobility patterns whose inter-meeting time distributions have power-law 'head' followed by exponential 'tail'.

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General Terms: Theory

Keywords: mobile ad-hoc network, inter-meeting time distribution, first passage time, stochastic ordering

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1. INTRODUCTION

As a crucial mobility metric and the main determinant of the link level dynamics in MANETs, the characteristics of inter-meeting time of mobile nodes have received ever-increasing attention. From the earlier work on the delay-capacity tradeoff [12] up to recent work in [20] and references therein, most studies in MANET have relied on exponentially distributed inter-meeting times for tractable analysis of network performance. Quite recently, the discovery of its non-exponential behavior inspires a new surge of research interest, e.g., the effect of power-law inter-meeting time on the performance of forwarding algorithms [7], the characteristic time of inter-meeting time [15], and the fundamental reason for the non-exponential behavior [5]. While there is now strong evidence showing that the distribution of inter-meeting time has a mixture of power-law and exponential behavior in reality [15] with partial explanation for its root [5], the following issue still remains unsolved: the effect of mobility patterns on the network performance such as network capacity or the end-to-end delay of forwarding algorithms through the mobility-induced inter-meeting time *distribution*.

The problem of the impact of mobility patterns on network performance is not new. However, it is facing new challenge due to the discovery of the mixture of power-law and exponential behaviors in the inter-meeting time distribution. While [15] maintains that current mobility models can still generate this mixture behavior, due to the complexity of the problem at hand, the study of effect of mobility patterns on the inter-meeting time distribution is largely limited to the estimation of the average inter-meeting time (first-order behavior) [11, 23, 15], or two extreme cases, exponential and power-law, separately. Under exponential inter-meeting time distribution, the performance of forwarding algorithms (e.g., epidemic routing [24], its K-hop variants [11], spray and wait [22], etc.) has been widely studied based on Markovian analysis [11, 22, 13, 21]. or so-called the mean-field approach based on ordinary differential equation (ODE) [13, 25]. On the other hand, under the assumption that inter-meeting time distribution has power-law tail, [7] analyzes the performance of generic two-hop forwarding algorithms and draws rather pessimistic conclusions.[1] The only exception is the very recent work [1] on the analysis and comparison of two-hop routing algorithms under exponential and hyper-exponential inter-meeting time distribution.

In this paper, we investigate how the stochastic nature of the mobility patterns affects the whole *distribution* of the

[1]It is shown in [7] that the mean end-to-end delay of any two-hop forwarding algorithm is infinite if the inter-meeting time between two nodes has power-law tail with infinite mean.

inter-meeting time. In contrast to existing works focusing largely on the mean inter-meeting time under a given specific mobility model, we directly quantify the impact of statistical mobility patterns on the *shape* of the inter-meeting time distribution, which in turns critically affects the performance of forwarding/routing algorithms running on top of the underlying nodes' mobility pattern. Motivated by recent findings in [15, 5] that shows clear transition behavior of the inter-meeting time distribution from power-law to exponential (which is called 'dichotomy' in [15]), our approach in this paper is to first divide the whole distribution of the inter-meeting time into *head* and *tail*, which is separated by the critical timescale τ_0, beyond which the inter-meeting time distribution shows exponential decay (tail).

Specifically, we first study the scaling relationship between τ_0, the size of the domain, and the statistical nature of general mobility patterns such as the degree of correlations or tendency of preserving the same direction in the mobility pattern. Second, for the 'head' of the inter-meeting time distribution ($t < \tau_0$), we prove via correlated random walk models that the inter-meeting time of mobile nodes with stronger correlations in their paths is *stochastically larger* [16] than that with weaker correlations. This stochastic ordering result, along with our scaling relationship between τ_0 and the degree of correlations, provides quantitative understanding of how different mobility models lead to different *shapes* of the inter-meeting time distribution and enables us to compare different degrees of power-law behavior in the head of the inter-meeting time. Third, we establish an *invariance property* for a class of mobility models. Based on reformulation of Kac's recurrence theorem [14, 10, 2] for any stationary ergodic process, we prove that the average inter-meeting time is invariant with respect to the degree of correlations in the mobility models. More interestingly, we also show that this invariance result holds not only for the inter-meeting time but also for many other variants of the contact-based metrics including contact time, inter-hitting time, inter-any-contact time, both under Boolean (distance-based) model and under physical interference models. The invariance result thus tightly binds the head and the tail of the inter-meeting time distribution and imposes a unifying constraint in that the mean inter-meeting time (and its variants) remains the same under a given domain size, *regardless of* the shape of the distribution and the location of τ_0. Last, we discuss the impact of our results on the study of mobility metrics and on the performance analysis of forwarding/routing algorithms in contact-based MANETs.

2. PRELIMINARIES

In this section we first collect a set of distinct mobility models to be used for our study and then give definitions of several contact-based metrics including inter-meeting time among nodes and the first passage time, which will be used throughout the paper.

2.1 Mobility Models

Correlated Random Walk on Grid (**CRW**) [3]: In each time step, a mobile node moves to one of its 2(4) neighbor sites on 1-D(2-D) grid. The initial direction of the node's step is randomly chosen from right and left (1-D case), or from right, left, up and down (2-D case). Then, the node takes a step in the chosen direction. After that, at each time slot, the node takes a step in the same direction with probability p, opposite direction with probability q and turns to other directions with probability r.[2] Hence, in 1-D case, $p + q = 1$, while in 2-D case, $p + q + 2r = 1$. Boundary condition is wrapping around.

Random Direction Mobility Models (**RD**) [19, 6]: a mobile node first chooses a uniform random direction, then moves until it hits the boundary of the domain in that direction.[3] After that, it chooses another random direction and the whole process repeats itself.

Isotropic Random Walk (**IRW**) [8, 6]: a mobile node first selects a random step-length L, a speed v from some well-defined distribution, and a direction ϕ taken uniformly and randomly from $[0, 2\pi)$. Then, it moves according to the chosen velocity for L with angle ϕ, and upon its completion of the step, the whole procedure repeats independently of all others. Note that by taking suitable step-length distribution, IRW can generate many different mobility patterns. For instance, Brownian motion mobility model can be approximated by taking very small step-lengths over small time intervals.

REMARK 1. *It is known that the stationary distributions of node positions in the steady-state for all the above mobility models exist and are given by a uniform distribution over* Ω *[17, 4, 3]. There are also many other mobility models whose stationary distributions of the node position are all uniform [4, 17]. In order to focus on the effect of mobility pattern on the network performance, in this paper, we do not consider the pause of mobile nodes.* □

2.2 Contact-based Metrics

Consider a set of mobile nodes $\{A, B_1, B_2, \ldots, B_N\}$ following some mobility models in a common domain Ω. For a node A, let $A(t) \in \Omega$ be the position of node A at time t. Similarly for $B_i(t)$.

We define by $\mathcal{N}_A(t) \subset \Omega$ the *contact set* of node A at time t, i.e., a node X can communicate with A at time t if and only if $X(t) \in \mathcal{N}_A(t)$. Exact definition of the contact set varies depending on the context: (i) If we consider A's contact with an arbitrary set of nodes $\Theta = \{B_i, i \in \mathcal{I}\}$ for some index set $\mathcal{I}$ under *Boolean* model with communication range d, then the contact set of Θ becomes

$$\mathcal{N}_\Theta(t) = \bigcup\nolimits_{i\in\mathcal{I}} \{x \in \Omega : \|x - B_i(t)\| \le d\}. \tag{1}$$

(ii) If we consider contact of node A under *SINR* (Signal-to-Interference-Noise Ratio) model [9], then the contact set of A at time t will be in the form of

$$\mathcal{N}_A(t) = \Big\{ x \in \Omega : \frac{P\|x - A(t)\|^{-\alpha}}{N_0 + \sum_i P\|x - B_i(t)\|^{-\alpha}} \ge \beta \Big\} \tag{2}$$

for some suitable threshold β (e.g., the minimum signal-to-interference noise ratio required for successful decoding at the receiver) and path loss exponent $\alpha \in [2,\ 4.5]$, i.e., node X can communicate with node A at time t ($X(t) \in \mathcal{N}_A(t)$) if and only if the channel condition between two nodes (given by SINR) is good enough.

