Heterogeneity in Contact Dynamics: Helpful or Harmful to Forwarding Algorithms in DTNs?

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Abstract*—In this paper we focus on how the heterogeneous contact dynamics of mobile nodes impact the performance of forwarding/routing algorithms in delay/disruption-tolerant networks (DTNs). To this end, we consider two representative heterogeneous network models, each of which captures heterogeneity among node pairs (individual) and heterogeneity in underlying environment (spatial), respectively, and examine the full extent of difference in delay performances they cause on forwarding/routing algorithms through formal stochastic comparisons. We first show that these heterogeneous models correctly capture non-Poisson contact dynamics observed in real traces. Then, we consider direct forwarding and multicopy two-hop relay protocol and rigorously establish *stochastic/convex ordering relationships* on their delay performances under these heterogeneous models and the corresponding homogeneous model, all of which have the same average inter-contact time over all node pairs. We show that heterogeneous models predict an entirely opposite ordering relationship in the delay performances depending on which of the two heterogeneities is captured. This suggests that merely capturing non-Poisson contact dynamics – even if the *entire distribution* of aggregated inter-contact time is precisely matched, is not enough and that one should carefully evaluate the performance of forwarding/routing algorithms under a properly chosen heterogeneous network setting. Our results will also be useful in correctly exploiting the underlying heterogeneity structure so as to achieve better performance in DTNs.

I. Introduction

In delay/disruption-tolerant networks (DTNs), frequent disruptions in end-to-end connectivity arise due to many factors such as random node mobility, power limitations, etc. Hence, to overcome the intermittent connectivity, mobile nodes relay or copy messages to other mobile nodes upon encounter, which ensures that the messages eventually reach their destinations. The performance of message delivery depends on how to relay or copy messages to mobile nodes. Thus, many forwarding/routing algorithms such as epidemic routing [23], two-hop relay [12], spray and wait [21], to name a few, have been proposed and commonly analyzed based upon a 'homogeneous' network model in which contacts between any pair of nodes occur according to a Poisson process. This model is justified in [12], [23] by observing that the inter-contact time[*] between two successive contacts for any node pair follows an exponential distribution via numerical simulations under synthetic mobility models. Recent works still resort to this homogeneous model for an analysis on the content distribution and for an analytical development of forwarding policies [13], [2].

However, measurement studies [19], [8], [15], [14], [3] from real traces reveal the existence of heterogeneity in contact dynamics, which all make the contact dynamics deviate from Poisson. In particular, [15], [6] shows that the inter-contact time distribution is a mixture of power-law and exponential distributions. In addition, [19], [8], [14], [3] show that there exists a significant degree of heterogeneity in mobile nodes' contact dynamics. [19], [14] further exploit empirically the observed heterogeneity for designing of new forwarding/routing algorithms. [8], [9], [3] introduce two heterogeneous network models, each of which captures the observed *individual* or *spatial* heterogeneity. In the individually heterogeneous network model [8], [9], the heterogeneity is characterized by allowing different contact rates for different node pairs, while the inter-contact time distribution of each pair is still exponential (but with different rates). On the other hand, in the spatially heterogeneous network model [3], the heterogeneity arises on each spatial cluster (site) in which mobile nodes reside, while they can move to the other spatial clusters. (See Section II for more details.) However, none of these works analytically investigates how the heterogeneity structure impacts the performance of forwarding/routing algorithms, not to mention whether the considered heterogeneity improves (helpful) or deteriorates (harmful) the performance.

In this paper, we examine how the forwarding/routing performance under the two representative heterogeneous network models [8], [9], [3] deviates from that under the aforementioned homogeneous model, aka the Poisson contact model. We first show that *each* of the two heterogeneous models correctly captures the non-Poisson contact dynamics (non-exponential inter-contact time distribution of a random pair of nodes) as observed in real traces. Then, we rigorously establish stochastic/convex ordering relationships among the delay performances of direct forwarding and multicopy two-hop relay protocol[†] [12], [23], [1] under the two heterogeneous models and the corresponding homogeneous model, all of

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[*]The inter-contact time of two mobile nodes is defined as the time interval from when their communication becomes unavailable to the time when the communication resumes. See [15], [5] for its formal definition.

[†]These algorithms are 'oblivious' to the underlying heterogeneous network structure.

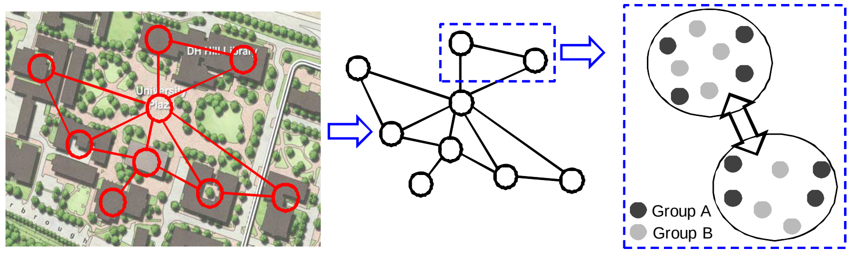


Fig. 1. An example for spatial and social (individual) heterogeneity in an opportunistic campus mobile network (or a campus-wise DTN).

which are *indistinguishable* from the average inter-contact time point of view. Our technical contributions in this paper can be summarized as follows.

- We prove that the message delivery delays of direct forwarding and multicopy two-hop relay protocol under the spatially heterogeneous model are *stochastically larger* than those under the corresponding homogeneous model, respectively.
- We also prove that the message delivery delay of direct forwarding under the individually heterogeneous model is *larger* than that under the corresponding homogeneous model *in convex ordering* (see Section IV for its formal definition), while the average delay of multicopy two-hop relay protocol under the individually heterogeneous model is *smaller* than that under the corresponding homogeneous model.
- As a special case of the above results, we show that if the average inter-contact time over all node pairs is the same for the three models, then $\mathbb{E}\{D^{[2]}_{\text{ind}}\} \leq \mathbb{E}\{D^{[2]}_{\text{hom}}\} \leq \mathbb{E}\{D^{[2]}_{\text{spa}}\}$, where $D^{[2]}_{\text{ind}}, D^{[2]}_{\text{spa}}$, and $D^{[2]}_{\text{hom}}$ are the message delivery delay of multicopy two-hop relay protocol under the individually and spatially heterogeneous models and the corresponding homogeneous model, respectively. The heterogeneity structure in the spatially heterogeneous model deteriorates the average delay performance of multicopy two-hop relay protocol, whereas the another heterogeneity structure in the individually heterogeneous model improves its average delay performance when compared with that under the corresponding homogeneous model. This implies that each of the two heterogeneous models predicts an entirely opposite average delay performance.
- We further show that the delay performance of direct forwarding under the spatially heterogeneous model is stochastically larger than that under the individually heterogeneous model, even when the *entire distributions* of aggregated inter-contact times over all node pairs under both heterogeneous models are precisely matched.

The rest of this paper is organized as follows. Section II gives preliminaries on the formal description of two representative (individually and spatially) heterogeneous network models. Section III presents the characteristic of inter-contact time under each of the two heterogeneous models. Sections IV and V provide our theoretical results on the stochastic comparison of message delivery delays for direct forwarding and multicopy two-hop relay protocol under each of the two heterogeneous models and the corresponding homogeneous model, respectively. We conclude in Section VI.

II. Preliminaries

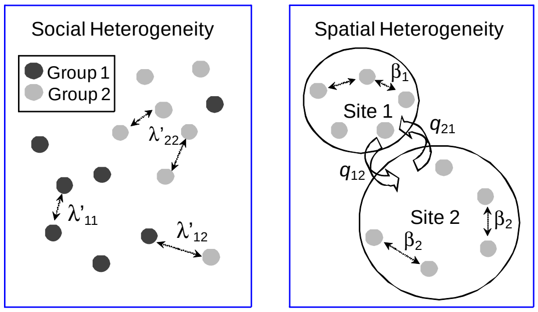
In this section, we present the details of two heterogeneous network models to be used for our paper. In general, mobile nodes typically belong to different societal groups, with different preferred sites following different mobility patterns. For example, in Figure 1, there exist several popular places (e.g., library, dormitory, or dining hall) in a campus and students may form spatially separate clusters around the popular places, while occasionally move to other clusters according to their own daily schedules. Further, in each spatial cluster, students from different groups (e.g., ECE/CS departments or undergraduate/graduate) typically mix together, but making more frequent contacts with others from the same group than from different groups. These social (or individual) and spatial heterogeneity structures can be captured under the following two heterogeneous network models [8], [9], [3], each of which directly characterizes the heterogeneity in mobile nodes' contact dynamics in a different manner, rather than defining detailed mobile trajectories inside a small domain or group (or social 'clique').

