Exploiting Heterogeneity in Mobile Opportunistic Networks: An Analytic Approach

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***Abstract*—Heterogeneity arises in a wide range of scenarios in mobile opportunistic networks and is one of key factors that govern the performance of packet forwarding algorithms. While the heterogeneity has been empirically investigated and exploited in the design of new forwarding algorithms, it has been typically ignored or marginalized when it comes to rigorous performance analysis of such algorithms. In this paper, we develop an analytical framework to *quantify* the performance gain achievable by exploiting the heterogeneity in mobile nodes' contact dynamics. In particular, we derive a delay upper bound of a heterogeneity-aware forwarding policy per a given number of message copies and obtain its closed-form expression, which enables our quantitative study on the benefit of leveraging underlying heterogeneity structure in the design of forwarding algorithms. We then analytically show that less than 20% of total (unlimited) message copies is only enough under various heterogeneous network settings to achieve the same delay as that obtained using the unlimited message copies when the networks become homogeneous. We also provide independent simulation results including real trace-driven evaluation to support our analytical results.**

I. Introduction

Mobile opportunistic networks (MONs) or delay/disruption tolerant networks (DTNs) have received much attention from networking research community as an extension of mobile ad-hoc networks (MANETs) toward several applications such as Pocket Switched Networks (PSNs) [5] or UMass Dieselnet [2]. In MONs/DTNs, network connectivity is changing over time and frequently disrupted due to random node mobility, power limitation, limited storage, among others. To overcome this intermittent connectivity nature, MONs/DTNs employ a *'store-carry-and-forward'* principle, in which mobile nodes can carry messages and copy and/or relay them to other nodes upon encounter, which in turn renders messages eventually delivered to their destinations.

Recently, many empirical studies indicate the presence of heterogeneity in a wide range of scenarios in MONs/DTNs. For example, [6], [15] investigate real mobility traces and disclose the characteristics of heterogeneity in mobile nodes' contact dynamics. Similarly, based on real mobility traces and survey data, [14], [20], [15], [19] uncover spatially and/or socially formatted community structures in nodes' mobility. The observed characteristics of underlying heterogeneity structure have been mainly used for the development of new mobility models [14], [19] and empirically exploited to the design of new forwarding/routing algorithms [20], [7], [15].

There are several analytical studies on the performance of a few forwarding/routing algorithms including epidemic routing [11], [16], [25], single-copy and multicopy two-hop relay protocols [12], [11], spray and wait [22], etc; however, these works are mainly based upon a homogeneous model in which any mobile node is making contacts with others according to a Poisson process. Other analytical works also fully rest on the homogeneous model for their investigation on the capacity-delay tradeoff [12], the cost-delay tradeoff [21], [18], the design of forwarding policy [1], and the content distribution [13]. The current literature still lacks analytical studies on exploiting the underlying heterogeneity structure to correctly understand the resulting performance gain.

In this paper, we analytically investigate how much benefit the heterogeneity in mobile nodes' contact dynamics can bring in the forwarding performance. To this end, we employ the heterogeneous network model used in [6], [9], [23], [17] in which the pairwise inter-contact time of a given node pair is exponentially distributed but with *different* rates over different pairs. (See Section II for its detailed description and justification.) Under this heterogeneous setting, we then consider a class of probabilistic two-hop forwarding policies in which a source node forwards a message with probability p_i to each relay node i upon encounter. Since message delivery delay and the number of (used) message copies are both mainly functions of p_i and the heterogeneity of contact rates over different node pairs, we are led to find an optimal forwarding policy $\{p_i^\star\}$, maximally exploiting the heterogeneity structure, to minimize the message delivery delay under a given constraint on the number of message copies.

Rather than directly solving the optimization problem, as a viable alternative, we derive a delay upper bound of any two-hop forwarding policy and find an optimal forwarding policy that minimizes the delay bound while satisfying the given constraint on the number of message copies. Although this solution is sub-optimal to the original problem, we are able to derive a *closed-form* expression of its guaranteed delay bound, which in turn enables to quantify the performance gain achievable by exploiting the heterogeneity structure in contact dynamics. In particular, when obtaining the closed-

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form expression of the delay bound, we provide an idea to *decompose* an original heterogeneous network into a set of several (partially) homogeneous networks, which makes the delay analysis more tractable.

We then analytically show that *less than 20%* of unlimited message copies is only enough under various heterogeneous network settings to achieve the same delay as the optimal delay (obtained at the expense of unlimited message copies by multicopy two-hop relay protocol) that any two-hop forwarding policy cannot exceed when the networks become homogeneous. Moreover, since the considerable performance improvement is still demonstrated through the delay upper bound of the sub-optimal two-hop forwarding policy, the maximally achievable performance gain by exploiting the heterogeneity in mobile nodes' contact dynamics will be much higher than expected. We also provide independent simulation results including real trace-driven evaluation to support our analytical results and to show the usefulness of the derived delay bound. Although there have been several empirical works [20], [7], [15] that propose heuristic forwarding/routing algorithms utilizing the underlying heterogeneity structure, to the best of our knowledge, this is the first work to analytically quantify the attainable performance gain by exploiting the underlying heterogeneity structure.

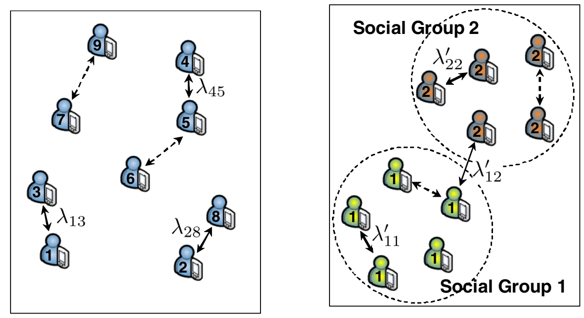
The rest of this paper is organized as follows. Section II gives preliminaries on a heterogeneous network model and related work. Section III presents a class of probabilistic two-hop forwarding policies and an optimization problem to find an optimal forwarding policy. Section IV provides an analysis on the achievable delay upper bound for any two-hop forwarding policy. Section V presents a sub-optimal forwarding policy which minimizes the derived delay upper bound, and provide analytical and simulation results on the attainable performance gain through exploiting the heterogeneity in contact dynamics. We conclude in Section VI.

