A Measurement-Analytic Framework for QoS Estimation Based on the Dominant Time Scale

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Abstract—**In this paper we describe a measurement-analytic framework for estimating the overflow probability, an important measure of quality of service (QoS), at a given multiplexing point in the network. A multiplexing point in the network could be a multiplexer or an output port of a switch where resources such as bandwidth and buffers are shared. Our approach impinges on using the notion of the dominant or critical time scale, which corresponds to the time-scale relevant for describing the queueing behavior based on particular network configurations. The dominant time-scale provides us with a measurement window for the statistics of the traffic, but is unfortunately itself defined in terms of the statistics of the traffic over all time. This in essence results in a *chicken and an egg* type of unresolved problem. For the dominant time scale to be useful for on-line measurements, we need to be able to break this chicken and egg type of cycle. In this paper, we present a stopping criterion to successfully break this cycle through online measurements and find a bound on the dominant time scale. Thus, the result has significant implications for network measurements. Our approach is quite different from other works in the literature that require off-line measurements of the entire trace of the traffic (since in our case, we need to measure only the statistics of the traffic up to a bound on the dominant time scale.) We also investigate the characteristics of this upper bound on the dominant time scale, and provide numerical results to illustrate the utility of our measurement analytic approach.**

Keywords— **Dominant time scale, Measurements, Gaussian processes, Stopping criterion, Overflow probability, Maximum Variance Asymptotics.**

I. INTRODUCTION

IN this paper we develop a *measurement-analytic* approach to estimate the overflow probability at a *multiplexing point* in the network. As shown in Figure 1, a multiplexing point in the network is where network resources are shared. (It could be a multiplexer or the output port of a switch.) The reason why we study the overflow probability, is that it is a fundamental measure of network congestion, and hence accurately estimating it is important both from a quality of service (QoS) and a network optimization perspective.

Traditionally, researchers have pursued the problem of QoS estimation from a purely analytical approach or a purely measurement-based approach. *We propose to use methods combining measurement and analysis.* For an illustrative example, assume that a network controller needs to estimate a QoS parameter that corresponds to a rare event, e.g., an overflow probability of 10^{-7}. A measurement-based approach that tries to estimate

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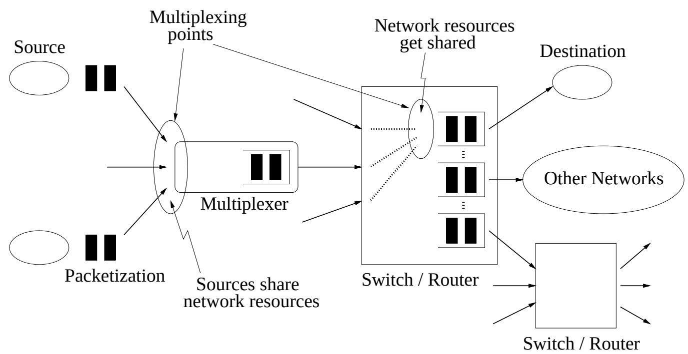

Fig. 1. A typical network

this probability directly will not work because it would take too long—any reasonable time-window would have too few samples (if any) to make an accurate estimation. On the other hand, if we had an accurate analytical approach to compute these rare events from the statistics of the arrival process, then we could simply measure the statistics of the traffic arrival process, and based on these measurements infer the overflow probability. This would solve the problem by working around the difficulty of having to measure rare events. Of course, there is still the problem of which statistics of the arrival stream to measure and over what window of time. These issues will be further explored in this paper.

In the last several years, the observed long range dependence (LRD) and self-similarity phenomenon of measured traffic has received significant attention. Various LRD and self-similar models have been proposed being more suitable than traditional Markovian models, in that they are able to capture the burstiness over the large time scales using a relatively small number of parameters [1], [2]. There have been in fact a number of arguments for and against the importance of LRD for traffic modeling [3], [4], [5]. However, recently, the notions of time and space scales have been able to fill in this divergence of opinions [5], [6], [7], [8]: Depending on the network configuration, there exists a time scale, called the Dominant Time Scale (DTS) over which the traffic should be characterized in order to estimate various QoS measures such as buffer overflow and delay distributions.

The concept of the dominant time scale is quite useful for queueing analysis and is closely related to the concept embodied in the statement "rare event occurs in the most probable way." To see this more precisely, let $X[s,t]$ be the amount of traffic arrived during the time interval $(s,t]$ and C be the service capacity of the link. If we define $X_t := X[-t,0] - Ct$, then the steady state overflow probability $\mathbb{P}\{Q > x\}$ is given by

$$\mathbb{P}\{Q > x\} = \mathbb{P}\{\sup_{t\geq 0} X_t > x\}, \quad (1)$$

where x is the buffer level under consideration. Now the above statement says that,

$$\mathbb{P}\{\sup_{t\geq 0} X_t > x\} \approx \sup_{t\geq 0} \mathbb{P}\{X_t > x\} \quad (2)$$

for sufficiently large x. This approximation is in fact a lower bound and shown to be quite accurate [9], [10], [11]. It thus leads to the notion of the *dominant time scale* (DTS) defined as the time index at which $\mathbb{P}\{X_t > x\}$ attains its maximum. From (2), it is easy to see that the DTS is the most probable duration of a busy period prior to overflow. As a concrete example of approximation (2), for a large class of Gaussian input processes with instantaneous arrival rate $\lambda = \mathbb{E}\{X[0,t]/t\}$ and service rate C, it has been rigorously shown [11], [12] that

$$\lim_{x\to\infty} \mathbb{P}\{\sup_{t\in E_x} Y_t > 1 | \sup_{t\geq 0} Y_t > 1\} = 1, \quad (3)$$

where the size of the interval E_x containing the DTS goes to 0 as x increases. Here, Y_t is defined as $Y_t := (X_{\frac{x}{C-\lambda}t} + xt)/x(t+1)$ such that the event $\{\sup_{t\geq 0} X_t > x\}$ is equivalent to $\{\sup_{t\geq 0} Y_t > 1\}$. An informal explanation of (3) is as follows: If X_t ever exceeds level x, then it exceeds level x within a relatively small interval around the DTS. For further results and explanations on this, see [12], [13] and references therein.