A. Individually Heterogeneous Network Model

An individually heterogeneous network model (simply, an individual model) is introduced in [8], [9] and described as follows. Consider a set of mobile nodes $\mathcal{N}$. The pairwise inter-contact time between nodes i and j is drawn from an exponential distribution with rate λ_{ij}. Thus, the heterogeneity is characterized by allowing different contact rates λ_{ij}, where $i, j \in \mathcal{N}$ and $i \neq j$.

If $\lambda_{ij} = \lambda$ for all $i, j \in \mathcal{N}$ and $i \neq j$, then the individual model reduces to the homogeneous model. A social heterogeneity structure can be also captured as a special case of the individual model. Suppose that there are K different social groups (memberships) G_i $(i = 1, \ldots, K)$ forming a partition of $\mathcal{N}$, i.e., $\mathcal{N} = \bigcup_{i=1}^{K} G_i$. Let λ'_{lk} be common contact rate between any member of G_l and another member of G_k, $l, k \in \{1, 2, \ldots, K\}$. That is, for any nodes $i \in G_l$ and $j \in G_k$, $\lambda_{ij} = \lambda'_{lk}$ $(l, k \in \{1, 2, \ldots, K\}$ and $i \neq j)$. Figure 2(a) shows an example with $K = 2$.

In [8], the individual model is validated by observing that empirical pairwise inter-contact time distributions for a



(a) A case of two social groups (b) A case of two spatial clusters

Fig. 2. Examples of the individual and spatial models.

large portion of node pairs can be well fitted by exponential distributions but with different rates.

B. Spatially Heterogeneous Network Model

A spatially heterogeneous network model (simply, a spatial model) is introduced in [3] and described formally as follows. Consider a set of mobile nodes $\mathcal{N}$. There are M different spatial clusters (or preferred sites). Let S_i be a spatial cluster (site) i, where $i \in \{1, \dots, M\}$. Then, each mobile node moves and encounters with others independently between sites and within each site as follows:

(i) Each mobile node in site S_i moves to site S_j with rate q_{ij} at any time t.

(ii) Any pair of mobile nodes in site S_i has Poisson contacts with rate β_i, i.e. the inter-contact time distribution for a node pair in site S_i is exponentially distributed with mean $1/\beta_i$.

Figure 2(b) depicts an example with two spatial clusters (sites). Let $X(t) \in \{S_1, \dots, S_M\} \triangleq \Omega$ be the site that a mobile node belongs to at time t. From the condition (i), $\{X(t)\}_{t\geq 0}$ is a continuous time Markov chain with infinitesimal generator $\mathbf{Q} = \{q_{ij}\}$. We assume $\{X(t)\}$ is irreducible, i.e., any mobile node can reach everywhere in finite time with positive probability. For analytical simplicity, we also assume that $q_{ij} = q_{ji}$, i.e., the transition rates of mobile nodes between sites S_i and S_j are the same.

In addition, the condition (ii) is supported in [3] by empirically observing that 90% of all the inter-contacts gathered in a confined area, a subset of whole network domain, approximately follows an exponential distribution but with different rates over different subsets. The Poisson contacts over a small confined area has been also theoretically justified in [5], regardless of the mobility pattern of each mobile node inside that small confined area. In this spatial model, the heterogeneity arises by allowing different contact rates β_i over different sites.

III. Inter-contact Time under Heterogeneous Network Models

In this section, we show that each heterogeneous network model can capture non-Poisson contact dynamics as observed in real traces. For notational simplicity, we enumerate each of node pairs and define an index set for the node pair as $\mathcal{I} = \{1, \dots, |\mathcal{N}|(|\mathcal{N}| - 1)/2\}$. We also define by I a random variable to indicate a random node pair, which is uniformly distributed over $\mathcal{I}$. Further, we define by T_I and T_i the *aggregate* inter-contact time over all node pairs and *pairwise* inter-contact time for a given node pair $i \in \mathcal{I}$, respectively. Here, the aggregate inter-contact time distribution can be obtained by randomizing the pairwise inter-contact time distributions over all node pairs, i.e.,

$$\mathbb{P}\{T_I > t\} = \mathbb{E}\{\mathbb{P}\{T_I > t|I\}\} = \sum_{i\in\mathcal{I}} \mathbb{P}\{T_i > t\}\frac{1}{|\mathcal{I}|}.$$

We will use different superscripts 'ind','spa', and 'hom' to distinguish T_I and T_i for the individual, spatial, and homogeneous models, respectively.

From the definition of the individual model, it follows that

$$\mathbb{P}\{T_i^{\mathrm{ind}} > t\} = e^{-\lambda_i t}, \text{ and } \mathbb{P}\{T_I^{\mathrm{ind}} > t\} = \sum_{i\in\mathcal{I}} e^{-\lambda_i t}\frac{1}{|\mathcal{I}|},$$

where λ_i is the contact rate of a given node pair $i \in \mathcal{I}$. We can rewrite this as

$$\mathbb{P}\{T_I^{\mathrm{ind}} > t\} = \mathbb{E}\{e^{-t/X_{\mathrm{ind}}}\}, \tag{1}$$

where X_{ind} is a discrete random variable taking values $\frac{1}{\lambda_i}$ with probability $\frac{1}{|\mathcal{I}|}$. Note that the actual distribution of X_{ind} can be quite general by suitably setting λ_i.[‡]

For the spatial model, we have the following results:

Proposition *1:* For the spatial heterogeneous model as defined earlier, we have for any $i \in \mathcal{I}$,

$$\mathbb{P}\{T_i^{\mathrm{spa}} > t\} = \mathbb{P}\{T_I^{\mathrm{spa}} > t\} = \mathbb{E}\{e^{-t/X_{\mathrm{spa}}}\}, \tag{2}$$

for some positive random variable X_{spa}. □

Proof: See Appendix A. ■

Proposition 1 says that the inter-contact time for spatial model follows a hyper-exponential distribution. Here, the random variable X_{spa} depends on $\mathbf{Q}$ and β_i. We refer to the proof of Proposition 1 for more details. From (1)–(2) and by noting that $\mathbb{E}\{T\} = \int_0^\infty \mathbb{P}\{T > t\}dt$, we have

$$\mathbb{E}\{T_I^{\mathrm{ind}}\} = \mathbb{E}\{X_{\mathrm{ind}}\} = \sum_{i\in\mathcal{I}} \frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}, \text{ and } \mathbb{E}\{T_I^{\mathrm{spa}}\} = \mathbb{E}\{X_{\mathrm{spa}}\}.$$

It was addressed in [10] how to approximate a power-law (heavy-tail) distribution in the regions of *primary interest* by a mixture of exponentials while the approximated distribution still has an exponential tail. This implies that the observed 'dichotomic' inter-contact time distribution with power-law and exponential mixture [15] can be approximated by hyper-exponential distributions within any desired degree of accuracy.

Throughout the rest of this paper, we focus on stochastic comparison of message delivery delays for direct forwarding

[‡]For example, setting $\lambda_1 = \lambda_2 \neq \lambda_i$ for $i \geq 3$ will give non-uniform distribution while setting $\lambda_i = \lambda$ for all i makes $X_{\mathrm{ind}} = 1/\lambda$, for which the aggregate inter-contact time follows a pure exponential distribution.

and multicopy two-hop relay protocol under the individual, spatial, and corresponding homogeneous models. Here, by the homogeneous model we mean that for all $i \in \mathcal{I}$,

$$\mathbb{P}\{T_i^{\text{hom}} > t\} = \mathbb{P}\{T_I^{\text{hom}} > t\} = e^{-t/\tau}, \quad (3)$$

where $\tau := \mathbb{E}\{X_{\text{ind}}\} = \mathbb{E}\{X_{\text{spa}}\}$ is the average inter-contact time over all node pairs for both heterogeneous models. Thus, under the constructed homogeneous model, the inter-contact time of any pair of nodes is exponentially distributed (thus giving Poisson contacts) with the same average aggregated inter-contact time as the other heterogeneous models.