II. Preliminaries

A. Network Model

The heterogeneous network model that we consider in this paper is used in [6], [9], [23], [17] and described as follows. There is a set of mobile nodes $\mathcal{N}$ in the whole network domain. The pairwise inter-contact time between mobile nodes i and j, denoted by T_{ij}, is independently drawn from an exponential distribution with rate $\lambda_{ij} > 0$ (i.e., contacts between nodes i and j occur according to a Poisson process with rate parameter λ_{ij}), where $i, j \in \mathcal{N}$ and $i \neq j$. Note that this contact process between nodes i and j is symmetric ($\lambda_{ij} = \lambda_{ji}$). The pairwise inter-contact times between any two node pairs are also mutually independent. In this model, the heterogeneity in mobile nodes' contact dynamics is captured by different contact rates λ_{ij}. Figure 1 (a) shows a general case of this heterogeneous network model.

If $\lambda_{ij} = \lambda$ for all $i, j \in \mathcal{N}$ and $i \neq j$, the heterogeneous network model reduces to the homogeneous model (a.k.a. Poisson contact model) in which contacts between *any pair* of mobile nodes occur according to a Poisson process with same rate parameter λ. This heterogeneous model also captures a social community structure [9] as a special case. Suppose that there are M different social groups G_i $(i = 1, \ldots, M)$ forming a partition of $\mathcal{N}$, i.e., $\mathcal{N} = \bigcup_{i=1}^{M} G_i$. Let λ'_{lk} be common contact rate between any member of G_l and another member of G_k for $l, k = 1, \ldots, M$. That is, $\lambda_{ij} = \lambda'_{lk}$ for all $i \in G_l$ and $j \in G_k$ where $l, k = 1, \ldots, M$. Figure 1 (b) shows a setting of two social groups as a special case of the heterogeneous network model.



(a) A general case (b) A special case with two social groups

Fig. 1. The heterogeneous network model

B. Related Work

In the literature, many empirical studies [14], [20], [6], [15], [19] focus on the presence of heterogeneity and its characteristics from real mobility traces and survey data. First, [6] shows heterogeneity in pairwise inter-contact time of each node pair. [15] also finds heterogeneity structure from individual node and (social) community viewpoints. In particular, it shows that human community can be divided into several social communities, and within each community there are several socially-active people (nodes) which make more frequent contacts with others. In addition, [20], [19] observe that spatial node distributions are heterogeneous (non-uniform) and there exist clustering points with high node density in which mobile nodes have higher chance to encounter other nodes. Similarly, [14] shows uneven spatial distribution of mobile nodes from survey data. All these empirical studies focus more on the development of a new mobility model [14], [19] and on the design of a new heuristic forwarding algorithm exploiting the underlying heterogeneity structure [20], [15].

On the other hand, there are a few of analytical works [10], [3], [23], [17] on the performance of forwarding/routing protocols in MONs/DTNs under heterogeneous network settings. In our previous work [17], we address how two different sources of heterogeneity (spatial and node heterogeneities) in mobile nodes' contact dynamics impact the delay performance. In addition, [10] analyzes an asymptotic capacity scaling property as the number of nodes grows to infinity under the presence of node and spatial heterogeneities. In [3], the authors also study how much network performance can be improved by adding infrastructures to MONs/DTNs under spatially heterogeneous network model. [23] proposes a class of heuristic routing algorithms which exploit the underlying heterogeneity structure. In contrast, in this paper, we analyze how much benefit the heterogeneity in mobile nodes' contact dynamics can bring in the forwarding performance in comparison with its homogeneous counterpart by *quantifying* the amount of reduction in the message delay per a given number of message copies, for *any* given *finite* number of mobile nodes in the network.

Lastly, the heterogeneous network model we adopt in this paper has also been recently used in several papers [9], [23], [17] for the design of forwarding protocol and/or performance analysis. In particular, the authors in [9] justify the exponential assumption for the pairwise inter-contact time distribution of a *given* node pair in the heterogeneous network model. Specifically, they show through statistical methods that over 85% of node pairs in *Infocom* and *MIT Reality** can be well fitted by exponential distributions with different rates.

III. Problem Formulation

In this section, we first explain a class of probabilistic two-hop forwarding policies with a given constraint on the number of message copies under the aforementioned heterogeneous model. We then formulate an optimization problem to find an optimal forwarding policy to minimize the message delivery delay while satisfying the constraint.

In the class of probabilistic two-hop forwarding policies, a source node forwards a message copy to each relay node r_i with probability $p_i \in [0, 1]$ upon encounter. Note that the source node has no benefit of forwarding a copy to each relay node upon the second or later encounter after skipping the first forwarding opportunity. Thus, the forwarding decision for each relay node is done only once upon the first encounter. Then, the forwarded message copies or an original message can be delivered to their destinations via relay nodes chosen in the forwarding decision or directly by the source, respectively. Figure 2 depicts this operation under the probabilistic two-hop forwarding policies.

Our focus in this paper is not to propose yet another forwarding algorithm but to analytically quantify the achievable performance gain by exploiting the heterogeneity in mobile nodes' contact dynamics. For tractable analysis (but still non-trivial), we do not consider two-hop forwarding policies which change relay paths or choose relay nodes on-the-fly upon encounter, i.e., the forwarding probability p_i for each relay node i is not changing over time but predetermined. As we shall show in Section V, a two-hop forwarding policy obtained from a class of policies considered, which is below explained, still demonstrates its significant performance improvement by exploiting the underlying heterogeneity structure. We also discuss a further benefit possible by changing relay paths on-the-fly upon encounter in Section V.