[2]Our definition is a bit different from [3], where a node takes step at time instance governed by a Poisson process.

[3]In fact, there are two variants of RD [6]. Here we describe the first variant; the second variant of RD belongs to the IRW described next.

Definition 1. *The* inter-meeting *(or* inter-contact*) time of node A with a set of nodes Θ is defined by*[4]

$$T_I \triangleq \inf_{t>0} \{t : A(t) \in \mathcal{N}_\Theta(t)\} \quad (3)$$

given that

$$A(0^-) \in \mathcal{N}_\Theta(0^-) \text{ and } A(0) \notin \mathcal{N}_\Theta(0). \quad (4)$$

If we replace $\mathcal{N}_\Theta(t)$ in (3) and (4) by $\mathcal{N}'_\Theta(t) = \Omega \setminus \mathcal{N}_\Theta(t)$, we obtain the *contact time* (or contact duration) of node A with a set of nodes Θ. Similarly, if the set of nodes in Θ are static (not moving), then we call the inter-meeting time as *inter-hitting time*. Further, if we remove the condition in (4), i.e., we start measuring the time until next encounter to Θ from a randomly chosen time instant $t = 0$, then T_I becomes the *first passage time* (FPT) of node A to the set Θ. In other words, the FPT T_F corresponds to the residual life time of the inter-meeting time [7, 15]. When the inter-meeting time T_I has a finite mean, the distribution of T_F is given by

$$\mathbb{P}\{T_F > t\} = \frac{1}{\mathbb{E}\{T_I\}} \int_t^\infty \mathbb{P}\{T_I > s\} ds, \quad (5)$$

i.e., T_F has the *equilibrium* distribution of T_I. Finally, we note that all these metrics can be similarly defined under a discrete time setting. For instance, for the inter-meeting time between nodes A and B, T_I becomes

$$T_I = \min_{t>0} \{t : A(t) \in \mathcal{N}_B(t),\ A(t+1) \notin \mathcal{N}_B(t+1)\}$$

given that $A(0) \notin \mathcal{N}_B(0)$ and $A(1) \in \mathcal{N}_B(1)$.

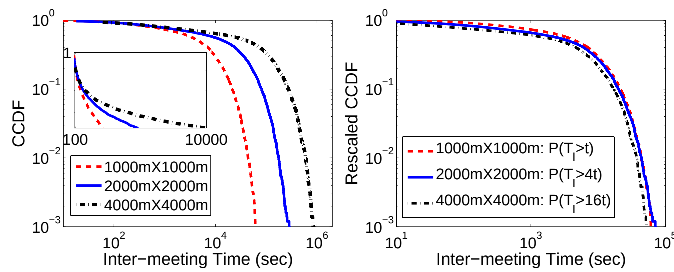
3. HEAD/TAIL OF INTER-MEETING TIME: A CLOSER LOOK AT DICHOTOMY

Recent studies in [5, 15] have pointed out that the inter-meeting time distribution is largely characterized by first a power-law until certain time scale (*head* of the inter-meeting time), beyond which the inter-meeting time distribution becomes of exponential type (*tail* of the inter-meeting time). Specifically, in [15], this dichotomic behavior is empirically observed and emphasized via a large set of real traces, while [5] suggests, via a simple *i.i.d.* uniform mobility model, that the transition from exponential to power-law behavior of the inter-meeting time arises from the coupled dynamics between the relevant time scale of interest and the size of the bounded domain. In this section, we give a closer look at this transition and quantify the timescale for transition in terms of the size of the domain and certain key characteristics of general mobility patterns, thereby providing precise distinction between the head and tail of the inter-meeting time distribution for any general mobility models under arbitrary sized domain.

To this end, first note that for a given mobility model $\mathcal{M}$ and the size of the domain[5] ('diameter' of the domain D), the node will forget where it was after it hits the boundary and bounces back to the 'center' of the domain. In other words, the timescale beyond which the inter-meeting time becomes of exponential type (tail) is on the same order of the typical amount of time it takes for the node to travel D distance. We call this timescale as *regenerative period* and write as

$$\tau_0 = \tau_0(\mathcal{M}, D)$$

to explicitly denote that it depends on the size of the domain as well as the characteristics of the mobility pattern. The concept of regenerative period is similar to the characteristic time of the inter-meeting time proposed in [15] except that here τ_0 is adopted to study the inter-meeting time distribution of general mobility models under arbitrary domain size. In light of the results in [5], for a given mobility model and under a given finite domain, the 'head' of $\mathbb{P}\{T_I > t\}$ is largely of power-law type for $t < \tau_0$ (essentially unbounded domain) while its 'tail' becomes exponential for $t > \tau_0$.



(a) $\mathbb{P}\{T_I > t\}$ (b) Rescaled $\mathbb{P}\{T_I > \eta t\}$

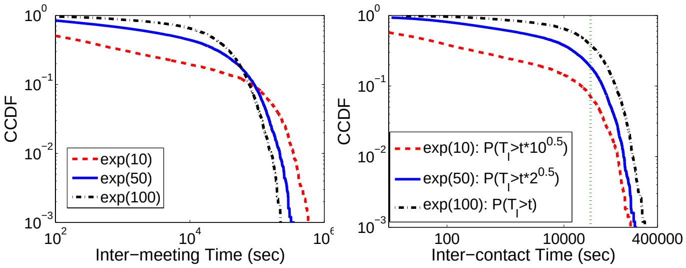
Figure 1: Effect of domain size on the regenerative period τ_0. IRW defined in Section 2.1 is used. Step length distribution is exponential with mean $100m$ and speed is set to constant ($1m/s$). Communication range $d = 50m$. Simulation time: 10^6 seconds. (a) $\mathbb{P}\{T_I > t\}$ in log-log scale; the inset is drawn in linear-log scale to show the power-law behavior more clearly. (b) shows the rescaled complementary cumulative distribution function (ccdf), i.e., $\mathbb{P}\{T_I > \eta t\}$ with factors $\eta = 1, 4, 16$, proportional to D^2.

Figure 1(a) shows $\mathbb{P}\{T_I > t\}$ of two *i.i.d.* mobile nodes under Boolean model with communication range $d = 50m$, each of which follows IRW with constant speed $1\ m/s$ whose step-length distribution is exponential with mean $100m$. As expected, the 'head' of the inter-meeting time distribution displays power-law behavior and is prolonged as the domain size increases. In order to grasp the relationship among τ_0, D, and $\mathcal{M}$, we consider the average displacement of a mobile node A at time t, which is given by $\sigma(t) = \sqrt{\mathbb{E}\{|A(t)|^2\}}$. From the aforementioned reasoning, we set $\sigma(\tau_0) = D$. Since $\sigma(t) \sim \sqrt{t}$ for the IRW with exponential step-length distribution, it follows that $\tau_0 \propto D^2$ for this mobility model. To quantitatively observe this scaling behavior, we consider $\mathbb{P}\{T_I > \tau_0 t\}$, a rescaled version of the ccdf of the inter-meeting time where the time scale is *normalized* such that the typical amount of time to travel D is equal to one time unit, regardless of the domain size. Figure 1(b) shows rescaled versions of $\mathbb{P}\{T_I > \eta t\}$ where η is chosen to be proportional to D^2. As clearly can be seen, the timescales beyond which the inter-meeting time distribution shows exponential decay all coincide, which asserts that τ_0 is indeed proportional to D^2 under the given IRW model.

Next, to quantify the impact of mobility pattern on the location of τ_0, we fix the domain size to $D = 2000m$ and consider a set of IRW models, but now with different mean step-length $\lambda = 10, 50, 100$m. Since the speed is always kept

[4] Note that our definition includes the pairwise inter-meeting time and the aggregate inter-contact time [15] by appropriately choosing the set Θ.

[5] This should be interpreted as a virtual boundary as in [5] in that mobile nodes tend to return to where they belong after reaching out beyond this virtual domain, either by actual reflection (wrapping around) as typically assumed in mobility models or by quasi-periodic returning behavior in reality.