In the following stochastic comparisons in Sections IV and V, we assume the followings as in other analytical works [12], [23], [1], [9], [3]. There is no resource constraint (i.e., infinite buffer) at each mobile node. Also, inter-contact times between any two node pairs are mutually independent. Further, message transfers between any two nodes are instantaneous at their contact epochs. This assumption is reasonable when the transmission time of a message between two nodes is relatively small (negligible) with respect to their inter-contact time.

IV. Impact of Heterogeneity in Delay Performance under Direct Forwarding

In this section, we first stochastically compare the delay performances of direct forwarding (i.e., a source node waits until it meets a destination node to deliver a message) under the three models. Although the direct forwarding is very simple and there certainly exist other algorithms with better performance, its performance can serve as a basis for performance evaluation or prediction of two-hop or multi-hop forwarding/routing algorithms. Let $D_{\text{ind}}^{[1]}$, $D_{\text{spa}}^{[1]}$, and $D_{\text{hom}}^{[1]}$ be the message delivery delay of direct forwarding under the individual, spatial, and homogeneous models, respectively. Source and destination pairs are uniformly chosen over $\mathcal{I}$. To proceed, we need the following definitions for the stochastic and convex orders:

Definition *1:* [20] For two random variables X and Y, X is said to be larger than Y *in the stochastic order* (denoted by $X \geq_{st} Y$) if $\mathbb{P}\{X > u\} \geq \mathbb{P}\{Y > u\}$ for all $u \in \mathbb{R}$, or equivalently if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ for any increasing function ϕ for which the expectation exists. □

Definition *2:* [20] For two random variables X and Y, X is said to be larger than Y *in the convex order* (denoted by $X \geq_{cx} Y$) if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ for any convex function ϕ for which the expectation exists. We also define *a concave order* (denoted by $X \geq_{cv} Y$) if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ for any concave function ϕ. □

Similarly, we further define an increasing convex (resp. increasing concave) order, written $X \geq_{icx} Y$ (resp. $X \geq_{icv} Y$), if $\mathbb{E}\{\phi(X)\} \geq \mathbb{E}\{\phi(Y)\}$ for any increasing convex (resp. increasing concave) function ϕ. It immediately follows from Definitions 1–2 that if $X \geq_{st} Y$, then $X \geq_{icx} Y$ and $X \geq_{icv} Y$. Similarly, if $X \geq_{cx} Y$ (resp. $X \geq_{cv} Y$), then $X \geq_{icx} Y$ (resp. $X \geq_{icv} Y$). Also, by noting that f is concave if $-f$ is convex, from Definition 2, $X \geq_{cx} Y$ implies $X \leq_{cv} Y$.

Further, from Definition 1, if $X \geq_{st} Y$, then $\mathbb{E}\{X\} \geq \mathbb{E}\{Y\}$. Also, from Definition 2, if $X \geq_{cx} Y$, then $\mathbb{E}\{X\} = \mathbb{E}\{Y\}$ and $\text{Var}\{X\} \geq \text{Var}\{Y\}$ by taking $\phi(\cdot) = (\cdot)^2$.

The message delivery delay for a given source and destination pair is nothing but their residual (or remaining) inter-contact time after the message is generated at the source node. First, for the homogeneous model, we have

$$\mathbb{P}\{D_{\text{hom}}^{[1]} > t\} = \mathbb{P}\{T_I^{\text{hom}} > t\} = e^{-t/\tau}$$

due to the memoryless property of the exponential inter-contact time distribution with mean τ for any pair of nodes. Similarly, for the individual model,

$$\mathbb{P}\{D_{\text{ind}}^{[1]} > t | I = i\} = \mathbb{P}\{T_I^{\text{ind}} > t | I = i\} = e^{-\lambda_i t}$$

for a given pair $i \in \mathcal{I}$, thus from (1), we have

$$\begin{aligned}\mathbb{P}\{D_{\text{ind}}^{[1]} > t\} &= \mathbb{E}\{\mathbb{P}\{D_{\text{ind}}^{[1]} > t | I\}\} = \mathbb{P}\{T_I^{\text{ind}} > t\} \\ &= \mathbb{E}\{e^{-t/X_{\text{ind}}}\}.\end{aligned}$$

However, for the spatial model, the inter-contact time of a given pair $i \in \mathcal{I}$ is no longer memoryless but of hyper-exponential form as in (2). Under stationary regime, note that the residual inter-contact time R_i of a pair $i \in \mathcal{I}$ follows the equilibrium distribution of T_i [7], [1], i.e., $\mathbb{P}\{R_i > t\} = \frac{1}{\mathbb{E}\{T_i\}} \int_t^\infty \mathbb{P}\{T_i > u\} du$. Then, from (2), we can write for any $i \in \mathcal{I}$,

$$\begin{aligned}\mathbb{P}\{R_i > t\} &= \frac{1}{\mathbb{E}\{T_i\}} \int_t^\infty \mathbb{E}\{e^{-u/X}\} du \\ &= \frac{1}{\mathbb{E}\{T_i\}} \mathbb{E}\left\{\int_t^\infty e^{-u/X} du\right\} = \frac{1}{\mathbb{E}\{X\}} \mathbb{E}\{X e^{-t/X}\}, \quad (4)\end{aligned}$$

where T_i and X here represent T_i^{spa} and X_{spa} for the spatial model, respectively. Since (4) holds for any $i \in \mathcal{I}$, we have

$$\mathbb{P}\{D_{\text{spa}}^{[1]} > t\} = \mathbb{P}\{R_i > t\} = \frac{\mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\}}{\mathbb{E}\{X_{\text{spa}}\}}. \quad (5)$$

Now, we present our results for stochastic comparison on the delay performances of direct forwarding under the individual, spatial, and homogeneous models.

Proposition *2:* Let $X_{\text{ind1}},\ X_{\text{ind2}}$ be random variables in (1) for two different scenarios under the individual model, and $D_{\text{ind1}}^{[1]},\ D_{\text{ind2}}^{[1]}$ be the corresponding message delivery delays of direct forwarding. Then, if $X_{\text{ind1}} \geq_{cx} X_{\text{ind2}}$, we have $D_{\text{ind1}}^{[1]} \geq_{cx} D_{\text{ind2}}^{[1]}$. □

Proof: By noting that $\mathbb{P}\{D_{\text{ind}}^{[1]} > t\} = \mathbb{E}\{e^{-t/X_{\text{ind}}}\}$ and $\mathbb{E}\{X_{\text{ind1}}\} = \mathbb{E}\{X_{\text{ind2}}\}$, we have $\mathbb{E}\{D_{\text{ind1}}^{[1]}\} = \mathbb{E}\{D_{\text{ind2}}^{[1]}\}$. Thus, in order to prove $D_{\text{ind1}}^{[1]} \geq_{cx} D_{\text{ind2}}^{[1]}$, it is enough to show [20]

$$\int_a^\infty \mathbb{P}\{D_{\text{ind1}}^{[1]} > t\} dt \geq \int_a^\infty \mathbb{P}\{D_{\text{ind2}}^{[1]} > t\} dt,$$

for all $a > 0$. It is equivalent to showing that

$$\mathbb{E}\{X_{\text{ind1}} e^{-a/X_{\text{ind1}}}\} \geq \mathbb{E}\{X_{\text{ind2}} e^{-a/X_{\text{ind2}}}\}, \quad (6)$$

for all $a > 0$.

Let $g(x) := xe^{-a/x}$. It is easy to check that $g(x)$ is an increasing convex function in $x > 0$ for all $a > 0$, i.e., $g'(x) > 0$ and $g''(x) \geq 0$. Thus, from $X_{\text{ind1}} \geq_{cx} X_{\text{ind2}}$ and Definition 2, the above inequality (6) holds for all $a > 0$ by taking $\phi(x) = xe^{-a/x}$. This completes the proof. ■

Proposition 2 says the message delivery delay gets stochastically larger in the sense of convex order as the underlying individual model becomes 'more heterogeneous' (in larger convex ordering of X). In particular, if $\mathbb{E}\{T_I^{\text{ind}}\} = \mathbb{E}\{T_I^{\text{hom}}\} = \tau$ (the same average aggregated inter-contact time under the individual and homogeneous models), we have

$$D_{\text{ind}}^{[1]} \geq_{cx} D_{\text{hom}}^{[1]},$$

since $X_{\text{ind}} \geq_{cx} \mathbb{E}\{X_{\text{ind}}\} = \mathbb{E}\{T_I^{\text{ind}}\} = \tau$. This means that the message delivery delay of direct forwarding under individual model is *more variable* than that under the homogeneous model, while the average delays under both models are the same.