For fairly general processes, there has recently been a large body of work that employs the notion of time scale based on large deviation theory [8], [14], [15]. This effort has focused on the asymptotic behavior of the buffer overflow probability when the number of sources, the queue length, and the service rate are all proportionally sent to infinity. Let $L(C,B,n)$ be the buffer overflow probability of a buffer size B with service rate C and $n = (n_1, \ldots, n_J)$ where n_j is the number of sources of type j. Then the result says that

$$\begin{aligned}\lim_{N\to\infty} \frac{1}{N} \log L(Nc, Nb, Nn) &= -I \\ &= -\inf_t \sup_s [s(ct+b) - st\sum_{j=1}^{J} n_j \alpha_j(s,t)], \quad (4)\end{aligned}$$

where $\alpha_j(s,t) := \log \mathbb{E}\{\exp(sX_j[0,t])\}/st$ is the effective bandwidth of a source of type j (see [16] for details). This equation is referred to as the *many sources asymptotic* and has been proven for discrete time in [15], and for continuous time in [14]. From this equation, the overflow probability can be written as

$$\mathbb{P}\{Q > Nb\} = e^{-NI + o(N)} \approx e^{-NI}. \quad (5)$$

In order to use the many sources asymptotic, we need to calculate the extremizing parameters in (4). Let s^* and t^* be the extremizing parameters over the sup and inf, respectively. The parameter s^* can easily be calculated due to the convexity of the $\alpha_j(s,t)$. Unfortunately, in the literature, there is no general property that we can take advantage of in order to find t^* (the parameter that is related to the DTS) [8], [17].

However, by looking at the problem from a slightly different viewpoint allows us to obtain very important properties of the DTS. In this scenario, which the paper will focus on, we characterize the input process by a Gaussian process. As in the many sources asymptotic, this Gaussian setting works especially well when a large number of sources are multiplexed.

Our goal in the paper will be to develop a measurement-analytic framework based on the DTS to estimate the buffer overflow probability. The DTS provides us with a measurement window for the statistics of the input process, but as will be discussed, is itself defined in terms of the statistics over all time t of the input process. Thus, for the DTS to be useful for on-line measurements, we need to be able to estimate it with only a bounded window of time over which the statistics of the input process are to be measured. This is really the central problem of the paper, and distinguishes our solution from other works in the literature that require off-line techniques to estimate the DTS.

II. PROBLEM DESCRIPTION AND BACKGROUND

As mentioned in the introduction, we will assume that the input process to a multiplexing point in the network (see Figure 1) is Gaussian. The motivation behind Gaussian traffic characterization is that it is very natural when a large number of sources are multiplexed (via the functional Central Limit Theorem (CLT) [18]), as is expected to be the case in future networks. In fact, extensive numerical results by us and other researchers have shown that aggregation of even a fairly small number of traffic streams is usually sufficient for the Gaussian characterization of the input process. Further, Gaussian processes are closed under superposition. Therefore, unlike the case of Markovian queueing models (e.g., see [19], [20] for difficulties with Markovian queueing models), analyzing a queue with a large number of Gaussian sources is no more difficult than analyzing a queue with a single Gaussian source. Gaussian processes are completely specified by their first two moments. This makes Gaussian traffic characterization ideal from a measurement point of view, since measuring statistics beyond the second moment is usually quite impractical. Also, Gaussian processes can have arbitrary correlation structure and this includes LRD (when the covariance function is not summable) processes. Hence, Gaussian processes cover all second-order LRD or second-order self-similar processes which have been shown to be good models for characterizing actual traffic [1].

Our multiplexing model constitutes a queueing system with fixed service rate C fed by a Gaussian process $\{X[0,t] : t \geq 0\}$ with stationary increments and mean rate $\lambda = \mathbb{E}\{X[0,t]/t\}$. Further, let $X_t := X[-t,0] - Ct$ and $\kappa := -\mathbb{E}\{X_t\}/t = C - \lambda$

and $\sigma^2(t) := \mathrm{Var}\{X_t\}$. If we define $f(t,x,\kappa)$ as

$$f(t,x,\kappa) := \frac{\sigma^2(t)}{(\kappa t + x)^2}, \tag{6}$$

then the DTS* $\hat{t}(x,\kappa)$ becomes

$$\hat{t}(x,\kappa) = \arg\max_{t\geq 0}\{f(t,x,\kappa)\}. \tag{7}$$

Note that this is the same time at which the right hand side of (2) achieves its maximum value. It turns out that for a very large class of Gaussian processes, it has been shown that

$$\log \mathbb{P}\left\{\sup_{t\geq 0} X_t > x\right\} + \frac{m(x,\kappa)}{2} \in O(\log x), \tag{8}$$

where

$$m(x,\kappa) := (\kappa\hat{t}(x,\kappa)+x)^2/\sigma^2(\hat{t}(x,\kappa)) = 1/\sup_{t\geq 0} f(t,x,\kappa). \tag{9}$$

This result is often called the Maximum Variance Asymptotic result or the MVA result. Further, and perhaps more importantly from a networking standpoint, although (8) is an asymptotic relation, the authors have also shown through extensive simulations that the following approximation

$$\mathbb{P}\left\{\sup_{t\geq 0} X_t > x\right\} \approx \exp(-\frac{m(x,\kappa)}{2}) \tag{10}$$

that naturally follows from (8) provides an accurate estimate of the overflow probability over a wide range of buffer levels and utilizations. Note that the MVA approximation of the buffer overflow probability (10) is completely determined by $\hat{t}(x,\kappa)$ and $\sigma^2(\hat{t}(x,\kappa))$. Here, it is interesting to see that, in the case of Gaussian sources, the parameter t^* in (4) is equivalent to the DTS $\hat{t}(x,\kappa)$, and that (5) gives the same expression as the MVA (10), when the input process is Gaussian [11], [12], [21].

In the literature there has been some work to calculate the extremizing point t^* and some numerical methods have been proposed. In [17], the entire source traffic was first recorded and then the whole time interval was divided into a sequence of time with fixed step Δ. Then, for each $t = n\Delta, n = 1,2,\ldots$, they computed the effective bandwidth at this point and finally performed a full search among those $n\Delta, n = 1,2,\ldots$ to find t^*. In [22], they proposed an iterative method to find (s^*, t^*) using appropriate traffic substitution at each step (We refer to the paper for details.) However, the rate of convergence (or convergence itself) of this method have not been analytically proven. What is important is that both these techniques require the entire recorded traffic trace and hence unsuitable for on-line measurements. In contrast, we will develop a scheme which will allow us to use on-line measurements to estimate the DTS.

As mentioned before, the MVA approximation given by (10) is supported theoretically by a variety of asymptotic results and empirically by extensive numerical studies [11], [12], [13], [23]. Hence, in this scenario we have a very good analytical tool to estimate the overflow probability. Further, the MVA result depends only on $\kappa = C - \lambda, \hat{t}(x,\kappa)$ and $\sigma^2(\hat{t}(x,\kappa))$. Now, we need to determine the mean rate λ and $\hat{t}(x,\kappa)$ as well as $\sigma^2(\hat{t}(x,\kappa))$, the variance up to the DTS, from measurements to estimate the overflow probability $\mathbb{P}\{Q > x\}$ (or equivalently $\mathbb{P}\{\sup_{t\geq 0} X_t > x\}$). Estimating λ is fairly straightforward since the rate of flow can be measured from traffic traces by counting the number of aggregate packets arrived in time intervals. However, estimating $\hat{t}(x,\kappa)$ and $\sigma^2(\hat{t}(x,\kappa))$, requires that we know a priori what the DTS is. But, from the definition of the DTS in (7), $\hat{t}(x,\kappa) = \arg\max_{t\geq 0}\sigma^2(t)/(\kappa t + x)^2$, i.e., the DTS itself requires taking the maximum over all t of the normalized variance, which means that we need knowledge of $\sigma^2(t)$ over all t. Hence, we are faced with what seems to be an unresolvable *chicken and an egg problem*, of which comes first: the measurement window $\hat{t}(x,\kappa)$ or the variance $\sigma^2(t)$? It actually turns out that we can indeed break this cycle, as shown in Section III, but first we provide a nice property of the DTS that is important from a measurement viewpoint.