(a) $\mathbb{P}\{T_I > t\}$ (b) Rescaled $\mathbb{P}\{T_I > \eta t\}$

Figure 2: Effect of 'correlation' of the mobility model on the regenerative period τ_0. Domain size is fixed to $2000m \times 2000m$. We use IRW models with exponential step-length distribution with means $\lambda = 10,\ 50,\ 100$ to represent different degrees of 'correlation' (stronger tendency of moving straight for longer duration) (a) $\mathbb{P}\{T_I > t\}$ in log-log scale; (b) shows the rescaled ccdf, $\mathbb{P}\{T_I > \eta t\}$ with factors $\eta = \sqrt{10}, \sqrt{2}, 1$, such that $\eta^2\lambda = \text{Const}$.

to 1 m/s, longer mean step-length naturally translates into stronger tendency of moving straight or preserving the same direction, i.e., stronger correlation over short time scale (until the current step finishes).

Figure 2(a) shows $\mathbb{P}\{T_I > t\}$ in a log-log scale for different mean step-lengths. We notice that larger step-lengths (stronger tendency of moving straight) make τ_0 smaller. This is well expected, since larger step-lengths mean that the mobile node will 'hit' the boundary earlier. Similarly as before, to quantify the scaling behavior with respect to different mean step-lengths, we consider how $\sigma(t)$ scales as the mean step-length increases under a given domain size. While exact computation of $\sigma(t)$ is quite involved, the scaling behavior can be obtained in a relatively simple manner as follows. Suppose $N(t)$ is the number of steps generated over $[0, t]$ under a given IRW model with mean step-length λ. Then, the position of the node at time t is approximated by $A(t) \approx \sum_{i=1}^{N(t)} L_i e^{j\phi_i}$ where L_i $(i = 1, 2, \dots)$ are *i.i.d.* step-lengths with $\mathbb{E}\{L_i\} = \lambda$ and ϕ_i are *i.i.d.* angles uniformly over $[0, 2\pi]$. Thus, $\sigma^2(t) \approx \mathbb{E}\{|\sum_{i=1}^{N(t)} L_i e^{j\phi_i}|^2\}$. Now, suppose that we increase the mean step-length λ to $k\lambda$ $(k > 1)$ and let $\hat{L}_i$ be the exponential random variable with mean $k\lambda$. Then, approximately, there will be $N(t)/k$ number of steps generated over $[0, t]$, while the mean of each step-length becomes k times larger, i.e., $\mathbb{E}\{\hat{L}_i\} = k\lambda$. Since L_i is exponentially distributed, this means that $\mathbb{E}\{\hat{L}_i^2\} = k^2\mathbb{E}\{L_i^2\}$. Thus, we have

$$\hat{\sigma}^2(t) \approx \mathbb{E}\Big\{\big|\sum_{i=1}^{N(t)/k} \hat{L}_i e^{j\phi_i}\big|^2\Big\} \approx k\sigma^2(t).$$

Hence, we have $\hat{\sigma}(t) \approx \sqrt{k}\sigma(t)$, i.e., the average displacement of mobile nodes with k times larger mean step-length grows $\sqrt{k}$ times faster, which implies that the regenerative period (τ_0) becomes $1/\sqrt{k}$ times smaller. In other words, while we increase the mean step-length by k times, $\tau_0^2 k$ always remains constant. As Figure 2(b) shows, after properly rescaling the ccdf $\mathbb{P}\{T_I > \eta t\}$ for different mean step-lengths λ with η chosen such that $\eta^2\lambda$ remains constant throughout, the regenerative periods τ_0 are all aligned around the same value.

The observations made from Figures 1 and 2 suggest that τ_0, the timescale that separates the head and tail of the inter-meeting time distribution, increases as the domain size increases and decreases as the degree of correlations (tendency of moving straight) increases, whereby the exact scaling relationship can be made through the average displacement $\sigma(t)$. Our finding here also matches with empirically observed exponential behaviors of the inter-meeting time over almost all time scale under several popular mobility models such as Random Waypoint (RWP) or RD, since these models have either very short regenerative period (for RWP) or very strong tendency of moving straight (RD), which both contribute to small τ_0, thus making the inter-meeting time distribution exponential virtually everywhere.[6]

While the tail behavior of the inter-meeting time distribution has received much attention in the literature, the quantitative analysis of the 'head' of $\mathbb{P}\{T_I > t\}$ in terms of mobility patterns has been uncharted territory; the only available result in the literature is that the head of $\mathbb{P}\{T_I > t\}$ basically follows a power-law distribution [5, 15], without any attempt to characterize or compare its 'shape' for different mobility models. Therefore, in the next section, we turn our attention to the behavior of the head of the inter-meeting time distribution, i.e., $\mathbb{P}\{T_I > t\}$ for $t < \tau_0$ over which the node rarely hits the boundary and thus the domain is essentially unbounded.

4. STOCHASTIC ORDERING FOR THE HEAD OF INTER-MEETING TIME

In this section, we show how correlations in mobility patterns affect the shape of the head of the inter-meeting time distribution. As mentioned earlier, in this regime, the mobile node practically resides in an unbounded domain (as it rarely hits the boundary). Consider first the inter-meeting time T_I of two independent mobile nodes A and B in an *unbounded* domain, each of which follows correlated mobile trajectory. Then, the difference $C(t) = A(t) - B(t)$ is also correlated over time t and the inter-meeting time of A and B reduces to the inter-hitting time of C to the origin. In order to properly capture the effect of correlations in the mobile trajectories on the inter-meeting time distribution in this regime, we employ a simple, yet effective CRW model introduced in Section 2.1 defined on 1-D grid (unbounded) and consider its inter-hitting time to origin. Recall that for 1-D CRW model, larger p means stronger correlation (stronger tendency to follow the same direction) between adjacent steps.

Define $\rho = p - q$ $(p + q = 1)$. To avoid trivialities, assume $-1 < \rho < 1$: when $\rho = -1$, the mobile node always bounces back and forth between two adjacent sites; when $\rho = 1$, it always goes straight line following the same direction. Note that $\rho = 0$ $(p = q = 0.5)$ corresponds to a simple random walk. Let $X_t = \pm 1$ be the direction (step) of node C. Then, it follows that $\mathbb{P}\{X_{t+1} = X_t\} = p$, $\mathbb{P}\{X_{t+1} = -X_t\} = q$, and the position of node C at time t is given by $S(t) = \sum_{k=1}^{t} X_k$. Note here that when $\rho \neq 0$, $S(t)$ itself is not a Markov chain, but a partial sum of Markov chain X_t. Assume that node C is initially at the origin, i.e., $S(0) = 0$ and we use $X_0 = \pm 1$ to denote the initial condition for the direction,

[6]We note that this is one of the key reasons why there have been many results assuming pure exponential distribution of inter-meeting time (thus leading to either Markovian or ODE based analysis), since those models indeed have exponential inter-meeting time distribution throughout and thus the theoretical results *do* match with the simulation in their setup, until recent empirical measurement of inter-meeting time reveals a different story.

e.g., $X_0 = -1$ means that C's first step, X_1, will be left (right) with probability p (q).

To proceed, we need the following definition:

$$\mathcal{W}_t \triangleq \sum_{k=1}^{t} 1_{\{\mathcal{E}(k)\}}, \tag{6}$$

where $1_{\{\cdot\}}$ is the indicator function and $\mathcal{E}(k) = \{S(k) > 0\} \cup \{\{S(k) = 0\} \cap \{S(k-1) > 0\}\}$ denotes the event that mobile node is on the *positive side* at time k, i.e., the mobile node is on site $S(k) > 0$ or on $S(k) = 0$ with $S(k-1) > 0$. Hence, $\mathcal{W}_t$ counts the number of steps the node is on the positive side out of t steps. From this definition, clearly $0 \le \mathcal{W}_t \le t$ and $t - \mathcal{W}_t$ is the number of steps the node is on the *negative side* out of t steps. The event $\{S(k) = 0\}$ is special here; $\{S(k) = 0\}$ alone cannot determine which side the node belongs to at time k. If the node has jumped from site 1 to site 0, i.e., $S(k-1) = 1$, then we say that the node is on the positive side at time k; or else, it is on the negative side.