Proposition *3:* If $\mathbb{E}\{T_I^{\text{spa}}\} = \mathbb{E}\{T_I^{\text{hom}}\}$, then $D_{\text{spa}}^{[1]} \geq_{st} D_{\text{hom}}^{[1]}$. □

Proof: Recall that

$$\mathbb{P}\{D_{\text{spa}}^{[1]} > t\} = \frac{1}{\mathbb{E}\{X_{\text{spa}}\}} \mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\},$$
$$\mathbb{P}\{D_{\text{hom}}^{[1]} > t\} = e^{-t/\tau}.$$

Let X_{hom} be a random variable that takes the value $\tau = \mathbb{E}\{X_{\text{spa}}\}$ with probability 1. Then, we can write

$$\mathbb{P}\{D_{\text{hom}}^{[1]} > t\} = \frac{1}{\mathbb{E}\{X_{\text{hom}}\}} \mathbb{E}\{X_{\text{hom}} e^{-t/X_{\text{hom}}}\}.$$

Since $X_{\text{spa}} \geq_{cx} X_{\text{hom}}$ and $xe^{-t/x}$ is an increasing convex function in $x > 0$ for all $t > 0$ as shown before, we have

$$\mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\} \geq \mathbb{E}\{X_{\text{hom}} e^{-t/X_{\text{hom}}}\}$$

for all $t > 0$, and thus $\mathbb{P}\{D_{\text{spa}}^{[1]} > t\} \geq \mathbb{P}\{D_{\text{hom}}^{[1]} > t\}$ for all $t > 0$. By Definition 1, this completes the proof. ■

Proposition 3 says that the message delivery delay of direct forwarding under the spatial model is stochastically larger than that under the homogeneous model, when the average inter-contact time under both models are matched. From Propositions 2 and 3, we see that the delay performance of direct forwarding under each heterogeneous model deviates from that under the homogeneous model in a different manner, though three models are the same in the average inter-contact time point of view over all node pairs.

Next, we compare the delay performances of direct forwarding under the spatial and individual models, when their *entire distributions* of the aggregate inter-contact time remain identical. This can be achieved by setting $X_{\text{spa}} \overset{d}{=} X_{\text{ind}}$ in (1)–(2). Still, our next result tells us that the delay of direct forwarding under the spatial model is always stochastically larger than that under the individual model.

Proposition *4:* If $T_I^{\text{spa}} \overset{d}{=} T_I^{\text{ind}}$, then $D_{\text{spa}}^{[1]} \geq_{st} D_{\text{ind}}^{[1]}$. □

Proof: Recall that

$$\mathbb{P}\{D_{\text{spa}}^{[1]} > t\} = \frac{1}{\mathbb{E}\{X_{\text{spa}}\}} \mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\},$$
$$\mathbb{P}\{D_{\text{ind}}^{[1]} > t\} = \mathbb{E}\{e^{-t/X_{\text{ind}}}\}.$$

Since $e^{-t/x}$ is increasing in $x > 0$ for any given $t > 0$, we have

$$\mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\} \geq \mathbb{E}\{X_{\text{spa}}\}\mathbb{E}\{e^{-t/X_{\text{spa}}}\}. \tag{7}$$

Then, from the assumption that

$$\mathbb{P}\{T_I^{\text{spa}} > t\} = \mathbb{E}\{e^{-t/X_{\text{spa}}}\} = \mathbb{E}\{e^{-t/X_{\text{ind}}}\} = \mathbb{P}\{T_I^{\text{ind}} > t\}$$

for any given $t > 0$, and from (5) and (7) we have

$$\begin{aligned}\mathbb{P}\{D_{\text{spa}}^{[1]} > t\} &= \frac{1}{\mathbb{E}\{X_{\text{spa}}\}} \mathbb{E}\{X_{\text{spa}} e^{-t/X_{\text{spa}}}\} \\ &\geq \mathbb{E}\{e^{-t/X_{\text{spa}}}\} = \mathbb{E}\{e^{-t/X_{\text{ind}}}\} = \mathbb{P}\{D_{\text{ind}}^{[1]} > t\},\end{aligned}$$

for any given $t > 0$. From Definition 1, the result follows. ■

To sum up, from Propositions 2–4, we observe that the performance of direct forwarding varies depending on which of the two heterogeneous models is chosen, i.e., how the non-Poisson contact dynamics observed in the real traces are modeled. In addition, the aggregated inter-contact time statistics (the whole distribution) are still insufficient to correctly predict the forwarding/routing performance, even though many existing works [7], [15], [3] have relied on the aggregated inter-contact time samples to uncover the characteristics of mobile nodes' contact patterns and justify their modeling choices.

V. Impact of Heterogeneity in Delay Performance under Multicopy Two-hop Relay

We now turn our attention to multicopy two-hop relay protocol [12], [23], [1] as a test case for a further investigation of the impact of the heterogeneity structure on the forwarding/routing performance. In this protocol, only source node can replicate a message and forward its copy to any relay node that does not have the message copy upon encounter.

Consider the delivery of a single message in the network with $|\mathcal{N}| = n + 2$. Given a pair of source s and destination d which is uniformly chosen over $\mathcal{I}$, there are n possible relay nodes $(r_1, r_2, \ldots, r_n)$. We denote T_{ij} and R_{ij} to be the pairwise inter-contact time of a given node pair (i, j) and their residual inter-contact time, respectively, where $i, j \in \{s, r_1, \ldots, r_n, d\}$ and $i \neq j$. Then, as shown in [1], the message delivery delay of the multicopy two-hop relay protocol (denoted by D) – the time interval from the time when the message is generated at a source node to the time when any copy of the message first reaches its destination, is given by

$$D \overset{d}{=} \min\{R_{sd}, R_{sr_1} + R_{r_1 d}, \ldots, R_{sr_n} + R_{r_n d}\}. \tag{8}$$

As before, $D_{\text{ind}}^{[2]}$, $D_{\text{spa}}^{[2]}$, and $D_{\text{hom}}^{[2]}$ denote the message delivery delay of multicopy two-hop relay protocol under individual, spatial, and homogeneous models, respectively. Here, we use superscript $D^{[2]}$ to indicate the multicopy two-hop relay protocol, whereby $D^{[1]}$ was used for the direct forwarding (single-hop) protocol in Section IV.

We first show that the stochastic ordering relationship in Proposition 3 still holds for the message delay delays of multicopy two-hop relay protocol under the spatial and homogeneous models.

Proposition *5:* If $\mathbb{E}\{T_I^{\text{spa}}\} = \mathbb{E}\{T_I^{\text{hom}}\}$, then $D_{\text{spa}}^{[2]} \geq_{st} D_{\text{hom}}^{[2]}$. □

Proof: Since the pairwise inter-contact time of any node pair is statistically equivalent to that of other node pairs under each of the spatial and homogeneous models, we only need to consider the message delivery delay of any given source and destination pair which is uniformly chosen over $\mathcal{I}$.