Proposition 1: The DTS $\hat{t}(x,\kappa)$ is an increasing function of x/κ.

Proof: See [24]. ■

What is important however is that for any given κ (or equivalently utilization $\rho = \lambda/C$), the DTS is increasing in x. Similarly, for any given x, the DTS is increasing in utilization (or decreasing in κ). These two results tell us that we only need to estimate the DTS for the largest load and buffer size of interest to us, and then we can immediately have an upper bound on the DTS for all lower values of x and the utilization. Moreover, the DTS depends on x and κ only by their ratio. This is very important because it reduces the degree of freedom from the DTS being a function of two variables, to being a function of one variable. This, from a practical point of view, means not having to take different measurements for each value of buffer size and utilization of interest.

III. Stopping criterion for finding DTS

In this section, we present a method of how to find a DTS using only a finite number of observation $t = t_1, t_2, t_3, \ldots$, for $\sigma^2(t)$, thus breaking the cycle mentioned earlier. Since the DTS itself is defined as a global maxima of a function $f(t,x,\kappa)$, we might have to estimate $\sigma^2(t)$ for all $t > 0$ in order to calculate the DTS. Thus, it is not suitable for an on-line measurement based analysis since it requires measurements over an infinite amount of time. To overcome this, we would really like to find a function such that measuring this function over a *finite* amount of time would enable us to find the global maximum. We show below that there indeed exists such a function and hence a stopping criterion for finding $\hat{t}(x,\kappa)$, telling us that once the stopping criterion is satisfied, we don't have to estimate $\sigma^2(t)$ for larger values of t. First we need the following results:

*This is often called relevant time scale or critical time scale in similar context. See [5], [6], [7] *Proposition 2:* Let $X[0,t]$ be a stochastic process with stationary increments. Then, for any convex function h, the function $S(t) := tE\{h(\frac{X[0,t]}{t})\}$ is subadditive, i.e., $S(s+t) \leq S(s)+S(t)$ for all $s,t \geq 0$.

Proof: Observe that

$$\begin{aligned} & h\left(\frac{X[0,t+s]}{t+s}\right) \\ = \;& h\left(\frac{X[0,t]+X[t,t+s]}{t+s}\right) \\ = \;& h\left(\frac{t}{t+s}(\frac{X[0,t]}{t}) + \frac{s}{t+s}(\frac{X[t,t+s]}{s})\right) \\ \leq \;& \frac{t}{t+s}h\left(\frac{X[0,t]}{t}\right) + \frac{s}{t+s}h\left(\frac{X[t,t+s]}{s}\right). \end{aligned}$$

Thus, by taking expectations, we get the result from stationary increments property. ■

Proposition 3: Let $\sigma^2(t) = Var\{X[0,t]\}$. Then, $r(t) := \sigma^2(t)/t$ is subadditive and $R(t) := \sigma^2(t)/t^2$ satisfies the following relation:

$$g(t) := \max_{u\in[t/2,t]} R(u) \geq R(s), \text{ for all } s \geq t. \tag{11}$$

Proof: In Proposition 2, setting $h(\cdot) = (\cdot)^2$ gives the first result. For the second result, let $I(k,t) := [2^{k-1}t, 2^k t]$ where k is a positive integer. Then by definition,

$$g(2^k t) = \max_{u\in I(k,t)} R(u). \tag{12}$$

First, note that for any $v \in I(k,t)$, there exist $p,q \in I(k-1,t)$ such that $v = p+q$. Since

$$\begin{aligned} R(v) &= R(p+q) \\ &\leq \frac{p}{p+q}R(p) + \frac{q}{p+q}R(q) \;\; \text{(subadditivity of } r(t)) \\ &\leq \max\{R(p),R(q)\}, \end{aligned}$$

hence, we have

$$R(v) \leq \max_{u\in I(k-1,t)} R(u) = g(2^{k-1}t). \tag{13}$$

Since (13) holds for any $v \in I(k,t)$, then $\max_{v\in I(k,t)} R(v) \leq g(2^{k-1}t)$. Therefore, from the definition of $g(2^k t)$ (see (12)), it follows that $g(2^k t)$ is decreasing in $k = 0,1,2,\ldots$, for any fixed t.

Now, for any $s \geq t$, we can take $k \geq 1$ such that $s \in I(k,t)$. Thus, we finally get

$$R(s) \leq \max_{u\in I(k,t)} R(u) = g(2^k t) \leq g(2^{k-1}t) \leq \cdots \leq g(t).$$

This completes the proof. ■

Now we are ready to prove our main theorem.

Theorem 1: Suppose that for a given $x/\kappa > 0$, there exist positive numbers t_s and p satisfying

$$g(t_s) = \max_{u\in[t_s/2,t_s]} R(u) \leq (\frac{p}{p+1})^2 R(\frac{px}{\kappa}), \tag{14}$$

then $\hat{t}(x,\kappa) \leq t_s$.

Proof: Observe that

$$\kappa^2 f(t,x,\kappa) = \frac{\sigma^2(t)}{(t+x/\kappa)^2} = R(t)(\frac{t}{t+x/\kappa})^2.$$

Then clearly $\kappa^2 f(t,x,\kappa) < R(t)$ for all $t, x/\kappa > 0$. For such p and t_s satisfying (14), suppose first that $t_s < px/\kappa$. Then by (14), we have $g(t_s) < R(px/\kappa)$ and this contradicts to (11). Thus we get $t_s \geq px/\kappa$. Now for all $t \geq t_s$, we have

$$\begin{aligned} \kappa^2 f(t,x,\kappa) &< R(t) \leq g(t_s) \leq (\frac{p}{p+1})^2 R(\frac{px}{\kappa}) \\ &= \kappa^2 f(px/\kappa, x, \kappa), \end{aligned}$$

by Proposition 3. Hence, $\hat{t}(x,\kappa) \leq t_s$. ■

Note that Theorem 1 holds for any $X[0,t]$ with stationary increments. In fact, since the Hurst parameter H is almost always assumed to be less than 1 (“$H = 1$” corresponds to the case that $X[0,t] = Xt$ for some random variable X), $R(t) \to 0$ as $t \to \infty$. Thus $\lim_{t\to\infty} g(t) = 0$ and hence t_s can always be chosen to be finite for any fixed $p, x/\kappa > 0$. Thus, in order to use Theorem 1 for a given buffer level x and utilization (or equivalently κ), first we estimate $R(t)$ for several values of $t = t_1, t_2, \ldots, t_n$. Then we set the positive numbers $p = p_1, p_2, \ldots, p_n$ such that $p_i x/\kappa = t_i$. For example, set $p = p_1$, and compare the right hand side of (14) with $g(t_i)$ for different values of i. If (14) is satisfied for a certain pair, then we immediately get an upper bound on $\hat{t}(x,\kappa)$, and the DTS can be found by searching $R(t)$ up to this bound.