Each forwarding probability p_i should be chosen to satisfy a constraint on the number of message copies, which in turn controls network cost or the amount of resource consumption incurred by additional message transfers.† At the same time, the message delivery delay critically depends on how we choose p_i for each relay node r_i. Thus, we can formulate the problem of finding an optimal forwarding policy $\vec{p}^{\star}$ under the constraint on the number of message copies in the heterogeneous model as an optimization problem. In what follows, we describe this formulation step by step.

Throughout the formulation and subsequent delay analysis, we assume the followings as in other analytical works [22], [11], [25], [1], [13]. The network is sparse and network traffic is light such that interference and contention [16] are not important factors. In other words, we assume that each node has infinite bandwidth and buffer. In fact, as will be shown in Section V, exploiting the heterogeneity in mobile nodes' contact dynamics is helpful in significantly reducing the number of message copies, which in turn keeps the network traffic low and thus decreases the effect of interference/contention. In addition, we assume that a message transfer between any two nodes at their contact instant takes a negligible time with respect to their inter-contact time.

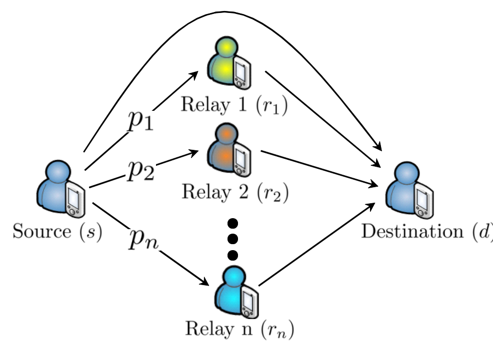


Fig. 2. A class of probabilistic two-hop forwarding policies. Source S forwards a message copy to relay node r_i with probability p_i.

Let $\mathcal{N} = \{s, r_1, \ldots, r_n, d\}$ with source s and destination d, and n possible relay nodes $r_1, r_2, \ldots, r_n$. Let $\{Y_i\}_{1\leq i\leq n}$ be the set of independent Bernoulli random variables with

$$\mathbb{P}\{Y_i = 1\} = p_i \ \text{ and } \ \mathbb{P}\{Y_i = 0\} = 1 - p_i,$$

to represent the forwarding decision to relay node r_i. For each i, we define a function I_{p_i} as

$$I_{p_i} = \begin{cases} 1 & \text{if } Y_i = 1, \\ \infty & \text{otherwise.} \end{cases} \tag{1}$$

Let $M = \sum_{i=1}^{n} Y_i$ denote the random variable to represent the number of message copies except the original message at the source node in the whole network. We also define $\vec{Y} \triangleq [Y_1, \ldots, Y_n]$ and $\vec{p} \triangleq [p_1, \ldots, p_n]$.

The message delivery delay D under a probabilistic two-hop forwarding policy with $\vec{p}$ can then be written as

$$D = \min\{T_{sd}, (T_{sr_1} + T_{r_1 d}) I_{p_1}, \cdots, (T_{sr_n} + T_{r_n d}) I_{p_n}\}. \tag{2}$$

We want to compute $\mathbb{E}\{D\}$ in terms of forwarding policy $\{p_i\}$ and the network parameter λ_{ij}. Since all the random variables inside the minimum operator in (2) are independent of each

*These are Bluetooth contact traces and we refer to [9] for details.

†As a special case, if $p_i = 1$ for all relay nodes (no resource constraint), then the probabilistic two-hop forwarding policy reduces to the multicopy two-hop relay protocol [11], [25].

others, we have

$$\mathbb{P}\{D > t\} = \mathbb{P}\{T_{sd} > t\} \prod_{i=1}^{n} \mathbb{P}\{(T_{sr_i} + T_{r_i d}) I_{p_i} > t\}. \quad (3)$$

By conditioning on I_{p_i}, we have

$$\begin{aligned}
&\mathbb{P}\{(T_{sr_i}+T_{r_i d}) I_{p_i} > t\} = \mathbb{E}\left\{\mathbb{P}\{(T_{sr_i}+T_{r_i d}) I_{p_i} > t | I_{p_i}\}\right\} \\
&= \mathbb{P}\{T_{sr_i}+T_{r_i d} > t\} \mathbb{P}\{I_{p_i}=1\} + \mathbb{P}\{t<\infty\} \mathbb{P}\{I_{p_i}=\infty\} \\
&= \mathbb{P}\{T_{sr_i}+T_{r_i d} > t\}\ p_i + (1-p_i), \quad (4)
\end{aligned}$$

where the last equality is from the definition of I_{p_i} in (1).

For notational simplicity, we define $f_0(t) \triangleq \mathbb{P}\{T_{sd} > t\}$ and $f_i(t) \triangleq \mathbb{P}\{T_{sr_i}+T_{r_i d} > t\}$, where T_{sd}, T_{sr_i}, and $T_{r_i d}$ are independent exponential random variables with rate $\lambda_{sd}, \lambda_{sr_i}$, and $\lambda_{r_i d}$, respectively. Then, from (4), (3) can be rewritten as

$$\mathbb{P}\{D > t\} = f_0(t) \prod_{i=1}^{n} \left[p_i f_i(t) + (1-p_i)\right], \quad (5)$$

and thus, by noting that $\mathbb{E}\{D\} = \int_0^\infty \mathbb{P}\{D > t\} dt$, we have

$$\mathbb{E}\{D\}_{\vec{p}} = \int_0^\infty f_0(t) \prod_{i=1}^{n} \left[p_i f_i(t) + (1-p_i)\right] dt, \quad (6)$$

where we use the subscript in $\mathbb{E}\{D\}_{\vec{p}}$ to clearly indicate that the average delay is a function of the forwarding policy $\vec{p} = [p_1, \ldots, p_n]$.

Now, we formally state our problem to find an optimal forwarding policy $\vec{p}^\star$ under the constraint on the average number of message copies, i.e., $\mathbb{E}\{M\} = \mathbb{E}\{\sum_{i=1}^n Y_i\} = \sum_{i=1}^n p_i$, as the following optimization problem: For $\mathbb{E}\{D\}_{\vec{p}} : [0,1]^n \to \mathbb{R}_+$,

$$(\mathbf{P1}) \qquad \begin{aligned} &\textbf{minimize} \quad \mathbb{E}\{D\}_{\vec{p}} \\ &\textbf{subject to} \quad \sum_{i=1}^n p_i \le K, \end{aligned}$$

where $\vec{p}$ denotes a forwarding policy and K is a positive integer ($1 \le K \le n$). As explicitly shown in (P1), the average number of copies (except the original message at the source node) allowed in the network is limited up to K-copies.