Let R_{ij}^{spa} and R_{ij}^{hom} be the residual inter-contact time of a given node pair (i,j) under the spatial and homogeneous models, respectively, where $i,j \in \{s, r_1, \ldots, r_n, d\}$ and $i \neq j$. From Proposition 3, we have $R_{ij}^{\text{spa}} \geq_{st} R_{ij}^{\text{hom}}$. The stochastic order is also closed under convolutions [20]. Thus, $R_{sd}^{\text{spa}} \geq_{st} R_{sd}^{\text{hom}}$ and $R_{sr_i}^{\text{spa}} + R_{r_id}^{\text{spa}} \geq_{st} R_{sr_i}^{\text{hom}} + R_{r_id}^{\text{hom}}$ $(i = 1, 2, \ldots, n)$. Then, it easily follows that these stochastic ordering relationships still hold for their first order statistic, i.e.,

$$\begin{aligned}\min\{&R_{sd}^{\text{spa}}, R_{sr_1}^{\text{spa}} + R_{r_1d}^{\text{spa}}, \ldots, R_{sr_n}^{\text{spa}} + R_{r_nd}^{\text{spa}}\}\\ &\geq_{st} \min\{R_{sd}^{\text{hom}}, R_{sr_1}^{\text{hom}} + R_{r_1d}^{\text{hom}}, \ldots, R_{sr_n}^{\text{hom}} + R_{r_nd}^{\text{hom}}\}.\end{aligned}$$

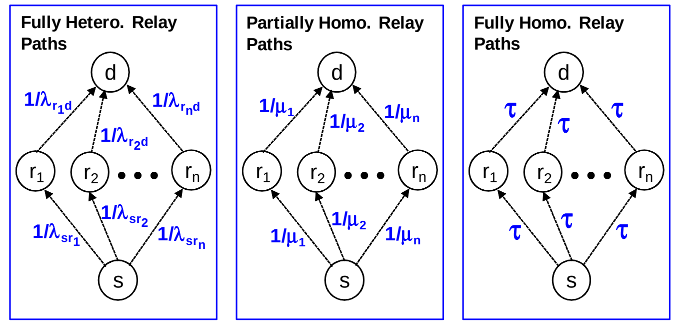
That is, $D_{\text{spa}}^{[2]} \geq_{st} D_{\text{hom}}^{[2]}$. This completes the proof. ■

Proposition 5 also implies that the hyper-exponential inter-contact time yields stochastically larger delay than the exponential inter-contact time for the mulicopy two-hop relay protocol when their average inter-contact times are matched.

In the following stochastic comparison, we assume that each message reaches its destination via relay nodes only and the direct path from source to destination is not considered. This may be the case with a moderate to large number of mobile nodes (e.g., campus-wide DTNs), i.e., the 'best' of n relay nodes is likely to reach the destination earlier than the source node does. We first compare the message delivery delays of mutlicopy two-hop relay protocol for *a given source and destination pair* under the individual and homogeneous models, as the pairwise inter-contact times are statistically different for different node pairs under the individual model. Later on, we will continue our stochastic comparison on the message delivery delays for *a uniformly and randomly chosen source and destination pair* under both models.

Let $D_{\text{ind}(s,d)}^{[2]}$ be the message delivery delay for a given source and destination (s,d) pair under the individual model. Here, for a proper comparison between $D_{\text{ind}(s,d)}^{[2]}$ and $D_{\text{hom}}^{[2]}$, we set the average inter-contact time for the corresponding homogeneous model as

$$\tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right]. \tag{9}$$



(a) (b) (c)

Fig. 3. Three different settings of n two-hop relay paths with varying degrees of heterogeneity: (a) a fully heterogeneous setting, (b) a partially homogeneous setting where each relay path is homogeneous. (i.e., two-hop components in each relay path have the same average inter-contact time as $1/\mu_i = \frac{1}{2}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$), and (c) a fully homogeneous setting where the average inter-contact time for any node pair is $\tau = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$.

That is, the average inter-contact time for any node pair $i \in \mathcal{I}$ under the constructed homogeneous model is simply the arithmetic mean of the average inter-contact times over all node pairs in n two-hop relay paths under the individual model. Let T_{ij}^{ind} and T_{ij}^{hom} be the inter-contact time of each node pair (i,j) under the individual and homogeneous models, respectively, where $i,j \in \{s, r_1, \ldots, r_n, d\}$ and $i \neq j$. Then, due to the memoryless property of exponential pairwise inter-contact time distributions under the individual and homogeneous models, $D_{\text{ind}(s,d)}^{[2]}$ and $D_{\text{hom}}^{[2]}$ are given by

$$D_{\text{ind}(s,d)}^{[2]} = \min\{T_{sr_1}^{\text{ind}} + T_{r_1d}^{\text{ind}}, \ldots, T_{sr_n}^{\text{ind}} + T_{r_nd}^{\text{ind}}\}, \tag{10}$$

$$D_{\text{hom}}^{[2]} = \min\{T_{sr_1}^{\text{hom}} + T_{r_1d}^{\text{hom}}, \ldots, T_{sr_n}^{\text{hom}} + T_{r_nd}^{\text{hom}}\}. \tag{11}$$

Instead of directly comparing $D_{\text{ind}(s,d)}^{[2]}$ with $D_{\text{hom}}^{[2]}$, we compare *each* of these message delivery delays with that under a *partially homogeneous* setting (a special case of the individual model). Figure 3(b) shows this partially homogeneous setting in which the delay over each relay path is now a sum of two *i.i.d.* exponential random variables with mean $\frac{1}{2}[1/\lambda_{sr_i} + 1/\lambda_{r_id}]$ (homogeneous for a given path, but heterogeneous over different paths). Let S_{sr_i} and S_{r_id} be *i.i.d.* exponential random variables with mean

$$\frac{1}{\mu_i} := \frac{1}{2}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_id}}\right], \tag{12}$$

where $i = 1, \ldots, n$. Then, the message delivery delay in this partially homogeneous model, $\tilde{D}_{\text{ind}(s,d)}^{[2]}$, is given by

$$\tilde{D}_{\text{ind}(s,d)}^{[2]} = \min\{S_{sr_1} + S_{r_1d}, \ldots, S_{sr_n} + S_{r_nd}\}. \tag{13}$$

Figure 3 depicts the aforementioned three different settings of n two-hop relay paths with varying degrees of heterogeneity over the average inter-contact times in the network.

To proceed, we collect several definitions on majorization [16] ordering. This is a partial order over vectors of real numbers and is useful in capturing the degree of heterogeneity in vector components.

Definition *3:* [16] For $\vec{x}, \vec{y} \in \mathbb{R}^n$, $\vec{x}$ is said to be *majorized* by $\vec{y}$, or $\vec{y}$ majorizes $\vec{x}$, (written $\vec{x} \prec \vec{y}$), if $\sum_{i=1}^{m} x_{[i]} \le \sum_{i=1}^{m} y_{[i]}$, $(m = 1, 2, \ldots, n-1)$, and $\sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]}$, where $x_{[1]} \ge \cdots \ge x_{[n]}$ $(y_{[1]} \ge \cdots \ge y_{[n]})$ denote the components of $\vec{x}$ (resp. $\vec{y}$) in decreasing order. □

From (12) and Definition 3, we have

$$\left(\frac{1}{\lambda_{sr_i}}, \frac{1}{\lambda_{r_i d}}\right) \succ \left(\frac{1}{\mu_i}, \frac{1}{\mu_i}\right) \tag{14}$$

for any $\lambda_{sr_i}, \lambda_{r_i d} > 0$, and $(1/\mu_i, 1/\mu_i)$ is the smallest in the sense of majorization ordering. Further, note that from (9) and (12),

$$\tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_i d}}\right] = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\mu_i}.$$

This implies that

$$\left(\frac{1}{\mu_1}, \ldots, \frac{1}{\mu_n}\right) \succ (\tau, \ldots, \tau) \tag{15}$$

for any $\mu_i > 0$.

Definition *4:* [16] For $\vec{x}, \vec{y} \in \mathbb{R}^n$, a real-valued function ψ defined on $\mathbb{R}^n$ is said to be *Schur-convex*, if $\vec{x} \prec \vec{y}$ implies $\psi(\vec{x}) \le \psi(\vec{y})$. Similarly, ψ is said to be *Schur-concave*, if $\vec{x} \prec \vec{y}$ implies $\psi(\vec{x}) \ge \psi(\vec{y})$. □

We also need the following result on the preservation of the increasing concave ordering.

Proposition *6:* [18, Proposition 9.5.4] If $X_1, \ldots, X_n$ are independent random variables and $Y_1, \ldots, Y_n$ are independent random variables, and $X_i \ge_{icv} Y_i$, where $i \in \{1, \ldots, n\}$, then

$$f(X_1, \ldots, X_n) \ge_{icv} f(Y_1, \ldots, Y_n)$$

for all increasing and componentwise concave function f. □

Now we present our main result on the stochastic comparison among $D^{[2]}_{\text{ind}(s,d)}$, $\tilde{D}^{[2]}_{\text{ind}(s,d)}$, $D^{[2]}_{\text{hom}}$ – the message delivery delay of multicopy two-hop relay protocol over the network setting in Figure 3(a), (b), (c), respectively.