From (6) and $S(0) = 0$, we have

$$\mathbb{P}\{T_I > t\} = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1\} = \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = -1\}. \tag{7}$$

To see the first equality in (7), note that from $X_0 = 1$ and $S(0) = 0$, we can define a virtual state $S(-1) = S(0) - X_0 = -1$. Then, $\mathbb{P}\{\mathcal{W}_t = t | X_0 = 1\}$ is the probability that node C starts from site -1 and goes right to the origin and then never returns back to -1 within t steps. Note that in any realization of $\{\mathcal{W}_t = t\}$, if there exists $0 < k \le t$ with $S(k) = 0$, then the node must be on the positive side at time k, since $S(k-1) = -1$ immediately implies $\mathcal{W}_t < t$. From the definition of the inter-hitting time in Section 2.2, this is exactly $\mathbb{P}\{T_I > t\}$.[7] Similarly, the second equality in (7) can also be derived by considering inter-hitting to site 1, with similar virtual state $S(-1) = 1$ and the event that node C always stays on the negative side up to time t.

Define $r(i, m, t) = \mathbb{P}\{\mathcal{W}_t = m | X_0 = i\}$ $(t = 0, 1, \ldots, i = \pm 1)$, with $r(i, 0, 0) = 1$ and $\mathcal{W}_t$ is given by (6). Then, from (7)

$$\mathbb{P}\{T_I > t\} = r(1, t, t) = r(-1, 0, t). \tag{8}$$

We only need to compute either $r(1, t, t)$ or $r(-1, 0, t)$. To this end, for $t \in \mathbb{Z}^+$ and $i, j = \pm 1$, we define

$$\begin{aligned}
&p(i,j,t) = \mathbb{P}\{S_t = 0, X_t = j | X_0 = i\},\\
&f(i,j,t) = \mathbb{P}\{S_t = 0, S_{t-1} \neq 0, \ldots, S_1 \neq 0, X_t = j | X_0 = i\},\\
&p(i,j,0) = 1_{\{i=j\}}, \quad f(i,j,0) = 0.
\end{aligned} \tag{9}$$

Here, $p(i, j, t)$ is the probability that node C returns to the origin at time t, and $f(i, j, t)$ is the probability of the *first* return to the origin at time t (after getting out of it). The event $\{S_t = 0, X_t = j | X_0 = i\}$, i.e., return to the origin with direction j at time $t > 0$ from initial direction i, can be decomposed into (i) the first return to the origin with direction j from initial direction i at some time $1 \le k \le t$ and (ii) a return to the origin with direction j from initial direction j. Hence, $p(i, j, t)$ and $f(i, j, t)$ satisfy the following recursion [18]

$$p(i,j,t) = \sum_{k=1}^{t} f(i,j,k) p(j,j,t-k) + 1_{\{t=0\}} p(i,j,t). \tag{10}$$

Similarly, suppose now that node C starting from the origin does not always stay on either positive or negative side until a given time t. Then, it must have returned to the origin at some earlier time l $(0 < l < t)$. Thus, the event of node C's stay on the positive side for m out of t steps $(0 < m < t)$ can be decomposed into (i) the first return to the origin with direction $j = \pm 1$ at time l from initial direction i and (ii) the node stays on the positive side for $m 1_{\{j=1\}} + (m-l) 1_{\{j=-1\}}$ out of $t - l$ steps. Hence, we have another recursive relationship between $r(i, m, t)$ and $f(i, j, l)$ for $0 < m < t$ as follows:

$$\begin{aligned}
r(i,m,t) = &\sum_{l=1}^{m} f(i,-1,l) r(-1, m-l, t-l)\\
&+ \sum_{l=1}^{t-m} f(i,1,l) r(1, m, t-l).
\end{aligned} \tag{11}$$

For simple (independent) random walk, similar recursion relationship exists [18], although many other nice properties of simple random walk do not apply to CRW model here.

From the 'convolution-like' structures in (10) and (11), it is convenient to use generating functions. Define

$$\begin{aligned}
&P(i,j,z) = \sum_{t=0}^{\infty} p(i,j,t) z^t, \; F(i,j,z) = \sum_{t=0}^{\infty} f(i,j,t) z^t\\
&R(i,z,\phi) = \sum_{m=0}^{\infty} \sum_{t=m}^{\infty} r(i,m,t) z^m \phi^{t-m}.
\end{aligned} \tag{12}$$

Then, multiplying both sides of (10) by z^t leads to

$$P(i,j,z) = 1_{\{i=j\}} + F(i,j,z) P(i,j,z). \tag{13}$$

Similarly, by multiplying $z^m \phi^{t-m}$ in both sides of (11) and considering the case of $m = 0$, t (since (11) holds only under $0 < m < t$), we obtain from (12)

$$\begin{aligned}
R(-1,z,\phi) &- R(-1,z,0) - R(-1,0,\phi) + 1\\
&= [R(-1,z,\phi) - R(-1,z,0)]\, F(-1,-1,z)\\
&\quad + [R(1,z,\phi) - R(1,0,\phi)]\, F(-1,1,\phi),
\end{aligned} \tag{14}$$

which corresponds to the initial condition $i = -1$ (i.e., $X_0 = -1$). Similarly, we can obtain another equation like (14) starting from $i = 1$.

We collect several useful properties for $R(i, z, \phi)$:

LEMMA 1. *We have* $R(-1, z, 0) = R(1, 0, z)$, $R(-1, 0, z) = R(1, z, 0)$, $R(i, z, z) = 1/(1 - z)$ *and*

$$(1 + \rho) [R(-1, z, 0) - 1] = (1 - \rho) [R(1, z, 0) - 1]. \tag{15}$$

PROOF. See Appendix. □

We are now ready to state our main result in this section.

THEOREM 1. *Let T_I be the inter-hitting time of a mobile node C to the origin in an unbounded 1-D grid. Then, for any given $t > 0$, $\mathbb{P}\{T_I > t\}$ is an increasing function of $\rho \in [0, 1)$.*

PROOF. We will first compute $P(i, j, z)$, the generating function of $p(i, j, t)$. Then, we can obtain $\mathbb{P}\{T_I > t\} = r(-1, 0, t)$ from (13), (14) and Lemma 1. Define

$$U(\theta) = \begin{bmatrix} \theta^{-1} p & \theta q \\ \theta^{-1} q & \theta p \end{bmatrix}, \tag{16}$$

and let $U^t(\theta)_{ij}$ be the element on the i^{th} row and j^{th} column of $U^t(\theta)$. For notational convenience, we set $i, j = \pm 1$ and let -1, 1 denote the first and the second row (column), respectively. Then it follows that

$$U^t(\theta)_{ij} = \sum_{k=-\infty}^{\infty} \mathbb{P}\{S_t = k, X_t = j | X_0 = i\} \theta^k.$$

[7] Since all sites are symmetric, the inter-hitting times of node C to any site (not necessarily the origin) must have the same distribution.

From (9), $p(i,j,t) = \mathbb{P}\{S_t = 0, X_t = j | X_0 = i\}$ is equal to the coefficient of θ^0 in $U^t(\theta)_{ij}$. Hence, $P(i,j,z)$ equals to the coefficient of θ^0 in $\sum_{t=0}^{\infty} z^t U^t(\theta)_{ij}$. Now, consider

$$M(z) = \begin{bmatrix} P(-1,-1,z) & P(-1,1,z) \\ P(1,-1,z) & P(1,1,z) \end{bmatrix},$$

which is equal to the coefficient of θ^0 in $\sum_{t=0}^{\infty} z^t U^t(\theta) = [I - zU(\theta)]^{-1}$. For example, we can write

$$P(-1,-1,z) = 1 + \frac{1-\rho z^2}{(1-z^2) f(z,\rho)},\ f(x,y) \triangleq \sqrt{\frac{1-x^2y^2}{1-x^2}}. \quad (17)$$

Recall that $r(-1,0,t)$ (or $r(1,t,t)$) is equal to $\mathbb{P}\{T_I > t\}$. Hence, we only need to derive $R(-1,0,\phi)$.

$$R(-1,0,\phi) = \sum_{m=0}^{\infty} \sum_{t=m}^{\infty} r(-1,m,t) 0^m \phi^{t-m}$$
$$= \sum_{t=0}^{\infty} r(-1,0,t)\phi^t \ \ (\text{since } 0^m = 0 \text{ for } m \neq 0.) \quad (18)$$

Set $z = \phi$ in (14), from (13), (17), and Lemma 1, we get

$$R(-1,0,\phi) = 1 + (1+\phi)\left[f(\phi,\rho) - 1\right] / (\phi(1-\rho)), \quad (19)$$

where $f(\cdot,\cdot)$ is defined in (17). From (18), $r(-1,0,t)$ is the coefficient of ϕ^t in $R(-1,0,\phi)$. Since $r(-1,0,0) = 1$ from initial condition, in what follows, we only consider $t \geq 1$.