IV. Delay Analysis

In this section, we derive an achievable upper bound on the delay in a tractable form, which leads us to find a sub-optimal solution to (P1). We also explain an intuition behind the delay upper bound by considering multicopy two-hop relay protocol [11], [25] as an importance special case.

A. An Upper Bound of Message Delivery Delay

A difficulty in solving the optimization problem (P1) arises, since it is not a convex optimization problem, which can be checked by showing that the Hessian matrix of $\mathbb{E}\{D\}_{\vec{p}}$ with respect to $\vec{p}$ is neither positive semidefinite nor negative semidefinite. Thus, we cannot resort to the standard convex optimization techniques [4] to find the optimal solution of (P1). Instead, we below derive an upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ from (6), which becomes mathematically more tractable.

First, by noting that T_{sd} is an exponential random variable with rate λ_{sd}, we rewrite (6) as

$$\begin{aligned}
\mathbb{E}\{D\}_{\vec{p}} &= \int_0^\infty e^{-\lambda_{sd} t} \prod_{i=1}^n \left[p_i f_i(t) + (1-p_i)\right] dt \\
&= \frac{1}{\lambda_{sd}} \int_0^\infty \left(\prod_{i=1}^n [p_i f_i(t) + (1-p_i)] \right) \lambda_{sd} e^{-\lambda_{sd} t} dt \\
&= \frac{1}{\lambda_{sd}} \mathbb{E}\left\{ \prod_{i=1}^n [p_i f_i(T_{sd}) + (1-p_i)] \right\}, \quad (7)
\end{aligned}$$

where the expectation is with respect to T_{sd}.

We denote $\|X\|_q$ to be the L_q norm of a (real-valued) random variable X, i.e., $\|X\|_q \triangleq [\mathbb{E}\{|X|^q\}]^{1/q}$ for $0 < q < \infty$, and $\|X\|_\infty \triangleq \inf[c \in \mathbb{R} :\ \mathbb{P}\{|X| > c\} = 0]$. We also define by $\mathcal{L}^q$ a set of all random variables X for which $\|X\|_q < \infty$.

To proceed, we need the following two inequalities that will be used to derive the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ from (7).

Theorem 1: [8], [24] *(Generalized Hölder's Inequality)* Let $1 \le q_i \le \infty$ with $\sum_{i=1}^n 1/q_i = 1$. If $X_i \in \mathcal{L}^{q_i}$ for $1 \le i \le n$, then $\prod_{i=1}^n X_i \in \mathcal{L}^1$ and

$$\|\prod_{i=1}^n X_i\|_1 \le \prod_{i=1}^n \|X_i\|_{q_i}. \qquad \square$$

Theorem 2: [8], [24] *(Minkowski's Inequality)* For $X, Y \in \mathcal{L}^q$ with $1 \le q \le \infty$,

$$\|X + Y\|_q \le \|X\|_q + \|Y\|_q. \qquad \square$$

Since $f_i(t) = \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\} \le 1$, it follows that $f_i(T_{sd}) \in \mathcal{L}^q$ and $p_i f_i(T_{sd}) + (1-p_i) \in \mathcal{L}^q$ for $1 \le q \le \infty$. Thus, for any $1 \le q_i \le \infty$ with $\sum_{i=1}^n 1/q_i = 1$, we have

$$\begin{aligned}
\mathbb{E}\{D\}_{\vec{p}} &\le \frac{1}{\lambda_{sd}} \prod_{i=1}^n \|p_i f_i(T_{sd}) + (1-p_i)\|_{q_i} \quad (8) \\
&\le \frac{1}{\lambda_{sd}} \prod_{i=1}^n [p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)], \quad (9)
\end{aligned}$$

where (8) is from the generalized Hölder's inequality and (9) is from Minkowski's inequality.

In contrast to the original form of $\mathbb{E}\{D\}_{\vec{p}}$ in (6) and (7), its upper bound in (9) is in a much more tractable form. More important, this upper bound leads us to find an optimal solution $\vec{p}^*$ that minimizes the upper bound, which is good enough to show the benefit of exploiting the heterogeneity in mobile nodes' contact dynamics, as will be shown in Section V.

B. A Special Case: Multicopy Two-hop Relay Protocol

We below consider the multicopy two-hop relay protocol as a special case ($K = n$) to get an intuition behind the delay upper bound in (9).

Let $T_{sr_i}^j$ ($j = 1, \ldots, K$) be *i.i.d.* exponential random variables with rate λ_{sr_i}, and similarly for $T_{r_i d}^j$ ($j = 1, \ldots, K$) with rate $\lambda_{r_i d}$. We define

$$\tilde{D}_i \triangleq \min\{T_{sd}, T_{sr_i}^1 + T_{r_i d}^1, \cdots, T_{sr_i}^K + T_{r_i d}^K\}, \quad (10)$$

for $i = 1, \ldots, n$. $\tilde{D}_i$ here is defined for general K-copies which will be used in the rest of our paper. By definition, $\tilde{D}_i$ can be interpreted as the message delivery delay of multicopy two-hop relay policy over a partially homogeneous network $\mathfrak{N}_i$ (as depicted in Figure 3) that is composed of a direct source-destination path and K *i.i.d.* two-hop relay paths, each of which has delay equal to the sum of two exponential random variables with rates λ_{sr_i} and $\lambda_{r_i d}$. In other words, K two-hop relay paths in $\mathfrak{N}_i$ are *i.i.d.* copies of the two-hop relay path via relay node i in the original heterogeneous network, and the direct path in both networks remains the same. Figure 3 shows this decomposition procedure from an original heterogeneous network to n partially homogeneous networks $\mathfrak{N}_i$ $(i = 1, \ldots, n)$