Theorem *1:* If $\mathbb{E}\{T_I^{\text{hom}}\} = \tau = \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{1}{\lambda_{sr_i}} + \frac{1}{\lambda_{r_i d}}\right]$, then $D^{[2]}_{\text{ind}(s,d)} \le_{icv} \tilde{D}^{[2]}_{\text{ind}(s,d)} \le_{st} D^{[2]}_{\text{hom}}$. □

Proof: **Proof of** $D^{[2]}_{\text{ind}(s,d)} \le_{icv} \tilde{D}^{[2]}_{\text{ind}(s,d)}$**:** Let U_1 and U_2 be *i.i.d.* exponential random variables with rate one. Then, observe that

$$T^{\text{ind}}_{sr_i} \overset{d}{=} \frac{1}{\lambda_{sr_i}}U_1, \text{ and } T^{\text{ind}}_{r_i d} \overset{d}{=} \frac{1}{\lambda_{r_i d}}U_2.$$

Similarly, we have

$$S_{sr_i} \overset{d}{=} \frac{1}{\mu_i}U_1, \text{ and } S_{r_i d} \overset{d}{=} \frac{1}{\mu_i}U_2.$$

Thus, from the independence of $T^{\text{ind}}_{sr_i}$ and $T^{\text{ind}}_{r_i d}$ and the independence of S_{sr_i} and $S_{r_i d}$, we have

$$\begin{aligned} T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d} &\overset{d}{=} \frac{1}{\lambda_{sr_i}}U_1 + \frac{1}{\lambda_{r_i d}}U_2, \\ S_{sr_i} + S_{r_i d} &\overset{d}{=} \frac{1}{\mu_i}U_1 + \frac{1}{\mu_i}U_2. \end{aligned} \tag{16}$$

Note that if $X_1, \ldots, X_n$ are exchangeable random variables, then for any convex function f, $\psi(\vec{a}) = \mathbb{E}\{f(\sum a_i X_i)\}$ is Schur-convex on $\mathbb{R}^n$ [16, p.287, Proposition B.2]. Thus, from (14), (16), and Definition 4, we have,

$$\begin{aligned} \mathbb{E}\{f(T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d})\} &= \mathbb{E}\left\{f\left(\frac{1}{\lambda_{sr_i}}U_1 + \frac{1}{\lambda_{r_i d}}U_2\right)\right\} \\ &\ge \mathbb{E}\left\{f\left(\frac{1}{\mu_i}U_1 + \frac{1}{\mu_i}U_2\right)\right\} = \mathbb{E}\{f(S_{sr_i} + S_{r_i d})\}, \end{aligned}$$

for any convex function f. Equivalently, from Definition 2, we have $T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d} \ge_{cx} S_{sr_i} + S_{r_i d}$ for each $i = 1, \ldots, n$. As mentioned in Section IV, it follows that

$$\begin{aligned} T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d} \ge_{cx} S_{sr_i} + S_{r_i d} &\Rightarrow T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d} \le_{cv} S_{sr_i} + S_{r_i d} \\ &\Rightarrow T^{\text{ind}}_{sr_i} + T^{\text{ind}}_{r_i d} \le_{icv} S_{sr_i} + S_{r_i d}, \end{aligned}$$

for any $i = 1, \ldots, n$. Then, since $\min\{x_1, \ldots, x_n\}$ is increasing on $\mathbb{R}^n$ and concave in each argument x_i, from (10), (13), and Proposition 6, we have

$$D^{[2]}_{\text{ind}(s,d)} \le_{icv} \tilde{D}^{[2]}_{\text{ind}(s,d)}. \tag{17}$$

Proof of $\tilde{D}^{[2]}_{\text{ind}(s,d)} \le_{st} D^{[2]}_{\text{hom}}$**:** Let $\nu_i := \frac{1}{\mu_i}$ and

$$g(\nu_i) := \mathbb{P}\{S_{sr_i} + S_{r_i d} > t\} = (1 + \frac{t}{\nu_i})e^{-t/\nu_i}$$

for any $t > 0$. We also define another function $h : \mathbb{R}^n_{++} \to \mathbb{R}_{++}$ by

$$h(\vec{\nu}) := \mathbb{P}\left\{\tilde{D}^{[2]}_{\text{ind}(s,d)} > t\right\} = \prod_{i=1}^{n} g(\nu_i),$$

where $\vec{\nu} := (\nu_1, \nu_2, \ldots, \nu_n)$. It is straightforward to check $\log g(\nu_i)$ is concave in $\nu_i > 0$ for all $t > 0$. Then, since $g(\nu_i)$ is log-concave, $h(\vec{\nu}) = \prod g(\nu_i)$ is Schur-concave on $\mathbb{R}^n_{++}$ [16, p.73, Proposition E.1]. Thus, from (11), (13), (15) and Definition 4, we have for any $\vec{\nu}$,

$$\begin{aligned} \mathbb{P}\left\{\tilde{D}^{[2]}_{\text{ind}(s,d)} > t\right\} &= h(\nu_1, \ldots, \nu_n) \\ &\le h(\tau, \ldots, \tau) = \mathbb{P}\left\{D^{[2]}_{\text{hom}} > t\right\}, \end{aligned}$$

for any given $t > 0$. In order words, by Definition 1,

$$\tilde{D}^{[2]}_{\text{ind}(s,d)} \le_{st} D^{[2]}_{\text{hom}}. \tag{18}$$

From (17) and (18), we are done. ■

By noting that $\le_{st} \Rightarrow \le_{icv}$, Theorem 1 implies that if $\mathbb{E}\{T_I^{\text{hom}}\} = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_i d}]$, then

$$D^{[2]}_{\text{ind}(s,d)} \le_{icv} D^{[2]}_{\text{hom}}.$$

Since $\phi(x) = x$ is increasing and concave, from the definition of increasing concave order, it further implies

$$\mathbb{E}\{D^{[2]}_{\text{ind}(s,d)}\} \le \mathbb{E}\{D^{[2]}_{\text{hom}}\}.$$

We now move on to the stochastic comparison on message delivery delays for a *uniform* source and destination pair. Note that the average message delivery delay of a uniform pair is nothing but the arithmetic mean of the average message delivery delays over all possible $|\mathcal{N}|(|\mathcal{N}|-1)/2$ source and destination (s,d) pairs. Then, since $\mathbb{E}\{D^{[2]}_{\text{ind}(s,d)}\}$ is upper-bounded by $\mathbb{E}\{D^{[2]}_{\text{hom}}\}$ for each (s,d) pair when $\mathbb{E}\{T_I^{\text{hom}}\} = \frac{1}{2n}\sum_{i=1}^{n}[1/\lambda_{sr_i} + 1/\lambda_{r_i d}]$, after computing the arithmetic mean of upper bounds on $\mathbb{E}\{D^{[2]}_{\text{ind}(s,d)}\}$ over all (s,d) pairs, we obtain the following corollary.

Corollary *1:* If $\mathbb{E}\{T_I^{\text{hom}}\} = \mathbb{E}\{T_I^{\text{ind}}\} = \sum_{i\in\mathcal{I}} \frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}$, then $\mathbb{E}\{D^{[2]}_{\text{ind}}\} \leq \mathbb{E}\{D^{[2]}_{\text{hom}}\}$. □

Proof: See Appendix B. ■

As shown in Theorem 1 and Corollary 1, the path diversity (heterogeneity) over n relay paths under the individual model results in better delay performance of multicopy two-hop relay protocol. It also turns out that Proposition 5 still holds under the same scenario (i.e., no direct path is used) considered in Corollary 1. Thus, under this scenario, if $\mathbb{E}\{T_I^{\text{spa}}\} = \mathbb{E}\{T_I^{\text{ind}}\} = \mathbb{E}\{T_I^{\text{hom}}\}$, we have

$$\mathbb{E}\{D^{[2]}_{\text{ind}}\} \leq \mathbb{E}\{D^{[2]}_{\text{hom}}\} \leq \mathbb{E}\{D^{[2]}_{\text{spa}}\}.$$

This means that the heterogeneity structure in the spatial model makes the average delay performance of multicopy two-hop relay protocol worse, whereas the another heterogeneity structure in the individual model is beneficial to its average delay performance when compared with that under the corresponding homogeneous model.