From Newton's generalized binomial theorem, we have

$$\sqrt{1-\rho^2\phi^2} = \sum_{m=0}^{\infty} b_m(-\rho^2\phi^2)^m,\ \frac{1}{\sqrt{1-\phi^2}} = \sum_{m=0}^{\infty} c_m(-\phi^2)^m,$$

where $b_m = d_m^{1/2}$, $c_m = d_m^{-1/2}$ with the *generalized binomial coefficient* $d_m^r = \frac{1}{m!}\prod_{i=0}^{m-1}(r-i)$ $(d_0^r = 1)$. Now, for any given $0 \leq \rho < 1$, expand $f(\phi,\rho) = \sum_{t=0}^{\infty} a_t(\rho)\phi^t$ where

$$a_{2k}(\rho) = \sum_{l=0}^{k} (-1)^k b_l c_{k-l} \rho^{2l},\ a_{2k+1}(\rho) = 0,\ (k \geq 0). \quad (20)$$

$a_{2k+1}(\rho) = 0$ for any $k \geq 0$, since $f(\phi,\rho)$ depends only on ϕ^2 for any given ρ.

Set $\rho = 1$ in $f(\phi,\rho)$, then we have

$$f(\phi,1) = \sqrt{(1-\phi^2)/(1-\phi^2)} = 1 = \sum_{t=0}^{\infty} a_t(1)\phi^t.$$

Since this is true for all $|\phi| < 1$, we have $a_t(1) = 0$ for any $t > 0$. From (20), this leads to $\sum_{l=0}^{k}(-1)^k b_l c_{k-l} = 0$, i.e., $b_0 c_k = -\sum_{l=1}^{k} b_l c_{k-l}$. Consequently,

$$a_{2k}(\rho) = (-1)^k b_0 c_k + \sum_{l=1}^{k} (-1)^k b_l c_{k-l} \rho^{2l}$$
$$= \sum_{l=1}^{k} (-1)^{k+1} b_l c_{k-l} (1-\rho^{2l}). \quad (21)$$

From (18), $\mathbb{P}\{T_I > t\} = r(-1,0,t)$ is the coefficient of ϕ^t in $R(-1,0,\phi)$. In light of (19), we have

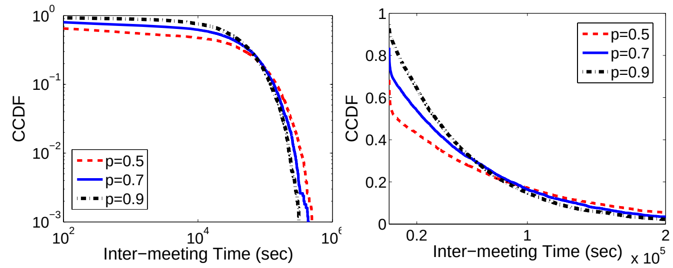
$$\mathbb{P}\{T_I > t\} = r(-1,0,t) = (a_t(\rho) + a_{t+1}(\rho)) / (1-\rho).$$

(20) denotes that only one of $a_t(\rho)$, $a_{t+1}(\rho)$ is non-zero. Suppose $t = 2k$ w.l.o.g. Then, from (21),

$$\mathbb{P}\{T_I > t\} = r(-1,0,t) = \frac{a_t(\rho)}{1-\rho} = \sum_{l=1}^{k} (-1)^{k+1} b_l c_{k-l} \frac{1-\rho^{2l}}{1-\rho}.$$

From direct computation, it is straightforward to see that $(-1)^{k+1} b_l c_{k-l}$ is always positive for any $1 \leq l \leq k$. Therefore, since $(1-\rho^{2l})/(1-\rho)$ is increasing in $\rho \in [0,1)$, $\mathbb{P}\{T_I > t\}$ is also increasing in ρ and we are done. $\square$

Theorem 1 says that the 'head' of the inter-meeting time distribution becomes heavier as the degree of correlation increases. While we provide rigorous proof via 1-D model, the same observation holds for 2-D as can be seen from Figure 3, where larger p (thus stronger tendency to keep the same direction) leads to heavier head of the inter-meeting time distribution $\mathbb{P}\{T_I > t\}$ until τ_0.



(a) log-log (b) linear-linear

Figure 3: Effect of correlations of the mobility model on the ccdf of inter-meeting time. CRW on 2-D grid (200×200) defined in Section 2.1 is used. $p = 0.5,\ 0.7,\ 0.9$ cases are simulated with $q = r = (1-p)/3$. (a) $\mathbb{P}\{T_I > t\}$ in a log-log scale; (b) $\mathbb{P}\{T_I > t\}$ in a linear-linear scale.

In a broader context, Theorem 1 also implies that the head of the inter-meeting time becomes stochastically larger [16] as the tendency of moving straight becomes stronger. Figure 2(a) reveals that this is indeed the case: larger mean step-length in IRW models makes $\mathbb{P}\{T_I > t\}$ larger for $t < \tau_0$.

5. INVARIANCE PROPERTY OF CONTACT-BASED METRICS

In this section we show the *invariance property* of contact-based metrics including the inter-meeting, contact, and inter-hitting time. In particular, the mean of these contact-based metrics does not depend on the degree of correlation ρ.

Consider three independent mobile nodes A, B and C following CRW in a bounded domain starting from their stationary distribution, i.e., uniform over the bounded domain. In contrast to the previous section, we now consider a bounded domain and the inter-meeting time distribution over all time t. For 1-D CRW on a ring, we assume N *is odd* to avoid trivial situation where two nodes never meet if they start from adjacent sites. For the same reason, we also assume $\sqrt{N}$ is odd when 2-D CRW on $\sqrt{N} \times \sqrt{N}$ finite square grid torus is under consideration.

To set the stage for the invariance results, consider a stationary $0-1$ valued process $\{Y_t\}$ $(t = 0,1,2,\ldots)$ with its stationary probability measure $\mathbb{P}$. Define *recurrence times* [2] $\{n_l\}_{l \geq 1}$ of the state 1 for $\{Y_t\}$ as follows:

$$T_1 \triangleq \min\{t \geq 0 : Y_t = 1\},$$
$$T_k \triangleq \min\{t > T_{k-1} : Y_t = 1\}\ (k \geq 2),$$
$$n_l \triangleq T_{l+1} - T_l. \quad (22)$$

Here, T_k denotes the k^{th} occurrence time instance of state 1. For example, for a sequence $\{Y_t\} = \{110010\ldots\}$, $T_1 = 0$, $T_2 = 1$, $T_3 = 4$ and so on. Accordingly, the recurrence times are $n_1 = 1$ and $n_2 = 3$. In this definition, $n_1 = 1$ denotes that the first recurrence time to state 1 is 1. This is somewhat different from what we commonly define the recurrence to state 1, which is usually the return to state

1 *conditioning* on starting from state 1 and subsequently getting out of state 1. For example, $n_2 = T_3 - T_2$ in the above example means the return to state 1 at time $t = 4$ conditioning on starting from state 1 at time $t = 1$ and subsequently getting out of state 1 at time $t = 2$.

The following will be used in the proof of our main result.

THEOREM 2. *[2] For a stationary $0-1$ process $\{Y_t\}$ $(t = 0, 1, 2, \ldots)$ with*

$$\lim_{n\to\infty} \mathbb{P}\{Y_0 = 0, Y_1 = 0, \ldots, Y_n = 0\} = 0, \tag{23}$$

the average recurrence time n_1 conditioning on $Y_0 = 1$ and $Y_1 = 0$ satisfies

$$\mathbb{E}\{n_1|Y_0 = 1,\ Y_1 = 0\} = 1 + \frac{1 - \mathbb{P}\{Y = 1\}}{\mathbb{P}\{Y = 1\} - \mathbb{P}\{Y_0 = 1, Y_1 = 1\}},$$

where Y denotes a random variable with distribution $\mathbb{P}$.