From $T^j_{sr_i} + T^j_{r_i d} \stackrel{d}{=} T_{sr_i} + T_{r_i d}$ and independence over $j = 1, \ldots, K$, for each i, we have

$$\mathbb{E}\{\tilde{D}_i\} = \int_0^\infty f_0(t)\big[f_i(t)\big]^K dt. \qquad (11)$$

We can compute a close-form expression of $\mathbb{E}\{\tilde{D}_i\}$ (See Appendix B for its derivation) as follows: for $\lambda_{sr_i} \neq \lambda_{r_i d}$

$$\mathbb{E}\{\tilde{D}_i\} = \frac{1}{(\lambda_{r_i d} - \lambda_{sr_i})^K} \sum_{j=0}^{K} \binom{K}{j} \frac{(-\lambda_{sr_i})^j \lambda_{r_i d}^{K-j}}{\lambda_{sd} + j\lambda_{r_i d} + (K-j)\lambda_{sr_i}},$$

and for $\lambda_{sr_i} = \lambda_{r_i d}$

$$\mathbb{E}\{\tilde{D}_i\} = \frac{1}{\lambda_{sr_i}} \sum_{j=0}^{K} \frac{K!}{(K-j)!(K + \lambda_{sd}/\lambda_{sr_i})^{j+1}}. \qquad (12)$$

Now, consider $K = n$ and set $q_i = n$ for $i = 1, \ldots, n$. Since the forwarding policy simply becomes $\vec{p} = \vec{1} = [1, \ldots, 1]$, (9) can be rewritten as

$$\begin{aligned}
\mathbb{E}\{D\}_{\vec{1}} &\leq \frac{1}{\lambda_{sd}} \prod_{i=1}^{n} \|f_i(T_{sd})\|_n \\
&= \frac{1}{\lambda_{sd}} \prod_{l=1}^{n} \left[\int_0^\infty \lambda_{sd} e^{-\lambda_{sd} t}\big[f_i(t)\big]^n dt\right]^{1/n} \\
&= \prod_{i=1}^{n} \left[\int_0^\infty f_0(t)\big[f_i(t)\big]^n dt\right]^{1/n} = \prod_{i=1}^{n} \Big[\mathbb{E}\{\tilde{D}_i\}\Big]^{1/n}. \qquad (13)
\end{aligned}$$

The delay upper bound in (13) is nothing but a geometric mean of $\mathbb{E}\{\tilde{D}_i\}$, the average message delivery delay of multicopy two-hop relay protocol under $\mathfrak{N}_i$. We observe that this delay upper bound still captures the underlying heterogeneity in mobile nodes' contact dynamics, as each decomposed network $\mathfrak{N}_i$ contains each of n different two-hop relay paths of the original heterogeneous network. In addition, a closed-form solution of this delay upper bound can be immediately obtained from (12).

Remark 1: When the network is homogeneous with $\lambda_{ij} = \lambda$, the upper bound in (13) becomes identical to the original expression of $\mathbb{E}\{D\}_{\vec{1}}$ from (6), i.e., all the intermediate inequalities that have led to (13) hold with equality for this special case of multicopy two-hop relay protocol under the homogeneous network setting. □

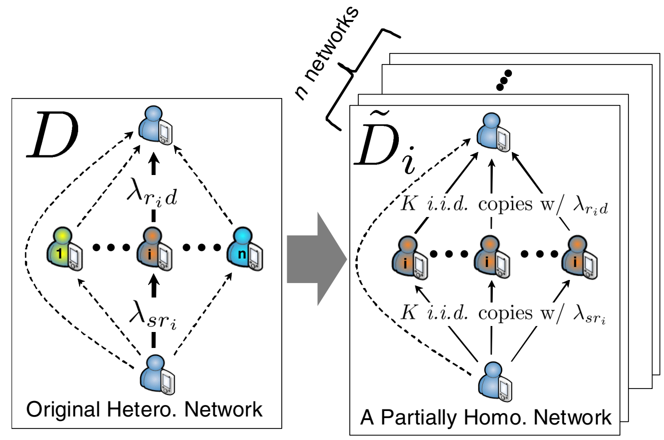


Fig. 3. Decomposition of a heterogeneous network into n partially homogeneous networks $\mathfrak{N}_i$ $(i = 1, \ldots, n)$. K two-hop relay paths in $\mathfrak{N}_i$ are *i.i.d.* copies of the two-hop relay path via relay node r_i in the original heterogeneous network.

Remark 2: Consider two homogeneous networks $\mathfrak{N}_U$ and $\mathfrak{N}_L$ with common contact rates for any node pair given by $\lambda_{\min} = \min_{i,j \in \mathcal{N}}\{\lambda_{ij}\}$ and $\lambda_{\max} = \max_{i,j \in \mathcal{N}}\{\lambda_{ij}\}$, respectively. Also, set the average message delivery delay of multicopy two-hop relay protocol under $\mathfrak{N}_U$ and $\mathfrak{N}_L$ by $\mathbb{E}\{\hat{D}_U\}$ and $\mathbb{E}\{\hat{D}_L\}$, respectively. Then, it is straightforward to see that $\mathbb{E}\{\hat{D}_L\} \leq \mathbb{E}\{D\}_{\vec{1}} \leq \mathbb{E}\{\hat{D}_U\}$. It is also known [11], [25] that the average delay of multicopy two-hop relay protocol under a homogeneous network is asymptotically $\frac{1}{\lambda}\sqrt{\frac{\pi}{2(n+1)}}$ as $n \to \infty$ where $|\mathcal{N}| = n+2$ and λ is a common contact rate for any node pair. Thus, if we were to focus on the asymptotic average delay of multicopy two-hop relay protocol, it would be $\mathcal{O}\left(\frac{1}{\sqrt{n+1}}\right)$, *regardless of* whether the underlying network is homogeneous or heterogeneous.[‡] This is precisely why we analyze the delay performance under a heterogeneous network setting with a given *finite* number of mobile nodes. □

V. Main Results

In this section, we obtain an optimal solution $\vec{p}^*$ which minimizes the delay upper bound derived in the previous section. We also show a closed-form expression of the guaranteed delay bound under this optimal solution $\vec{p}^*$. Finally, we quantify the performance gain of exploiting the heterogeneity in mobile nodes' contact dynamics under heterogeneous network settings using the closed-form expression of the guaranteed delay bound.