From our theoretical results, we expect that the delay performance of other two-hop and multi-hop forwarding/routing protocols under each heterogeneous model differs considerably from that under the homogeneous model and there exists a significant performance gap between the two heterogeneous models as we observed, *even when the entire aggregate inter-contact time distributions are precisely matched.* Further studies are still required as to how to correctly capture and model the underlying heterogeneity in mobile nodes' contact dynamics to correctly compare and accurately estimate the actual forwarding/routing performance under more general protocols. We leave this as a future work.

VI. Conclusion

In this paper we have mainly focused on how the underlying heterogeneity structure in mobile nodes' contact dynamics impacts the performance of forwarding/routing algorithms in DTNs. Based upon two representative heterogeneous network models, we have investigated their non-Poisson contact dynamics and stochastically compared their delay performances of direct forwarding and multicopy two-hop relay protocol with those under the homogeneous model. In particular, our findings show that each heterogeneous model predicts an entirely opposite delay performance when compared with that under the homogeneous model. This calls for much more careful studies on the forwarding/routing performance under non-Poisson contacts, and perhaps more importantly, under properly chosen heterogeneous models.

Appendix A
Proof of Proposition 1

Here, we prove that the inter-contact time of any node pair which is uniformly chosen in $\mathcal{I}$ has a hyper-exponential distribution for the spatial model. By the definition of the spatial model, the distribution of inter-contact time of a node pair is identical to the others, it is enough to show the inter-contact time distribution of a given node pair $i \in \mathcal{I}$.

Let T_{AB} be inter-contact time between randomly chosen nodes A and B. Without loss of generality, we assume that a contact between nodes A and B occurs at time 0. We denote $\{A(t)\}_{t\geq 0}$ and $\{B(t)\}_{t\geq 0}$ to be Markov chains for a state (a spatial cluster) that nodes A and B belong to at time t, respectively. In this proof, we use state i to indicate state S_i for notational simplicity ($\Omega = \{1,2,\ldots,M\}$). Since the Markov chain is irreducible and its state space is finite, it is ergodic and thus there exists a stationary probability π_i, where $i \in \Omega$ (i.e., $\vec{\pi}$ such that $\vec{\pi}\mathbf{Q} = \vec{0}$) [4]. We assume that the system is in the steady-state with its stationary distribution $\vec{\pi}$. Recall that the infinitesimal generator $\mathbf{Q}$ of the Markov chains is given by

$$\mathbf{Q} = \begin{bmatrix} -q_1 & q_{12} & \cdots & q_{1M} \\ q_{21} & -q_2 & \cdots & q_{2M} \\ \vdots & \vdots & \vdots & \ddots \\ q_{M1} & q_{M2} & \cdots & -q_M \end{bmatrix},$$

where $q_i = \sum_{k\neq i} q_{ik}$. We also define a matrix $\mathbf{B}$ by $\mathbf{B} = \text{diag}\{\beta_1, \beta_2, \ldots, \beta_M\}$. From the definition of the spatial model, we know that a contact process based on $\mathbf{B}$ between nodes A and B is modulated by $\{A(t)\}_{t\geq 0}$ and $\{B(t)\}_{t\geq 0}$. In order words, a contact between nodes A and B happens according to a Poisson process with rate β_i, only when two nodes reside in the same state i ($i \in \Omega$). One can expect the similarity as a point process between the contact process under the spatial model and an arrival process governed by the Markov Modulated Poisson Process (MMPP) [11] widely used in teletraffic engineering. In the MMPP, packet arrivals occur according to a Poisson process with a different rate which is modulated by an irreducible continuous time Markov chain.

To show the clear relationship with the MMPP, we first define a stochastic process $C(t) := (A(t), B(t))$ to represent a pair of the states that nodes A and B belong at time t, as the contact process is modulated by the pair of the states. The process $C(t)$ is a continuous time Markov chain $\{C(t)\}_{t\geq 0}$ with state space $S = \{1,\ldots,M\}\times\{1,\ldots,M\}$ and infinitesimal generator $\mathbf{Q}' = [\ q'(\underline{u},\underline{v})\ ]_{\underline{u},\underline{v}\in S}$. In particular, $\mathbf{Q}'$ is given by

$$\mathbf{Q}' = \begin{bmatrix} -q'_1 & q_{12} & q_{13} & \cdots & 0 \\ q_{21} & -q'_2 & q_{23} & \cdots & 0 \\ q_{31} & q_{32} & -q'_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -q'_{M^2} \end{bmatrix}.$$

Here, the entries of the generator $\mathbf{Q}'$ are ordered lexicographically, i.e., $(1,1),(1,2),\ldots,(1,M),(2,1),\ldots,(M,M)$, and

$q'_l = \sum_{k\neq i} q_{ik} + \sum_{k\neq j} q_{jk}$, where $l = M(i-1)+j$ and $i,j \in \{1,2,\ldots,M\}$. The steady state probability vector $\vec{\pi}'$ for $\{C(t)\}_{t\geq 0}$ can be represented in terms of π_i and is given by $\vec{\pi}' = [\pi_1^2\ \pi_1\pi_2\ \cdots\ \pi_1\pi_M\ \pi_2\pi_1\ \cdots\ \pi_M^2]$.

For notational convenience, we define another $M^2 \times M^2$ matrix $\mathbf{B}'$ from the $M \times M$ matrix $\mathbf{B}$, as the generator $\mathbf{Q}'$ is a $M^2 \times M^2$ matrix. The $\mathbf{B}'$ is a diagonal matrix with $\mathbf{B}'_{jj} = \beta_i$ if $j = M(i-1)+i$, otherwise zero, where $j \in \{1,2,\ldots,M^2\}$ and $i \in \{1,2,\ldots,M\}$. Thus, the contact process based on $\mathbf{B}'$ between nodes A and B is modulated by $\{C(t)\}_{t\geq 0}$. That is, when the Markov chain $\{C(t)\}_{t\geq 0}$ is in the state (i,i), contacts occur according to the Poisson process of β_i. Therefore, it has exactly the same structure of MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) [11].

Consider the epochs of successive contacts in the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) to obtain the pairwise inter-contact time distribution between nodes A and B, i.e., $\mathbb{P}\{T_{AB} > t\}$. As mentioned above, the contact process between nodes A and B starts at an "arbitrary contact epoch", i.e., $t = 0$ is a contact epoch. It is called *interval-stationary* process in the MMPP [11]. We denote J_n, $n \geq 0$, to be the state of the Markov chain $\{C(t)\}_{t\geq 0}$ associated with the n^{th} contact (J_0 is the state at $t = 0$). We also denote X_n, $n \geq 1$, to be the inter-contact time between the $(n-1)^{st}$ and the n^{th} contacts ($X_0 = 0$). Then, the sequence $\{(J_n, X_n), n \geq 0\}$ is a Markov renewal sequence with transition probability matrix [11], [17]

$$\mathbf{F}(\mathbf{t}) = \int_0^t e^{(\mathbf{Q}'-\mathbf{B}')u} du \mathbf{B}' = \left[\mathbf{I} - e^{(\mathbf{Q}'-\mathbf{B}')t}\right](\mathbf{B}'-\mathbf{Q}')^{-1}\mathbf{B}'$$
$$= \left[\mathbf{I} - e^{(\mathbf{Q}'-\mathbf{B}')t}\right]\mathbf{F}(\infty), \quad (19)$$

where the element $F_{ij}(t)$ of $\mathbf{F}(\mathbf{t})$ is the conditional probability $\{J_n = j, X_n \leq t \mid J_{n-1} = i\}$ for $n \geq 1$, and $\mathbf{I}$ is a $M^2 \times M^2$ identity matrix.