REMARK 2. *Kac's recurrence theorem [14, 10] relates the average of n_1 conditioning on $\{Y_0 = 1\}$, i.e., $\{T_1 = 0\}$, to the probability of a specific event, e.g., $Y_t = 1$. In particular, Kac's recurrence theorem gives $\mathbb{E}\{n_1|Y_0 = 1\} = 1/\mathbb{P}\{Y = 1\}$. Here, $\mathbb{E}\{n_1|Y_0 = 1\} = \mathbb{E}\{T_2 - T_1|Y_0 = 1\} = \mathbb{E}\{T_2|Y_0 = 1\}$. In other words, $1/\mathbb{P}\{Y = 1\}$ is the expected time interval between two adjacent recurrences of state 1. Theorem 2 reformulates Kac's Recurrence Theorem by relating the average of n_l to the probability of events $\{Y_t = 1\}$ and $\{Y_t = 1,\ Y_{t+1} = 1\}$. Specifically, Theorem 2 gives $\mathbb{E}\{n_1|Y_0 = 1, Y_1 = 0\}$, the expected time interval between two recurrences of state 1 given that Y_t first gets out of state 1 (and subsequently moves into state 0 before returning to 1 again). Note that if the state 1 is always of 'short' duration in $\{Y_t\}$, i.e., there is no consecutive 1 in the sequence, $\mathbb{P}\{Y_0 = 1, Y_1 = 1\} = 0$, and thus $\mathbb{E}\{n_1|Y_0 = 1, Y_1 = 0\} = 1/\mathbb{P}\{Y = 1\}$. In this case, Theorem 2 reduced to the original Kac's recurrence theorem.* □

We now present the main result in this section.

THEOREM 3. *Let A, B be two independent mobile nodes following 1-D CRW on a ring with N sites (N odd), and C be a static node sitting on an arbitrary site on this ring. Let (i) T_H be the inter-hitting time of node A to C; (ii) T_I be the inter-meeting time of nodes A and B; (iii) T_C be the contact time of nodes A and B. Then, the average of T_H, T_I and T_C are all* invariant *with respect to the correlation coefficient $\rho \in (-1, 1)$. Specifically,*

$$\mathbb{E}\{T_H\} = N - 1, \tag{24}$$

$$\mathbb{E}\{T_I\} = 2(N - 1), \tag{25}$$

$$\mathbb{E}\{T_C\} = 2. \tag{26}$$

REMARK 3. *For a simple random walk on 1-D ring, i.e., $\rho = 0$, [15] shows (24)*[8] *and (25). N is assumed to be even in [15] while we require it to be odd. This is because in [15] two nodes start from the same site, while in our case two nodes start from anywhere and the inter-meeting time starts once they meet (which is guaranteed only under odd N). We here emphasize that our Theorem 3 holds for CRW with* any *degree of correlation ρ while [15] only considers a simple random walk. It is also interesting to note that stronger correlations, i.e., larger ρ, lead to heavier head of the inter-meeting time as shown in Section 4, but at the same time Theorem 3 says that correlations do* not *affect the mean of the inter-meeting time defined on a bounded domain.* □

[8]The average hitting time given in [15] is N, since [15] includes the last time slot when node A hits C in the definition of the hitting time, while we do not. Hence, there is a difference of 1 between their result and (24).

PROOF. Without loss of generality, assume nodes A, B, C start from their stationary distribution, i.e., uniform over the domain [3]. Let $A(t)$, $B(t) \in \{1, 2, \ldots, N\}$ be the position of nodes A and B at time t $(t = 0, 1, 2, \ldots)$. Since node C is static, we use $C(0)$ to denote its position.

Inter-hitting time T_H: Define Z_t as

$$Z_t = 1_{\{A(t)=C(0)\}}. \tag{27}$$

Since $\{A(t)\}$ is stationary, the sequence $\{Z_t\}$ is also stationary. In order to apply Theorem 2, we need to show that Z_t satisfies (23). To see this, note that while the CRW $A(t)$ itself is not Markov for $\rho \neq 0.5$, $\Gamma_A(t) \triangleq \{A(t), X(t)\}$ (augmented with the direction $X(t)$ of the CRW $A(t)$) becomes a Markov chain with $\mathcal{T} = \{1^-, 1^+, 2^-, 2^+, \ldots, N^-, N^+\}$ as the state space, where m^-, m^+ means that node is at site m with direction $-1, +1$, respectively. Since this chain is aperiodic and irreducible and has finite states ($2N$), it is also ergodic. Thus, the chain $\{\Gamma_A(t)\}$ will visit any set of state infinitely often, including $\{C(0)^-, C(0)^+\}$, which asserts that the condition in (23) is satisfied.

Now, define $\{T_k\}_{k\geq 1}$ and recurrence times $\{n_l\}_{l\geq 1}$ for sequence $\{Z_t\}$ by replacing Y with Z in (22). Then, conditioning on $Z_0 = 1$ and $Z_1 = 0$, $n_1 - 1$ gives the inter-hitting time T_H of node A to C. Thus, from Theorem 2,

$$\mathbb{E}\{T_H\} = \frac{1 - \mathbb{P}\{Z = 1\}}{\mathbb{P}\{Z = 1\} - \mathbb{P}\{Z_0 = 1,\ Z_1 = 1\}},$$

where Z is a random variable with the same distribution as Z_t. Observe that (i) $\mathbb{P}\{Z = 1\} = 1/N$ since $C(0)$ is randomly chosen from N sites (each site has equal probability $1/N$) and (ii) $\mathbb{P}\{Z_0 = 1, Z_1 = 1\} = \mathbb{P}\{Z_1 = 1|Z_0 = 1\}\mathbb{P}\{Z_0 = 1\} = 0$ since once node A meets C, A leaves site $C(0)$ at the next time slot. Therefore, (24) follows.

Inter-meeting time T_I: The proof is quite similar to the case of inter-hitting time as above. First, define $W_t = 1_{\{A(t)=B(t)\}}$. From the stationarity of $A(t)$ and $B(t)$ and from their independence, W_t is also stationary. To check the condition in (23), similarly as before, construct a chain $\Sigma(t) \triangleq \{\Gamma_A(t), \Gamma_B(t)\}$ with state space $\mathcal{T} \times \mathcal{T}$ (total $(2N)^2$ states). From the independence of $\Gamma_A(t)$ and $\Gamma_B(t)$ and since odd N ensures the chain is aperiodic, it follows that $\Sigma(t)$ is also ergodic. Thus, following the same arguments in the proof for inter-hitting time, we can show that W_t also satisfies the condition in (23). Now, we have

$$\mathbb{E}\{T_I\} = \frac{1 - \mathbb{P}\{A(0) = B(0)\}}{\mathbb{P}\{A(0) = B(0)\} - \mathbb{P}\{A(0) = B(0), A(1) = B(1)\}}. \tag{28}$$

Note that $\mathbb{P}\{A(0) = B(0)\} = 1/N$ (since $A(0)$ and $B(0)$ are independent and uniform over N sites). Further, since the direction of a node (A or B) is ± 1 with equal probability under the stationary distribution, it follows that $\mathbb{P}\{A(1) = B(1) \mid A(0) = B(0)\} = 1/2$ and thus (25) holds.

Contact time: Set $V_t = 1 - W_t$. Then, the average contact time can be calculated as a 'dual' of the average inter-meeting time in (28).

$$\mathbb{E}\{T_C\} = \frac{1-\mathbb{P}\{A(0)\neq B(0)\}}{\mathbb{P}\{A(0)\neq B(0)\} - \mathbb{P}\{A(0)\neq B(0), A(1)\neq B(1)\}}$$
$$= \frac{1-(N-1)/N}{(N-1)/N - (N-1)/N \times (1-1/(2N-2))} = 2. \quad (29)$$

In (29), $\mathbb{P}\{A(1) = B(1)|A(0) \neq B(0)\} = 1/(2N-2)$ since conditioning on $A(0) \neq B(0)$, $A(1) = B(1)$ is true only if there is one and only one site between $A(0)$ and $B(0)$ (with probability $2/(N-1)$ and both nodes A, B jump to it at time $t = 1$ (with probability 1/4). □

Much in the same way as in Theorem 3, we can show the invariance property for a set of 2-D CRW models as follows, whose proof is omitted for brevity.