Now in order to investigate the tightness of the bound, we consider the following two special cases.

A. Fractional Brownian motion case

Suppose that $\sigma^2(t) = Vt^{2H}$ for $t > 0$, i.e., the input process $X(0,t)$ is a fractional Brownian motion process with Hurst parameter $H \in [0.5, 1)$. Then, from (14), and since we can freely choose the positive constant p, the smallest t_s can be found as

$$t_s = 2\min_{p>0}[(1+\frac{1}{p})^{\frac{1}{1-H}} p]\frac{x}{\kappa}. \tag{15}$$

Direct calculations show that the right hand side of (15) is minimized when $p = p^* = \frac{H}{1-H}$ and that the resulting t_s is

$$\begin{aligned} t_s &= 2(\frac{1}{H})^{\frac{1}{1-H}} \frac{H}{1-H}\frac{x}{\kappa} \\ &= 2(\frac{1}{H})^{\frac{1}{1-H}} \hat{t}(x,\kappa) = b(H)\hat{t}(x,\kappa), \end{aligned}$$

where $b(H) := 2(\frac{1}{H})^{\frac{1}{1-H}}$. Fig. 2 shows that $b(H)$ is a strictly decreasing function of H and that $2e < b(H) \leq 8$. This means that for a fBm process, in the worst case scenario, the stopping criterion will give a measurement window that is eight times the DTS.

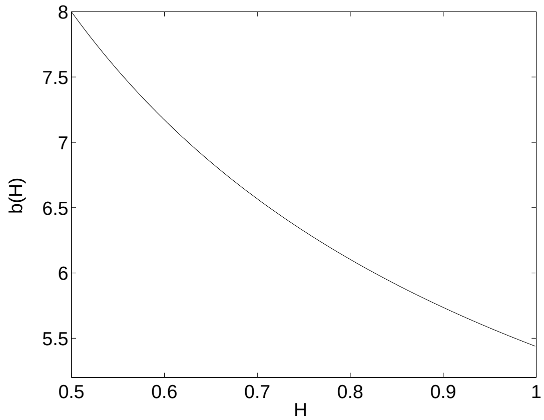

Fig. 2. $t_s/\hat{t}(x,\kappa)$ for fractional Brownian motion

B. Large buffer case

In this section, we will see the behavior of $t_s/\hat{t}(x,\kappa)$ as x increases to infinity. First, suppose that $\sigma^2(t) \sim Vt^{2H}$, i.e.,

$$\lim_{t\to\infty} \frac{\sigma^2(t)}{Vt^{2H}} = 1, \tag{16}$$

where $0.5 \le H < 1$. Then, we have the following.

Proposition 4: Let $t_s(x,\kappa)$ be the number satisfying (14) with equality,[†] that is,

$$g(t_s(x,\kappa)) := \max_{p>0}(\frac{p}{p+1})^2 R(\frac{px}{\kappa}). \tag{17}$$

Then, for any fixed $\kappa > 0$,

$$\lim_{x\to\infty} \frac{t_s(x,\kappa)}{\hat{t}(x,\kappa)} = b(H) = 2(\frac{1}{H})^{\frac{1}{1-H}}. \tag{18}$$

Proof: Clearly, $t_s(x,\kappa) \ge px/\kappa$ for any p that satisfies (14), and note that

$$(\frac{p}{p+1})^2 R(\frac{px}{\kappa}) = \frac{\sigma^2(px/\kappa)}{(px/\kappa + x/\kappa)^2}.$$

Let $p^*(x)$ be the maximizer of the right hand side of (17) for a given x/κ, then we already know that $p^*(x)x/\kappa = \hat{t}(x,\kappa) \sim \frac{H}{1-H}\frac{x}{\kappa}$ [12]. So, we have

$$\lim_{x\to\infty} p^*(x) = \frac{H}{1-H}. \tag{19}$$

Thus $t_s(x,\kappa) \uparrow \infty$ as $x \uparrow \infty$. Also, from (16), we know that

$$\lim_{t\to\infty} \frac{R(t)}{Vt^{2H-2}} = 1. \tag{20}$$

In other words, for any given $\epsilon > 0$, we can choose $M > 0$ such that

$$(1-\epsilon)Vt^{2H-2} \le R(t) \le (1+\epsilon)Vt^{2H-2}$$

for all $t \ge M$. Thus by definition of $g(t)$, we have

$$(1-\epsilon)V(\frac{t}{2})^{2H-2} \le g(t) \le (1+\epsilon)V(\frac{t}{2})^{2H-2}$$

for all $t \ge 2M$. Hence

$$\lim_{x\to\infty} \frac{g(t_s(x,\kappa))}{V(t_s(x,\kappa)/2)^{2H-2}} = 1. \tag{21}$$

Now, from (17), $g(t_s(x,\kappa))$ can be represented as

$$g(t_s(x,\kappa)) = (\frac{p^*(x)}{p^*(x)+1})^2 R(\frac{p^*(x)x}{\kappa}).$$

By rewriting this as

$$\frac{g(t_s(x,\kappa))}{V(t_s(x,\kappa)/2)^{2H-2}} = \left(\frac{p^*(x)}{p^*(x)+1}\right)^2 \frac{R(p^*(x)x/\kappa)}{V(p^*(x)x/\kappa)^{2H-2}} \left(\frac{p^*(x)x/\kappa}{(t_s(x,\kappa)/2)}\right)^{2H-2},$$

we have

$$\lim_{x\to\infty} \frac{t_s(x,\kappa)}{2\hat{t}(x,\kappa)} \frac{\hat{t}(x,\kappa)}{p^*(x)x/\kappa} (\frac{p^*(x)}{p^*(x)+1})^{\frac{1}{H-1}} = 1 \tag{22}$$

from (20) and (21) . Therefore, we have (18) from (19). ■

From (18), we know that there exists $M > 0$ such that $t_s(x,\kappa)/\hat{t}(x,\kappa) \le M$ for all $x > 0$. Further, as x increases, this function converges to a constant $b(H)$. Although this is an asymptotic result, we see in the next section using real traces that $t_s(x,\kappa)/\hat{t}(x,\kappa)$ is no larger than 6 for most of the buffer levels x.