The matrix $\mathbf{F}(\infty) = (\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}'$ is stochastic and its stationary vector $\vec{p}$ is given by [11], [17]

$$\vec{p} = \vec{p}\ (\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}' = \frac{1}{\vec{\pi}'\vec{\beta}'}\vec{\pi}'\mathbf{B}',$$

where $\vec{\beta}'_{M^2\times 1} := [\beta'_1\ \beta'_2\ \cdots\ \beta'_{M^2}]^T$. Here, $\beta'_j = \beta_i$ if $j = M(i-1)+i$, otherwise zero, where $j \in \{1,\ldots,M^2\}$ and $i \in \{1,\ldots,M\}$. Thus, an element of $\vec{p}$ is $\frac{\pi_i^2\beta_i}{\sum_{k=1}^M \pi_k^2\beta_k}$ if the element is for the state (i,i), otherwise 0. Here, since the contact process between nodes A and B governed by the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) is interval-stationary, the initial probability vector $\{J_0\}$ of the MMPP with ($\mathbf{Q}'$, $\mathbf{B}'$) is chosen to be $\vec{p}$. Thus, since $\mathbb{P}\{T_{AB} > t\} = \mathbb{P}\{X_n > t\}$ for $n \geq 1$, from (19) with $\vec{p}$, we have

$$\mathbb{P}\{T_{AB} > t\} = \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}(\mathbf{B}' - \mathbf{Q}')^{-1}\mathbf{B}'\vec{e}$$
$$= \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}\vec{e}, \quad (20)$$

where $\vec{e}_{M^2\times 1} = [1\ 1\ \cdots\ 1]^T$. The second equality is from the fact that the matrix $(\mathbf{B}'-\mathbf{Q}')^{-1}\mathbf{B}'$ is stochastic. Note that (20) is the marginal distribution of an inter-contact time between two successive contact epochs, and it is the form of a matrix exponential.

Recall that $q_{ij} = q_{ji}$ in the generator $\mathbf{Q}$, where $i,j \in \Omega$. Hence, it is easy to see that the matrix $\mathbf{Q}'-\mathbf{B}'$ is symmetric, and thus its eigenvalues and eigenvectors are real. By the spectral theorem [22], the matrix $\mathbf{Q}'-\mathbf{B}'$ can be diagonalized by an orthogonal matrix. In other words, $\mathbf{Q}'-\mathbf{B}' = \mathbf{M}\mathbf{U}\mathbf{M}^{-1}$, where $\mathbf{M}$ is a $M^2 \times M^2$ orthogonal matrix containing orthonormal eigenvectors of $\mathbf{Q}'-\mathbf{B}'$, and $\mathbf{U}$ is a $M^2 \times M^2$ diagonal matrix in which each diagonal element is its an eigenvalue. Thus, (20) becomes

$$\mathbb{P}\{T_{AB} > t\} = \vec{p}\ e^{(\mathbf{Q}'-\mathbf{B}')t}\vec{e} = \vec{p}\ \mathbf{M}e^{\mathbf{U}t}\mathbf{M}^{-1}\vec{e}. \quad (21)$$

Here, since all the eigenvalues of $\mathbf{Q}' - \mathbf{B}'$ are real and $e^{(\mathbf{Q}'-\mathbf{B}')t} \to \mathbf{0}$ as $t \to \infty$ in (19), all the eigenvalues (in $\mathbf{U}$) should be negative [22]. Therefore, (21) becomes a weighted sum of exponentials (i.e., hyper-exponential). This completes the proof. ■

APPENDIX B
PROOF OF COROLLARY 1

Recall that Theorem 1 implies that if $\mathbb{E}\{T_I^{\text{hom}}\} = \tau = \frac{1}{2n}\sum_{i=1}^n [1/\lambda_{sr_i} + 1/\lambda_{r_id}]$, then

$$\mathbb{E}\{D^{[2]}_{\text{ind}(s,d)}\} \leq \mathbb{E}\{D^{[2]}_{\text{hom}}\}. \quad (22)$$

Here, from this result, we show that if $\mathbb{E}\{T_I^{\text{hom}}\} = \mathbb{E}\{T_I^{\text{ind}}\} = \sum_{i\in\mathcal{I}} \frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}$, then $\mathbb{E}\{D^{[2]}_{\text{ind}}\} \leq \mathbb{E}\{D^{[2]}_{\text{hom}}\}$. First, observe that

$$\mathbb{E}\{D^{[2]}_{\text{hom}}\} = \int_0^\infty \mathbb{P}\{\min\{T^{\text{hom}}_{sr_1} + T^{\text{hom}}_{r_1d}, \ldots, T^{\text{hom}}_{sr_n} + T^{\text{hom}}_{r_nd}\} > t\}dt$$
$$= \int_0^\infty \prod_{i=1}^n \mathbb{P}\{T^{\text{hom}}_{sr_i} + T^{\text{hom}}_{r_id} > t\}dt = \int_0^\infty (1+t/\tau)^n e^{-nt/\tau}dt$$
$$= \frac{\tau}{n}\sum_{i=0}^n \frac{n!}{(n-i)!n^i}.$$

For notational simplicity, let $f(n) := \frac{1}{n}\sum_{i=0}^n \frac{n!}{(n-i)!n^i}$. Hence, we have

$$\mathbb{E}\{D^{[2]}_{\text{hom}}\} = \tau f(n). \quad (23)$$

As mentioned earlier, the average message delivery delay of a uniform source and destination pair is equivalent to the arithmetic mean of the average message delivery delays over all possible $|\mathcal{N}|(|\mathcal{N}|-1)/2$ source and destination (s,d) pairs. Here, since the average message delivery delay for each (s,d) pair is different depending on which (s,d) pair is chosen, we use (i,j) instead of (s,d) to clearly distinguish each source and destination pair, where $i,j \in \mathcal{N} \triangleq \{1,2,\ldots,n+2\}$. Then, the average message delivery delay of a uniform source and destination pair is given by

$$\begin{aligned}
\mathbb{E}\{D_{\rm ind}^{[2]}\} &= \frac{2}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j>i}^{n+2}\mathbb{E}\{D_{{\rm ind}(i,j)}^{[2]}\} \\
&= \frac{1}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\mathbb{E}\{D_{{\rm ind}(i,j)}^{[2]}\} \\
&\leq \frac{1}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{1}{2n}\sum_{k\neq i,j}^{n+2}\left(\frac{1}{\lambda_{ik}}+\frac{1}{\lambda_{kj}}\right)f(n) \\
&= \frac{f(n)}{(n+2)(n+1)2n}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\left(\frac{1}{\lambda_{ik}}+\frac{1}{\lambda_{kj}}\right), \qquad (24)
\end{aligned}$$

where the inequality is from (22)–(23). Further, summation terms in (24) can be simplified as follows. Observe that

$$\begin{aligned}
\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{ik}} &= \sum_{i=1}^{n+2}\left(\sum_{k\neq i}^{n+2}\frac{n+1}{\lambda_{ik}}-\sum_{j\neq i}^{n+2}\frac{1}{\lambda_{ij}}\right) \\
&= \sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{n}{\lambda_{ij}},
\end{aligned}$$

and

$$\begin{aligned}
\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{kj}} &= \sum_{i=1}^{n+2}\sum_{j=1}^{n+2}\sum_{k\neq i,j}^{n+2}\frac{1}{\lambda_{kj}}-\sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{1}{\lambda_{kj}} \\
&= \sum_{j=1}^{n+2}\left(\sum_{k\neq j}^{n+2}\frac{n+2}{\lambda_{kj}}-\sum_{h\neq j}^{n+2}\frac{1}{\lambda_{hj}}\right)-\sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{1}{\lambda_{kj}} \\
&= \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{n}{\lambda_{kj}} = \sum_{j=1}^{n+2}\sum_{k\neq j}^{n+2}\frac{n}{\lambda_{jk}}.
\end{aligned}$$

Thus, (24) can be rewritten as

$$\begin{aligned}
\mathbb{E}\{D_{\rm ind}^{[2]}\} &\leq \frac{f(n)}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j\neq i}^{n+2}\frac{1}{\lambda_{ij}} \\
&= \left(\frac{2}{(n+2)(n+1)}\sum_{i=1}^{n+2}\sum_{j>i}^{n+2}\frac{1}{\lambda_{ij}}\right)f(n) \\
&= \left(\sum_{i\in\mathcal{I}}\frac{1}{\lambda_i}\frac{1}{|\mathcal{I}|}\right)f(n) = \mathbb{E}\{T_I^{\rm ind}\}f(n), \qquad (25)
\end{aligned}$$

where the equalities are from the definition of the individual model. See also (1). Then, from the assumption that $\mathbb{E}\{T_I^{\rm ind}\} = \mathbb{E}\{T_I^{\rm hom}\} = \tau$, and from (23) and (25) we have

$$\mathbb{E}\{D_{\rm ind}^{[2]}\} \leq \mathbb{E}\{T_I^{\rm ind}\}f(n) = \tau f(n) = \mathbb{E}\{D_{\rm hom}^{[2]}\}.$$

This completes the proof. ■

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