A. A Sub-Optimal Forwarding Policy

We first fix $q_i \geq 1, i = 1, \ldots, n$ that satisfy $\sum_{i=1}^{n} 1/q_i = 1$, and then derive an optimal forwarding policy $\vec{p}^*$ that minimizes the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ in (9). To this end, we consider the following optimization problem, whose solution will be sub-optimal to the original problem (P1).

[‡]Only the constant coefficient $1/\lambda_{\min}$ or $1/\lambda_{\max}$ will change and the order term in n remains the same.

$$(\mathbf{P1'}) \quad \begin{array}{ll} \textbf{minimize} & \prod_{i=1}^{n} \left[p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i) \right] \\ \textbf{subject to} & \sum_{i=1}^{n} p_i \leq K. \end{array}$$

We define a sequence of $a_i(q_i)$ for a given q_i by

$$a_i(q_i) \triangleq \|f_i(T_{sd})\|_{q_i} = \left[\int_0^\infty \lambda_{sd} e^{-\lambda_{sd} t} \left[f_i(t)\right]^{q_i} dt \right]^{1/q_i}, \quad (14)$$

where $f_i(t) = \mathbb{P}\{T_{sr_i} + T_{r_i d} > t\}$. For notational simplicity, we will use a_i instead of $a_i(q_i)$ unless it is necessary to specify the given q_i. Rearrange a_i in an increasing (non-decreasing) order and set $a_{[i]}$ to be the i^{th} smallest one among $a_1, \cdots, a_n$, i.e., $a_{[1]} \leq a_{[2]} \leq \cdots \leq a_{[n]}$. Also, let $c_1, \cdots, c_n$ be a permutation of $\{1, \ldots, n\}$ which satisfies $a_{c_l} = a_{[l]}$ for all $l = 1, \ldots, n$. Then, we have the following proposition for the optimal solution $\vec{p}^*$ of (P1$'$).

Proposition 1: For any arbitrarily fixed $q_i \in [1, \infty]$ such that $\sum_{i=1}^n 1/q_i = 1$, the optimal solution $\vec{p}^*$ of (P1$'$) is always of the following form:

$$p_i^* = \begin{cases} 1 & \text{if } i \in \{c_1, \cdots, c_K\}, \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

Proof: See Appendix A. ■

Remark 3: Proposition 1 implies that, even though we start with a constraint on the average number of message copies $\mathbb{E}\{M\} = \sum_{i=1}^n p_i \leq K$, interestingly enough, the optimal forwarding policy $\vec{p}^*$ under this average constraint actually attains $M = \sum_{i=1}^n Y_i = K$ with probability 1. □

Proposition 1 says that the optimal forwarding policy $\vec{p}^*$ of (P1$'$) is to choose K relay nodes $r_{c_1}, \ldots, r_{c_K}$ and to forward message copies to them. Since the optimal forwarding policy $\vec{p}^*$ in Proposition 1 holds for a given $\{q_i\}$, the choice of K relay nodes under the optimal forwarding policy $\vec{p}^*$ still depends on $\{q_i\}$, which will be specified. In addition, from Proposition 1 and (9), the guaranteed delay bound by the optimal forwarding policy $\vec{p}^*$ becomes

$$\mathbb{E}\{D\}_{\vec{p}^*} \;\leq\; \frac{1}{\lambda_{sd}} \prod_{l=1}^{K} a_{[l]}. \quad (16)$$

Since $f_i(\cdot) \leq 1$, from (14), $a_i \leq 1$ for all i. Thus, we can interpret $\prod_{l=1}^K a_{[l]}$ in (16) as a delay discounting factor by K additional message copies in the heterogeneous network setting, whereby $1/\lambda_{sd}$ is simply the message delivery delay of direct forwarding from source to destination.

We explain how to choose a set of free variables $\{q_i\}$ under constraints $\sum_{i=1}^n 1/q_i = 1$ and $q_i \geq 1$ $(i = 1, \ldots, n)$. First, observe that if $p_i = 0$, a_i will not contribute to the upper bound of $\mathbb{E}\{D\}_{\vec{p}}$ in (9) regardless of the choice of q_i, and thus (9) simply becomes (16). Also, for any random variable X, $\|X\|_q$ is monotone increasing in q if $X \in \mathcal{L}_q$ [8], [24]. Then, since $f_i(T_{sd}) \in \mathcal{L}_q$ for $1 \leq q \leq \infty$ as mentioned before, $a_i(q_i)$ is monotone increasing in $q_i \geq 1$ for $i = 1, \ldots, n$. Also, from $f_i(\cdot) \leq 1$ and the definition of L_∞ norm (i.e., $\|\cdot\|_\infty$), we have $a_i(\infty) = 1$ for $i = 1, \ldots, n$. Thus, if we can assign $q_i = \infty$, jointly with $p_i^* = 0$, we can put the smallest possible values to other q_i's while satisfying $\sum_{i=1}^n 1/q_i = 1$, which in turn gives smaller delay upper bound. Hence, we arrive to the following assignment rule for $\{q_i\}$ that makes the guaranteed delay bound in (16) tighter.