IV. Experimental Results

Using (14), we can find the DTS for different values of x and κ. Since the DTS is a function of x/κ, we will fix κ in the remainder of the paper and focus on the DTS as the buffer size x increases. Similarly, we could apply our result to the case of fixed buffer size and different utilizations (or κ). In this section we propose a simple algorithm to find $t_s(x,\kappa)$ and show that $t_s(x,\kappa)/\hat{t}(x,\kappa)$ is usually small as suggested by Proposition 4, for several different input traffic scenarios. Since the point of the paper is not to develop new estimation schemes for calculating the mean or the variance at a given point (which can be done by using a variety of standard methods), we assume that our estimates of λ and $R(t)$ are accurate. Let the smallest time scale available be normalized to one. For example, if input traffic is an MPEG-encoded video sequence with 25 frames/sec and only frame level data are available, then one time unit corresponds to 40msec. Suppose that we measure $R(1), R(2), \ldots$ as time goes on and that we want to find $\hat{t}(x,\kappa)$ for $x = i\Delta, i = 0,1,2,\ldots$ where Δ is the buffer step size. Then, although Theorem 1 is stated for the continuous time case, it is straightforward to see that we can still use it for the discrete time case as shown below.

[†] If $\lim_{t\to 0}\sigma^2(t) = 0$, it is easy to show that $R(t)$ is continuous for all $t > 0$ using Cauchy-Schwarz inequality and stationary increment property. Hence $g(t)$ is also continuous and the equality in (14) can be achieved.

A. Stopping algorithm

We can use (14) in the following form:

Discrete version of Theorem 1: If there exist positive integers i, j and even number n satisfying

$$g(n) := \max_{n/2 \le u \le n} R(u) \le (\frac{j}{\frac{i\Delta}{\kappa} + j})^2 R(j), \quad (23)$$

then $\hat{t}(i\Delta, \kappa) \le n$.

Suppose we know $R(1), R(2), \ldots, R(n)$. Then in order to apply our stopping criterion, we first compute $g(n)$ and we compare every pair of these values to see if (23) holds. If (23) is satisfied for a certain pair, then we immediately get the upper bound and hence we can find $\hat{t}(i\Delta, \kappa)$ without knowing all $R(n)$ for $n \ge 1$.

Let $t_s(i, \kappa)$ be the upper bound on $\hat{t}(i\Delta, \kappa)$ from (23) and let $a(i, j) := (j/(i\Delta/\kappa + j))^2$. Fig. 3 shows the algorithm for finding $t_s(i, \kappa)$ for $i = 1, 2, \ldots, M$ while measuring $R(n)$ as n increases. This algorithm can be explained as follows. Let $X = \{x_{i,j}\}$ be a matrix where $x_{i,j} := a(i, j)R(j)$ for $i = 1, 2, \ldots, M$. Note that $x_{i,j}$ decreases as i increases and that $R(n)$ completely specifies n^{th} column of X. First, set $i = 1$. Whenever a new value of $g(n)$ is available, we compare it with $x_{i,j}$ for $j = 1, 2, \ldots, n$. If (23) is satisfied for some $\tilde{j}$, $1 \le \tilde{j} \le n$, then we get $t_s(i, \kappa) = n$. In this case, we fix $\tilde{j}^{th}$ column and continue to compare $x_{l,\tilde{j}}$ with $R(n)$ for $l = i+1, i+2, \ldots$, until the condition (23) with i replaced by l is violated. This will give $t_s(l, \kappa) = n$. Once we have found $t_s(i, \kappa)$ for $i = 1, 2, \ldots, M$, the DTS $\hat{t}(i\Delta, \kappa)$ can be obtained by simply searching $x_{i,j}$ for $\hat{t}((i-1)\Delta, \kappa) \le j \le t_s(i, \kappa)$ using Proposition 1. This can be done by any sophisticated searching method, or we can integrate this with the above algorithm.

B. Simulations

To experimentally validate our stopping criterion and numerically investigate the bound, we use MPEG-1 and JPEG encoded *Star Wars* video traffic sequences whose frame interval is 40msec. The first set is 20 multiplexed JPEG sources and the other one is 40 multiplexed MPEG sources. We set the utilization parameter $\rho = \lambda/C$ to 0.9 throughout our simulations, unless otherwise noted. For the JPEG case, we have observed (after calculating all the $\sigma^2(t)$!) that the function $f(t, x, \kappa)$ in (6) is a uni-modal function for all $x > 0$. However, for the MPEG case, due to its periodic nature, the function $f(t, x, \kappa)$ generally has more than one local maximum for all the values of $x > 0$. Thus, without our stopping criterion, we have to measure $\sigma^2(t)$ for all $t > 0$ to find the global maximum of $f(t, x, \kappa)$.

Figs. 4 and 5 show the resulting DTS and its upper bound from the stopping criterion for the JPEG and MPEG sequences, respectively. We see that $\hat{t}(x, \kappa)$ and $t_s(x, \kappa)$ increase as the buffer size increases in both cases. Although Fig. 5 is obtained under $\rho = 0.9$, we can use this figure for different utilizations. Suppose that our new utilization is ρ and the maximum buffer delay is $d = 15$ms. Then the buffer size required for this utilization ρ becomes $\lambda d/\rho$. Let x be the corresponding buffer size in

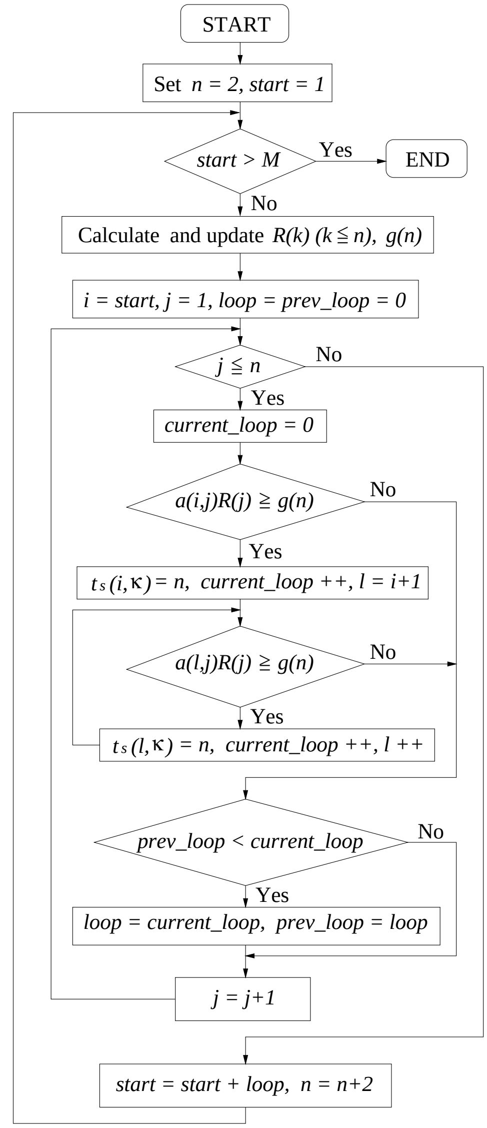

Fig. 3. Algorithm for finding $t_s(i, \kappa)$, the upper bound on the dominant time scale