PROPOSITION 1. *For 2-D CRW model with $p+q+2r = 1$, the average of T_H, T_I and T_C are all* invariant *with respect to p, q and r. Specifically,*

$$\mathbb{E}\{T_H\} = N-1, \ \mathbb{E}\{T_I\} = 4(N-1)/3, \ \mathbb{E}\{T_C\} = 2.$$

Recall that the main ingredient for the proof of Theorem 3 is to construct stationary sequence $\{Z_t\}$, $\{W_t\}$, $\{V_t\}$ from $A(t), B(t)$ and show that they all satisfy the condition in (23). Suppose now that all the mobile nodes are independent and have stationary distribution for their positions. Let $\Phi(t) = \{A(t), B_1(t), \ldots, B_N(t)\}$ be the collection of all the positions of mobile nodes in the steady-state. Then, since $f(\Phi(t))$ is also stationary for any measurable function f, we can readily extend the invariance property as in Theorem 3 to at least the following two more general cases:

General inter-contact to set Θ: For $t \geq 0$, define $W_t^{any} = 1_{\{A(t)\in\mathcal{N}_\Theta(t)\}\}}$, where $\mathcal{N}_\Theta(t)$ is the contact set of an arbitrary set of node $\Theta = \{B_i, i \in \mathcal{I}\}$ at time t defined in (1).

General SINR interference model: For $t \geq 0$, define $W_t^{\text{SINR}} = 1_{\{B(t)\in\mathcal{N}_A(t)\}\}}$, where $\mathcal{N}_A(t)$ is the contact set of node A at time t under SINR interference model in (2).

Now, recall from the end of Remark 2 that if the duration of contact is short enough when compared to the typical duration of inter-contact,[9] the average statistics of the above general contact-based metric can be computed *only* from the stationary distribution of the mobile nodes, independently of their detailed mobility structure or the degree of correlations. Since there are many mobility models whose stationary distributions are identical (e.g., RD or IRW with general step-length distribution), our observation suggests that we can enjoy a great deal of simplification as long as the average statistics are concerned. However, there is a caveat; the system must be in stationary regime before we compute those average statistics. Take a sparse network with large domain for example, then unless the initial positions of all the mobile nodes are close to their stationary distributions, it may take very long for them to converge their stationary distribution[10] and it could be that we want packets to be forwarded/routed to other nodes *before* the convergence to stationary regime occurs. Our observation of this invariance property thus suggests an interesting relationship between the role of stationarity of the system (or the assumption that system is in the steady-state) and the first-order statistics of contact-based metrics.

[9]This is a reasonable assumption in sparse MANET with low-density mobile nodes or delay tolerant networks.

[10]This is called *mixing time* in the Markov process literature [10].

	IRW (10)	IRW (50)	IRW (100)	RD
$(200m)^2$	456/83	443/80	448/78	451/80
$(500m)^2$	2103/357	2115/345	2111/362	2123/366
$(1000m)^2$	8375/1557	8403/1501	8241/1526	8401/1453

Table 1: Avg. inter-meeting/inter-any-contact time (Boolean)

Table 1 shows the average inter-meeting/inter-any-contact time in seconds under the same mobility models used in Figure 2 defined over three different domain sizes. We use Boolean model for contact as defined in (1) with communication range set to $d = 50m$. In addition to the set of IRW models with different mean step-lengths, we have also run RD model (see Section 2.1) to represent very strong degree of correlation in the mobility pattern, since under RD model a mobile node always goes straight until it hits the boundary. For inter-meeting time, we consider a pair of mobile nodes. For inter-any-contact time, we consider the inter-contact of a given mobile node A to a set of five given mobile nodes $\{B_i\}$ $(i = 1, 2, \ldots, 5)$. Table 1 clearly shows that, under a given domain size, the average inter-meeting and inter-any-contact time do not depend on mobility patterns with different degrees of correlation, as expected from Theorem 3.

SNR(dB)	IRW (10)	IRW (50)	IRW (100)	RD
0	11.9/514.6	12.4/552.3	12.3/537.1	12.1/522.8
10	21.5/303.0	21.1/327.4	21.4/322.4	20.8/302.4
20	35.0/190.1	33.2/207.5	34.8/192.1	33.1/187.5

Table 2: Avg. contact/inter-meeting time (SINR)

Table 2 shows the average contact/inter-meeting time of a pair of nodes in seconds under the same mobility models as in Table 1. We now use SINR interference model for contact as defined in (2), with parameters $\alpha = -4$, $\beta = 5(\approx 7dB)$. Domain size is set to $400m \times 400m$ throughout. We define $\text{SNR} = P/N_0$ as the value measured at distance $d_0 = 15m$.[11] We vary the signal-to-noise ratio such that $\text{SNR} \in \{0, 10, 20\}dB$. As expected from our Theorem 3 and its extension, the average contact and inter-meeting time remain invariant with respect to the mobility patterns with different degrees of correlation.

REMARK 4. *[23] shows through direct computation that the average inter-meeting time of IRW model under Boolean contact model does not depend on its step-length distribution. In contrast, in this paper, we consider CRW to explicitly capture the degree of correlations in the mobility patterns and rigorously prove the invariance property for not only the inter-meeting time of a given pair, but a set of very general contact-based metrics such as contact time and inter-any-contact time. Note that our results apply equally to Boolean as well as SINR interference model for contact, which is made possible by utilizing the powerful family of Kac's recurrence theorem. Finally, as mentioned earlier, we note that IRW models with longer mean step-lengths can be interpreted as models with stronger correlations in the mobility pattern (stronger tendency to preserve the same direction). In view of this, Tables 1 and 2 confirm that our invariance results hold true for a broader class of mobility models whenever the concept of correlations in the mobility patterns can be applied.* □

[11]d_0 varies according to the communication channel quality requirement.

6. DISCUSSION

Having discovered a number of stochastic properties of contact-based metrics under general mobility models, we now turn our attention to their implication on the performance of network protocols over mobile nodes. First, recall that the invariance property of average statistics for contact-based metrics holds when the system is in stationary regime. Suppose that the system is already in the steady-state, e.g., the initial positions of mobile nodes are drawn from their stationary distributions. Then, the following question naturally arises: will the invariance property lead to the same (or similar) system performance?

The answer to this question is yes when the inter-meeting time is exponentially distributed, or equivalently, when the regenerative period τ_0 is very small, which is the case for RWP and RD models. More precisely, for a class of mobility models whose τ is small enough, the invariance result tells us that their inter-meeting time *distributions* are all the same, which will lead to similar network performance. This is also supported by results in [11, 22, 13, 25, 21], in that the average inter-meeting time critically determines the performance of forwarding algorithms.