We set $q_i = K$ to a_i in (14), i.e., $a_i(K)$, for $i = 1, \ldots, n$, temporarily. After finding $c_1, \ldots, c_n$ as mentioned above, we assign

$$q_i = \begin{cases} K & \text{if } i \in \{c_1, \cdots, c_K\}, \\ \infty & \text{otherwise.} \end{cases} \quad (17)$$

Then, $\sum_{i=1}^n 1/q_i = \sum_{i=1}^K 1/K = 1$. Since $a_{c_{K+1}}(\infty) = \cdots = a_{c_n}(\infty) = 1$ as mentioned above, $a_{c_i}(K)$ $(i = 1, \ldots, K)$ will fully contribute to the delay upper bound in (16). Also, from Proposition 1, the optimal solution $\vec{p}^*$ in (15) is still $p_i^* = 1$ for the same $i \in \{c_1, \cdots, c_K\}$, otherwise $p_i^* = 0$. Hence, the optimal forwarding policy $\vec{p}^*$ becomes to choose K relay nodes $r_{c_1}, \ldots, r_{c_K}$ based on $a_i(K)$ for all i. Also, the assignment rule for $\{q_i\}$ does not change the optimal forwarding policy $\vec{p}^*$ itself, but makes the delay upper bound in (16) smaller.

We now derive a closed-form expression of the guaranteed delay bound under the optimal forwarding policy $\vec{p}^*$. Observe that from the definitions of $\mathbb{E}\{\tilde{D}_i\}$ and $a_i(K)$ in (11) and (14), respectively, for $i = 1, \ldots, n$, we have

$$a_i(K) = \|f_i(T_{sd})\|_K = \lambda_{sd}^{1/K} \left[\mathbb{E}\{\tilde{D}_i\}\right]^{1/K}. \quad (18)$$

We denote $\mathbb{E}\{\tilde{D}_{[l]}\}$ to be the l^{th} smallest one among $\mathbb{E}\{\tilde{D}_i\}$ $(i = 1, \ldots, n)$. Also, let $d_1, \cdots, d_n$ be another permutation of $\{1, \ldots, n\}$ which satisfies $\mathbb{E}\{\tilde{D}_{d_l}\} = \mathbb{E}\{\tilde{D}_{[l]}\}$ for all l. By noting that $g(x) = x^{1/K}$ is monotone increasing in $x \geq 0$, from (18), we have $d_i = c_i$ for all i. It implies that the optimal forwarding policy $\vec{p}^*$ is equivalent to choosing K relay nodes $r_{c_1}, \ldots, r_{c_K}$ based on $\mathbb{E}\{\tilde{D}_i\}$ $(i = 1, \ldots, n)$. Thus, from (16) and (18), the guaranteed delay bound under the optimal forwarding policy $\vec{p}^*$ becomes

$$\mathbb{E}\{D\}_{\vec{p}^*} \;\leq\; \prod_{l=1}^{K} \left[\mathbb{E}\{\tilde{D}_{[l]}\}\right]^{1/K}. \quad (19)$$

Similar to a special case $K = n$ in IV-B, as in (19), the guaranteed delay bound by the optimal forwarding policy $\vec{p}^*$ under K-copies constraint is nothing but a geometric mean of the K smallest ones among n message delivery delays of multicopy two-hop relay protocol, each of which is obtained under each decomposed network $\mathfrak{N}_i$ (as shown in Figure 3) that consists of a direct path and K *i.i.d.* two-hop relay paths with parameters λ_{sr_i} and $\lambda_{r_i d}$. Then, from the closed-form expression of $\mathbb{E}\{\tilde{D}_i\}$ in (12), we immediately have a closed-form expression of the guaranteed delay upper bound in (19). Hence, by decomposing an original heterogeneous network into n partially homogeneous networks, we are able to measure the performance gain achieved through exploiting the heterogeneity in mobile nodes' contact dynamics by using the closed-form expression of the derived delay bound.

Remark 4: Similar to Remark 1, for $K < n$, when the network is homogeneous with $\lambda_{ij} = \lambda$, the delay upper bound by the forwarding policy $\vec{p}^*$ in (19) becomes equal to the expression of $\mathbb{E}\{D\}_{\vec{p}^*}$ from (6). Since the forwarding policy $\vec{p}^*$ is a form of $[1, \ldots, 1, 0, \ldots, 0]$ and the expression of $\mathbb{E}\{\tilde{D}_i\}$ in (11) is the same for all i, the equality in (19) holds. □

B. Performance Gain of Exploiting Heterogeneity

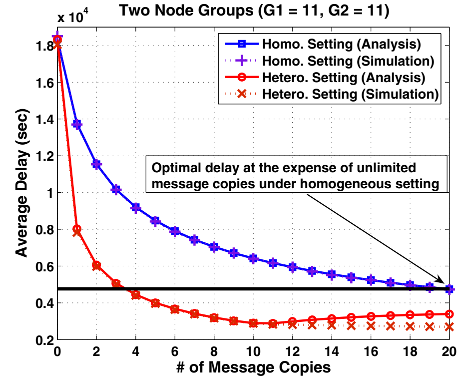
We start our quantitative study on the benefit of exploiting the heterogeneity in contact dynamics under a setting of two social groups as shown in Figure 1 (b), a special case of the heterogeneous network model. We consider 2 social groups G_1 and G_2 forming $\mathcal{N} = \bigcup_{i=1}^{2} G_i$, and $|\mathcal{N}| = 22$. Recall that λ'_{lk} is a common contact rate between any member of G_l and another member of G_k for $l, k = 1, 2$. Note that the total number of possible relay nodes for any given source-destination pair is 20. In the heterogeneous network setting, we set $\lambda'_{11} = 2\times10^{-4}$, $\lambda'_{22} = 2\times10^{-5}$, and $\lambda'_{12} = 10^{-4}$. This setting reflects a scenario that mobile nodes in G_1 are more socially active than those in G_2 and make frequent contacts with other group members as well as same group members. On the other hand, in the corresponding homogeneous network setting, we set $1/\lambda_{ij} = \bar{\mu}$ for all $i, j \in \mathcal{N}$ and $i \neq j$ where $\bar{\mu}$ is the overall average inter-contact times over all node pairs, given by

$$\bar{\mu} = \left[\sum_{i=1}^{2} \frac{1}{\lambda'_{ii}} \binom{|G_i|}{2} + \frac{1}{\lambda'_{12}} \binom{|G_1|}{1} \binom{|G_2|}{1}\right] \Big/ \binom{|\mathcal{N}|}{2}.$$

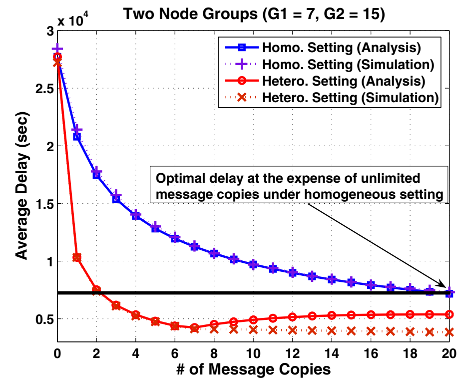
Thus, the overall average inter-contact time over all node pairs is the same for both settings.