Fig. 5. Then we have

$$\frac{x}{(\frac{1}{0.9} - 1)\lambda} = \frac{\lambda d/\rho}{(\frac{1}{\rho} - 1)\lambda},$$

where $\lambda = 1625 \cdot 48$ bytes/40ms. For example, take $\rho = 0.5$. Then we get $x = 135 \cdot 48$ bytes in the above equation and we can obtain the corresponding DTS and its upper bound from Fig. 5 as $\hat{t}(x, \kappa) \approx 80$ms and $t_s(x, \kappa) \approx 400$ms. Thus, in the above setting, we only need to measure $\sigma^2(t)$ or correlations up to 400ms or 10 frame intervals. Fig. 6 shows $t_s(x, \kappa)/\hat{t}(x, \kappa)$ as a function of buffer size. For the JPEG case, the ratio is well behaved for all buffer sizes under consideration. In contrast, for the MPEG case, the ratio $t_s(x, \kappa)/\hat{t}(x, \kappa)$ is irregular because of the periodic nature of MPEG traffic. For a buffer size of $400 \cdot 48$ bytes, the ratio reaches 17.5 and it is omitted from the figure. However, the upper bound $t_s(x, \kappa)$ is still relatively small at this

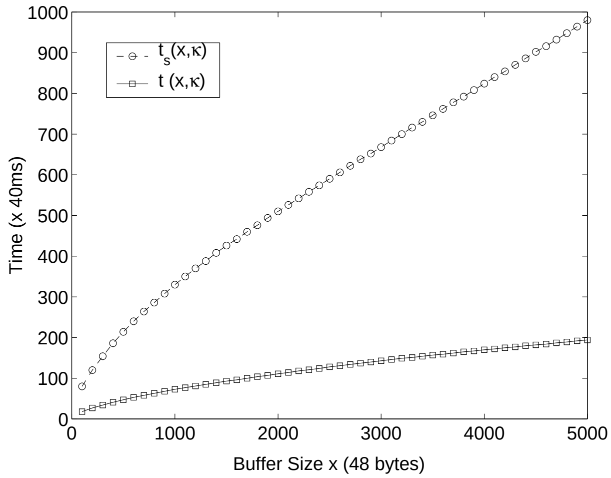

Fig. 4. $\hat{t}(x,\kappa)$ and $t_s(x,\kappa)$ for 20 multiplexed JPEG video traffic

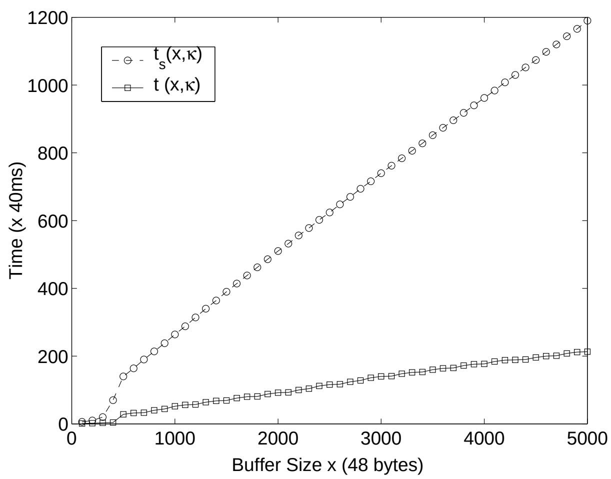

Fig. 5. $\hat{t}(x,\kappa)$ and $t_s(x,\kappa)$ for 40 multiplexed MPEG video traffic

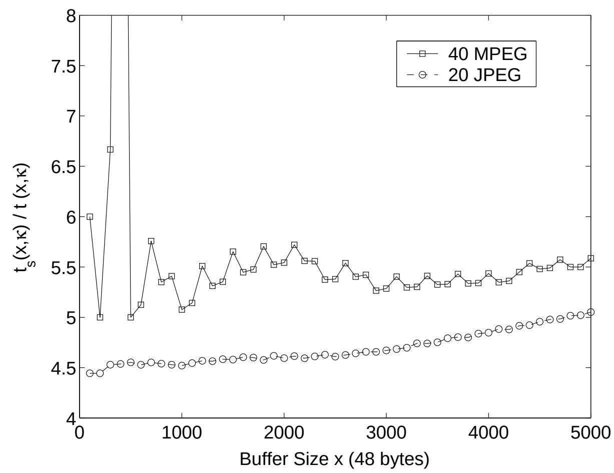

Fig. 6. $t_s(x,\kappa)/\hat{t}(x,\kappa)$ as a function of x for JPEG and MPEG video traffic

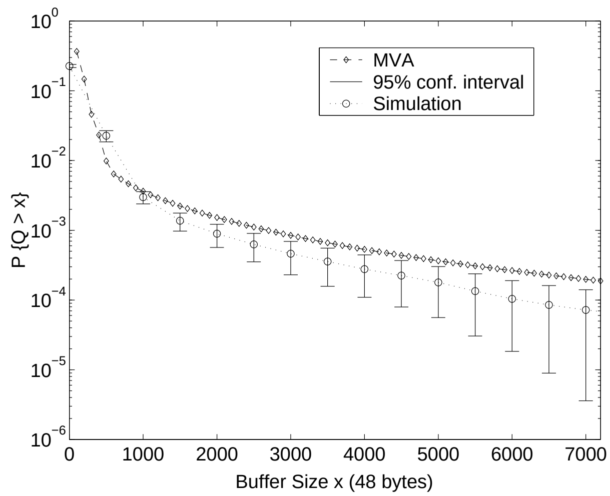

Fig. 7. Buffer overflow probability and its MVA approximation for 40 multiplexed MPEG video traffic. Utilization is set to $\rho = 0.85$

point. Here, it should be pointed out that our stopping criterion is guaranteed to work for any stationary input traffic and that, as predicted from Proposition 4, the ratio $t_s(x,\kappa)/\hat{t}(x,\kappa)$ is no larger than 6 for most of the buffer sizes used in our simulation. This property enables us to apply our stopping criterion to an on-line measurement framework. Fig. 7 shows the buffer overflow probability for 40 multiplexed MPEG sequence with 95% confidence interval and the MVA approximation (given by (10)) using our stopping criterion for the calculation of DTS. Since the accuracy of the MVA approximation has already been verified for general sets of real traffic [11], [12], [23], we do not provide further results on this.

From the structure of the MVA approximation and our stopping criterion, we can use Fig. 7 to solve different network problems. Let $C = 620$Mbps and the maximum buffer delay be $d =$ 15ms. Suppose we want to determine the maximum number of sources n such that the probability of delay violation is no more than $P_D = 10^{-6}$. We assume that the sources are i.i.d and that the 40 multiplexed MPEG sequence represents one source.