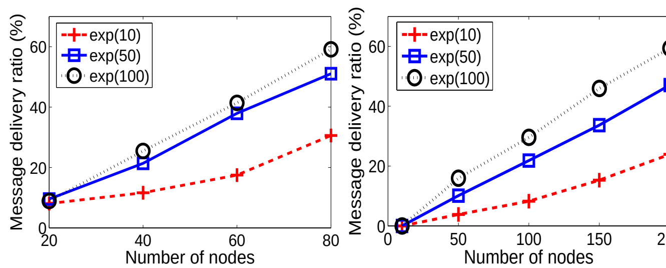
However, when the inter-meeting time distribution severely deviates from exponential,[12] which is the case in reality as shown in [7, 15], the 'head' of the distribution starts to kick in with non-negligible τ_0. In particular, our results in Sections 4 and 5 suggest that *stronger correlations* lead to *heavier head* of the ccdf of the inter-meeting time, which must be offset by *lighter tail* so as to maintain the same average[13] from invariance property for any given domain size. This corresponds to a *less dangerous* ordering [16]:

Definition 2. *X is said to be* less dangerous *than Y if: (i) $\mathbb{E}\{X\} \le \mathbb{E}\{Y\}$, and (ii) their CCDF $\mathbb{P}\{X > t\}$ and $\mathbb{P}\{Y > t\}$ have a unique intersection $t_0 \in \mathbb{R}$ such that $\mathbb{P}\{X > t\} \ge \mathbb{P}\{Y > t\}$ for all $t < t_0$ and $\mathbb{P}\{X > t\} \le \mathbb{P}\{Y > t\}$ for all $t \ge t_0$.* □

For example, in view of the invariance result, Figure 2(a) shows that the inter-meeting time of IRM with exponential step-length distribution of mean $100m$ (say, T_{I100}) is less dangerous than the inter-meeting time of IRM with exponential step-length distribution of mean $10m$ (say, T_{I10}). Figure 3(a) also shows that there exists a dangerous ordering for CRW models as well. Our next step is to employ the following property [16]:

Proposition 2. *If X is less dangerous than Y and $\mathbb{E}\{X\} = \mathbb{E}\{Y\}$, then $X \le_{cv} Y$, i.e., $\mathbb{E}\{\varphi(X)\} \le \mathbb{E}\{\varphi(Y)\}$ for all convex function φ.* □

Thus, from our invariance property, we have $T_{I100} \le_{cv} T_{I10}$. The convex ordering relationship can be used to facilitate system performance analysis. For example, consider the first passage time (FPT) (or the residual/remaining inter-meeting time) defined in (5). FPT is an important mobility metric to evaluate the system performance [7, 15, 1]. As a simple illustration, consider a single packet from a source that is being broadcast to all other N nodes in the network (similar to the epidemic routing [24, 25] but without recovery process). Let t_1 and t_2 ($t_1 < t_2$) be the time instants at which the number of 'infected' nodes (those that have already received a copy of the packet) becomes N_1 and N_1+1, respectively. Thus, $t_2 - t_1$ is the amount of time it takes to increase the number of infected nodes by 1. It is also the residual inter-contact time of the set of infected nodes $\mathcal{B}$ at time t_1 to the set of uninfected nodes $\bar{\mathcal{B}}$ at the same time instant, i.e., $t_2 - t_1$ is the minimum residual inter-contact time of a sequence of residual inter-contact times between one node, say, $A \in \mathcal{B}$ and another node $C \in \bar{\mathcal{B}}$. Such two nodes A and C may or may not have met before t_1. However, when we look at their residual inter-contact time at t_1, as long as the initial positions of all nodes are independent, t_1 is a randomly accessed time instant. In other words, $t2 - t1$ precisely becomes the first passage time to a set of nodes, whose statistics are governed by the equilibrium distribution as in (5). Although this argument does not apply precisely when there are many packets being transmitted at the same time, we expect that FPT still plays a major role[14] in the performance study of network protocol under various configurations, as long as transmissions of different packets do not heavily interfere with each other.[15]



(a) $500m \times 500m$ (b) $2000m \times 2000m$

Figure 4: Effect of correlation in mobility patterns on system performance. Simulation setting is the same as Figure 2(a) and Table 1 with $500m \times 500m$ domain (a) and $2000m \times 2000m$ domain (b). A fixed amount of data (1800 packts) are transmitted using epidemic routing protocol. We use ns-2 with different number of nodes, and measure the message delivery ratio at 600 seconds in (a) and 6000 seconds in (b).

To numerically support the aforementioned argument, we present ns-2 simulation results in Figure 4(a) and (b) for the message delivery ratio of epidemic routing protocol [24] under a set of IRW mobility models as in Figure 2(a). The domain size is set to $500m \times 500m$ (a) and $2000m \times 2000m$ (b). Clearly, different degrees of correlation lead to very different network performance. In this figure, 600 and 6000 seconds are chosen such that the message delivery ratios of one mobility pattern (here we choose exp(100)) in (a) and (b) at the chosen time instants are the same. Note that the difference in network performance is amplified with larger domain size.

While the average inter-meeting time is invariant as seen in Table 1, the average FPT could be adopted to predict the performance ordering shown in Figure 4. Specifically, consider the FPT of T_{I100} and T_{I10} and denote them as T_{F100} and T_{F10}, respectively. Since $T_{I100} \le_{cv} T_{I10}$, we have $\mathbb{E}\{T_{I100}^2\} \le \mathbb{E}\{T_{I10}^2\}$ by taking $\varphi(\cdot) = (\cdot)^2$, which gives

$$\mathbb{E}\{T_{F100}\} = \frac{\mathbb{E}\{T_{I100}^2\}}{2\mathbb{E}\{T_{I100}\}} \le \frac{\mathbb{E}\{T_{I10}^2\}}{2\mathbb{E}\{T_{I10}\}} = \mathbb{E}\{T_{F10}\},$$

[12] We mean the whole shape of the distribution, not just the tail, since the tail of the inter-meeting has been shown to be always exponential [15, 5].

[13] Note that $\mathbb{E}\{T_I\} = \int_0^\infty \mathbb{P}\{T_I > t\}dt$.

[14] Note that if the inter-meeting time is exponentially distributed, so is the FPT with the same mean.

[15] e.g., when incoming traffic density is low, which is typical for MANET.

from (5) and the invariance property. In other words, mobility pattern with stronger correlation produces smaller average FPT (or smaller variance of the inter-meeting time), which could be translated into larger message delivery ratio.

7. CONCLUSION

In this paper we studied the effect of correlation in mobility patterns on the inter-meeting time distribution. We quantified the time scale for transition in inter-meeting time distribution with respect to both domain sizes and correlation in mobility patterns. We then studied the head part of inter-meeting time and proved that stronger correlation in mobility patterns leads to heavier head of inter-meeting time. Further, we derived invariance property for contact-based metrics such as inter-meeting time, contact time and inter-any-contact time under both Boolean and SINR interference models. Numerical findings support our theoretical analysis and also suggest a convex ordering relationship between inter-meeting time distributions produced by mobility patterns with different degrees of correlation. We expect that our study of stochastic properties of contact-based metrics under general mobility models will provide a first step toward detailed performance analysis of various routing/forwarding algorithms and shed light on better design of network protocols under realistic mobility patterns.

Appendix: Proof of Lemma 1

Due to the space limit, we provide sketch of the proof here: (i) the first two equalities can be shown from $r(-1,t,t) = r(1,0,t)$ by (7) and the definition of $R(i,z,\phi)$ in (12). For the third equality, write out $R(i,z,z)$ and exchange the order of double summation (m and t) and then note that $\sum_{m=0}^{t} r(i,m,t) = 1$.

To show (15), since $S(0) = 0$, for $t \geq 1$, if $\mathcal{W}_t = 0$, in all t steps, the node is always on the negative side. Hence, its first step X_1 must be -1. Consequently,

$$\begin{aligned} r(1,0,t) &= \mathbb{P}\{\mathcal{W}_t = 0, X_1 = -1 | X_0 = 1\} \\ &= \mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1, X_1 = -1\} \cdot q. \end{aligned}$$

Similarly $r(1,t,t) = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1, X_1 = 1\} \cdot p$. By symmetry, we have

$$\mathbb{P}\{\mathcal{W}_t = 0 | X_0 = 1, X_1 = -1\} = \mathbb{P}\{\mathcal{W}_t = t | X_0 = 1, X_1 = 1\}.$$

Since $r(-1,t,t) = r(1,0,t)$, for $t \geq 1$,

$$p \cdot r(-1,t,t) = q \cdot r(1,t,t). \tag{30}$$

Note that when $t = 0$, $r(-1,t,t) = r(1,t,t) = 1$. Then, $R(-1,z,0) - 1 = \sum_{m=1}^{\infty} r(-1,m,m) z^m$ and $R(1,z,0) - 1 = \sum_{m=1}^{\infty} r(1,m,m) z^m$. From (30), this immediately lead to $p\,[R(-1,z,0) - 1] = q\,[R(1,z,0) - 1]$. Lastly, from $p = (1+\rho)/2$ and $q = (1-\rho)/2$, (15) follows.

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