Then our problem is to find the largest n ($0 < n < C/\lambda$) such that

$$\sup_{t\geq 0} \frac{n\sigma^2(t)}{((C-n\lambda)t + Cd))^2} \leq -\frac{1}{2\log P_D} := A, \qquad (24)$$

where λ and $\sigma^2(t)$ are the mean and the variance function of the source. This kind of problem is very important for Admission Control. Let n^* be the largest number of such sources that can be admitted without violating the overflow probability constraint. Then, since the left hand side of (24) increases as n increases, n^* must satisfy (24) with equality, and the resulting equation has a unique solution n^* for any given A.‡ Rewriting (24) gives us

$$\sup_{t\geq 0} \frac{\sigma^2(t)}{(\kappa t + \frac{\kappa C d}{C-n\lambda})^2} = A\frac{(C-n\lambda)^2}{\kappa^2 n}, \qquad (25)$$

‡We assume that n is a real valued number in (24). The actual number then becomes $\lfloor n \rfloor$.

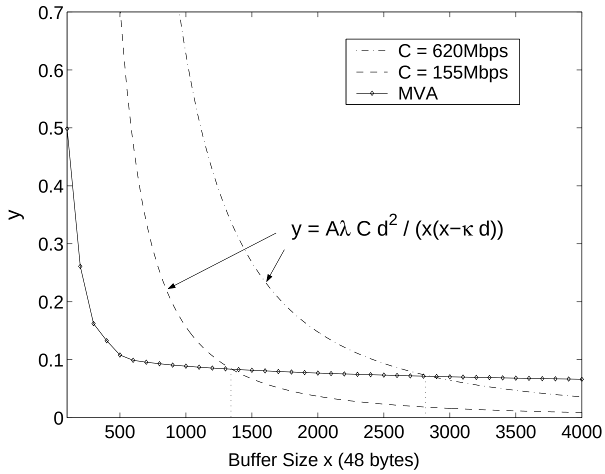

Fig. 8. Calculating the maximum number of sources to be accepted under $P_D \leq 10^{-6}$ and maximum buffer delay = 15ms for two different service capacity C.

where $\kappa = (1/\rho - 1)\lambda$ and ρ is the utilization used in Fig. 7, i.e., $\rho = 0.85$. Now let x and y be defined as

$$x = \frac{\kappa C d}{C - n\lambda}, \text{ and } y = A\frac{(C - n\lambda)^2}{\kappa^2 n}. \tag{26}$$

Substituting x in terms of n into y gives

$$y = \frac{A\lambda C d^2}{x(x - \kappa d)}, \tag{27}$$

where $x > \kappa d$ since $0 < n < C/\lambda$. However, note that the x and y also should satisfy the following equation (from (25))

$$y = \sup_{t \geq 0} \frac{\sigma^2(t)}{(\kappa t + x)^2}, \tag{28}$$

and that the MVA curve in Fig. 7 is nothing but $\exp(-1/2y)$ as a function of x. Thus, the solution x^* is the point at which (27) and (28) coincide, and the corresponding n^* can be obtained from (26)

Fig. 8 shows how we can find such an n^* using the MVA approximation and the stopping criterion. The solid line corresponds to (28), which can be calculated on-line by the stopping criterion. The remaining two curves correspond to (27) for different service capacities. Note that the meeting points are approximately $x^* = 2800 \cdot 48$ bytes and $x^* = 1350 \cdot 48$ bytes for $C = 620$ and $C = 155$Mbps, respectively. Hence from (26), we obtain $n^* = 40$ for $C = 620$Mbps and $n^* = 9$ for $C = 155$Mbps. Note that the link utilization $n^*\lambda/C$ increases from 0.92 to 0.96 as the service capacity C increases from 155Mbps to 622Mbps, as expected from statistical multiplexing.

V. Discussion

In this section, we discuss the implications of $\hat{t}(x,\kappa)$ and $t_s(x,\kappa)$ for traffic modeling in more detail. First, we make some observations:

- the value of $\hat{t}(x,\kappa)$ for any fixed x and κ depends only on the shape of the function $R(t)$ within a finite time interval $[0, t_s(x,\kappa)]$, and not on the limiting behavior of $R(t)$ (and hence the Hurst parameter).
- In order to produce the same $\mathbb{P}\{Q > x\}$, the model should be able to capture the correlations of lags up to $t_s(x,\kappa)$, not the DTS. Specifically, suppose that we know all the statistics for the Gaussian input process X_t and that, for a given buffer size and utilization, we calculate the DTS $\hat{t}(x,\kappa)$ and its upper bound $t_s(x,\kappa)$. Now, we generate another stationary Gaussian process Y_t whose mean and correlations are the same as those of X_t up to the DTS. Then, in this case, the undetermined region of $R(t)$ for $t > \hat{t}(x,\kappa)$ might change the location of the DTS for the process Y_t. However, if we match the mean and variance of the Y_t process up to the upper bound $t_s(x,\kappa)$ of X_t, instead of the DTS itself, then by Theorem 1, these two processes will give the same DTS $\hat{t}(x,\kappa)$ and $\sigma^2(\hat{t}(x,\kappa))$, no matter how the correlations of lags beyond $t_s(x,\kappa)$ behave.

For LRD processes, we generally have

$$R(t) \sim V t^{2H-2},$$

where $0.5 \leq H < 1$ is the Hurst parameter. Qualitatively, we can see that $R(t)$ tends to decrease slowly as H increases, which makes $\hat{t}(x,\kappa)$ and $t_s(x,\kappa)$ larger. (For instance, $\hat{t}(x,\kappa) = \frac{H}{1-H}\frac{x}{\kappa}$ for fBm processes.) Since $t_s(x,\kappa)$ is finite for any given x and κ, the long range dependence property by itself does not change the buffer distribution. Instead, it exerts its influence on the value of $\hat{t}(x,\kappa)$ by which the buffer behavior is determined. This explains why the overflow probability is so different for a *fixed* Short Range Dependent (SRD) model and LRD model. Note that for any given x, we can always find an SRD model that has similar correlations up to $t_s(x,\kappa)$ and hence induces a similar overflow probability. In this sense the LRD model should be considered as just one of the different ways of modeling real traffic, rather than the only model we have in order to match $\mathbb{P}\{Q > x\}$.

Another observation worth mentioning is that we can use the parameter $t_s(x,\kappa)$ as a unit time of traffic reordering while maintaining the same $\mathbb{P}\{Q > x\}$. Suppose that we divide the incoming packet stream into successive blocks of size $t_s(x,\kappa)$ and then rearrange the blocks while the order within a block remains untouched. These operations called "external shuffling" [3] have been used to verify the impact of LRD on queueing performance. Note that these operations do not change the its marginal distribution and correlations up to $t_s(x,\kappa)$, hence the buffer overflow probability is also the same, as suggested by the MVA approximation. Also note that the order of blocks need not be random since correlations of lags larger than $t_s(x,\kappa)$ have no effect on $\hat{t}(x,\kappa)$ and $\sigma^2(\hat{t}(x,\kappa))$.

Our results also find some applications in LRD traffic modeling. Previous attempts to find a suitable model of the real LRD traffic and to analyze the buffer based on the model always require the estimation of the Hurst parameter H, even for the simplest model such as fractional Brownian motion. However, our results suggest that we do not have to estimate H at all in order to find the buffer distribution. Further, it is also implied that the so-called "variance-time plot," which has been used for the estimation of H [1], already contains all the information we need for overflow probability. Hence, estimating H is more costly than all the steps we use for calculation of the DTS and the resulting buffer overflow probability, via the stopping criterion.

Finally, since we use a measurement-analytic approach to estimate the overflow probability, it should be pointed out that our work can be applied to any multiplexing point in a network. In other words, our method enables us to calculate the performance by directly measuring the traffic at the point of interest, without having to worry about the complications in a purely analytical approach of having to characterize the departure processes, routing, etc.

VI. Conclusion

In this paper we have developed a measurement-analytic approach for estimating the overflow probability at the multiplexing point of interest, in the network. Our approach assumes that the input traffic can be characterized by a very general class of Gaussian processes, and hence is applicable to systems where a moderate to large number of traffic sources are multiplexed. Our work is motivated by the fact that for estimating QoS parameters that correspond to rare events such as the overflow probability, we require a combined measurement and analytical framework. The dominant time scale is useful in our framework because it makes it possible to transforms the problem of measuring rare events to one of measuring the input traffic itself over a finite time window. However, a seemingly impossible difficulty with this approach is that the dominant time scale is itself defined in terms of the variance of the traffic over all time. We show how to break this "chicken and an egg" type of cycle and find a bound for the dominant time scale. We investigate the tightness of this bound and provide numerical examples to illustrate our measurement-analytic approach. We also provide some interesting insights gained from using our approach. Since our approach only requires a finite window of measurements (usually small, depending of course on the dominant time scale and its bound), it has significant value from an on-line measurement point of view and differs from works in the literature that require knowledge of the entire trace of the traffic.

We are currently investigating ways of extending our results for on-line estimation of delay distributions in the case of priority and generalized processor sharing types of service disciplines.

References

[1] W. E. Leland, M.S. Taqqu, W. Willinger, and D. V. Wilson, "On the Self-Similar Nature of Ethernet Traffic (Extended Version)," *IEEE/ACM Transactions on Networking*, vol. 2, no. 1, pp. 1–15, Feb. 1994.

[2] J. Beran, R. Sherman, M. S. Taqqu, and W. Willinger, "Long-range dependence in variable-bit-rate video traffic," *IEEE Transactions on Communications*, vol. 43, no. 2–4, pp. 1566–1579, Feb.–April 1995.

[3] A. I. Erramilli, O. Narayan, and W. Willinger, "Experimental queueing analysis with long-range dependent packet traffic," *IEEE/ACM Transactions on Networking*, vol. 4, pp. 209–223, 1996.

[4] D. P. Heyman and T. V. Lakshman, "What are the implications of long-range dependence for VBR video traffic engineering?," *IEEE/ACM Transactions on Networking*, vol. 4, pp. 301–317, 1996.

[5] B. K. Ryu and A. Elwalid, "The importance of long-range dependence of VBR video traffic in ATM traffic engineering: myths and realities," in *Proceedings of ACM SIGCOMM*, San Francisco, CA, Aug. 1996.

[6] M. Montgomery and G. De Veciana, "On the Relevance of Time Scales in Performance Oriented Traffic Characterization," in *Proceedings of IEEE INFOCOM*, San Francisco, CA, 1996, pp. 513–520.

[7] M. Grossglauser and J.-C. Bolot, "On the relevance of long-range dependence in network traffic," *IEEE/ACM Transactions on Networking*, pp. 629–640, Oct. 1999.

[8] C. Courcoubetis, V. A. Siris, and G. D. Stamoulis, "Application of the many sources asymptotic and effective bandwidths to traffic engineering," *Telecommunication Systems*, vol. 12, pp. 167–191, 1999.

[9] N. G. Duffield and Neil O'Connell, "Large deviations and overflow probabilities for the general single server queue, with application," *Proc. Cambridge Philos. Soc.*, vol. 118, pp. 363–374, 1995.

[10] P. W. Glynn and W. Whitt, "Logarithmic asymptotics for steady-state tail probabilities in a single-server queue," *Journal of Applied Probability*, pp. 131–155, 1994.

[11] J. Choe and N. B. Shroff, "A Central Limit Theorem Based Approach for Analyzing Queue Behavior in High-Speed Networks," *IEEE/ACM Transactions on Networking*, vol. 6, no. 5, pp. 659–671, Oct. 1998.

[12] J. Choe and N. B. Shroff, "Queueing Analysis of High-Speed Multiplexers including Long-Range Dependent Arrival Processes," in *Proceedings of IEEE INFOCOM*, New York, NY, March 1999.

[13] J. Choe and N. B. Shroff, "Use of Supremum Distribution of Gaussian Processes in Queueing Analysis with Long-Range Dependence and Self-Similarity," *Stochastic Models*, vol. 16, no. 2, Feb. 2000.

[14] D. D. Botvich and N. Duffield, "Large deviations, the shape of the loss curve, and economies of scale in large multiplexers," *Queueing Systems*, vol. 20, pp. 293–320, 1995.

[15] C. Courcoubetis and R. Weber, "Buffer overflow asymptotics for a buffer handling many traffic sources," *Journal of Applied Probability*, vol. 33, pp. 886–903, 1996.

[16] F.P. Kelly, "Notes on effective bandwidths ," *Stochastic networks: Theory and Applications*, pp. 141–168, Oxford University Press, 1996.

[17] C. Courcoubetis and V. A. Siris, "Measurement and analysis of real network traffic," in *7th Hellenic Conference on Informatics (HCI'99), Ioannina, Greece*, 1999.

[18] J. Jacod and A. N. Shiryaev, *Limit Theorems for Stochastic Processes*, Springer-Verlag, June, 1987.

[19] G. L. Choudhury, D. M. Lucantoni, and W. Whitt, "Squeezing the Most Out of ATM," *IEEE Transactions on Communications*, vol. 44, pp. 203–217, Feb. 1996.

[20] N. B. Shroff and M. Schwartz, "Improved Loss Calculations at an ATM Multiplexer," *IEEE/ACM Transactions on Networking*, vol. 6, no. 4, pp. 411–422, Aug. 1998.

[21] J. Choe and N. B. Shroff, "On the supremum distribution of integrated stationary Gaussian processes with negative linear drift," *Advances in Applied Probability*, vol. 31, no. 1, pp. 135–157, 1999.

[22] C. Courcoubetis, V. A. Siris, and G. D. Stamoulis, "Traffic equivalence and substitution in a multiplexer," in *Proceedings of IEEE INFOCOM*, 1999.

[23] E. W. Knightly and N. B. Shroff, "Admission Control for Statistical QoS: Theory and Practice," *IEEE Network*, vol. 13, no. 2, pp. 20–29, March 1999.

[24] J. Choe and N. B. Shroff, "Efficient Gaussian Traffic Modeling in High-Speed Networks," Tech. Rep., Purdue University, West Lafayette, IN, 2000.