Figure 4 (a) and (b) depict analytical and simulation results for an average delay obtained using the above optimal forwarding policy $\vec{p}^*$ of (P1′), a sub-optimal policy (solution) to the original problem (P1), per a given number of message copies under the heterogeneous and corresponding homogeneous network settings where we set the size of each group as $(|G_1|, |G_2|)$ = (11,11) and (7,15) for each heterogeneous setting, respectively. We obtain the analytical result of an average delay for a uniformly chosen source-destination pair, i.e., a statistical average of average delays for total 231 source-destination pairs, per each given number of message copies by computing the guaranteed delay bound of the sub-optimal forwarding policy in (19). We also implement a custom event-driven simulator using C++ where random contacts of each node pair occur according to a Poisson process with its contact rate, and provide independent simulation results for the actual average delay of the sub-optimal forwarding policy for the uniform source-destination pair per each given number of message copies.

Figure 4 shows that very few message copies (*2-copies* or *3-copies*) with the sub-optimal forwarding policy under the heterogeneous network settings are only needed to achieve the same delay performance as the performance limit of any two-hop relay forwarding policies under their homogeneous counterparts. Here, the performance limit is the minimum delay achieved at the expense of unlimited message copies (20 additional message copies) by multicopy two-hop relay protocol[§]. Figure 4 also implies that we can save *more than 80%* of message copies under the heterogeneous network settings, which is significantly helpful in reducing overall resource consumption over the network. Along with this observation, since most of the performance gain (i.e., large reduction of delay) takes place with first few message copies under the heterogeneous network settings, we deduce that few relay nodes with high contact rate in G_1 play a dominant role in actually delivering a message to its destination.



(a) $|G_1| = |G_2| = 11$



(b) $|G_1| = 7$ and $|G_2| = 15$

Fig. 4. Average delay achieved via the sub-optimal two-hop forwarding policy per each given number of message copies under heterogeneous and corresponding homogeneous network settings. The horizontal thick line indicates the optimal delay that any two-hop forwarding policy cannot exceed under the homogeneous settings.

In addition, simulation results in Figure 4 exhibit that the delay upper bound of the sub-optimal forwarding policy (i.e., the RHS of (19)) is very closed to its actual performance for a small to moderate number of message copies under the heterogeneous network settings, while it is the exact delay under the homogeneous network settings as in Remarks 1 and 4. Also, although an increasing behavior of the delay upper bound exists, it can be explained as follows. Recall that the RHS of (19) can be written in a form of the product of L_q norms

[§]Although an optimal two-hop forwarding policy under a constraint of K message copies in any homogeneous network setting is to forward K message copies to the first K encountered nodes, its performance still cannot exceed the performance limit.

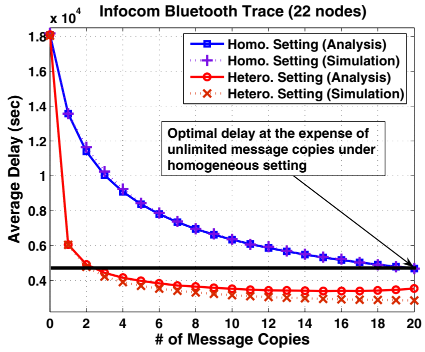


Fig. 5. Average delay achieved via the sub-optimal two-hop forwarding policy per each given number of message copies under a heterogeneous network setting (*Infocom* trace) and its homogeneous counterpart. The horizontal thick line indicates the optimal delay that any two-hop forwarding policy cannot exceed under the homogeneous setting.

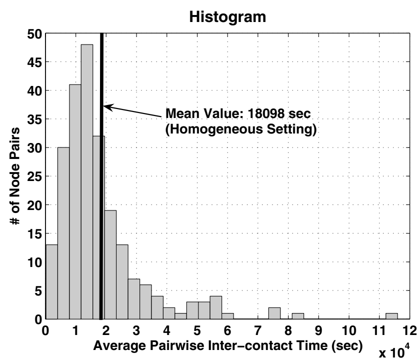


Fig. 6. Histogram of average *pairwise* inter-contact times over all node pairs in *Infocom* trace.

as in (16), i.e., $\prod_{l=1}^{m} a_{[l]}(m)$ where $a_i(m) = \|f_i(T_{sd})\|_m$ for $i = 1, \dots, n$. Also, since $a_i(m)$ is monotone increasing in $m \geq 1$ and $a_i(m) \leq 1$ as mentioned in Section V-A, it is possible that $\prod_{l=1}^{m} a_{[l]}(m) \leq \prod_{l=1}^{m} a_{[l]}(m{+}1) \cdot a_{[m+1]}(m{+}1)$ when delay reduction by adding $a_{[m+1]}(m{+}1)$ becomes not significant. However, as shown in Figure 4, the delay upper bound does not keep increasing and the delay upper bound for a large number of message copies is still useful when compared to its actual performance.

We further continue our investigation over a real Bluetooth contact trace (*Infocom*) [5] which was collected during the IEEE INFOCOM'05 conference. It contains 41 nodes' contact information over 3 days where we here use 22 nodes' information ($|\mathcal{N}| = 22$). In order to adopt this *Infocom* trace under the heterogeneous network model, we extract the average pairwise inter-contact time of all node pairs and use them under the heterogeneous network model. Similar to previous test case, for the homogeneous network setting, we set $\bar{\mu} = 18098sec$ as shown in Figure 6, where the overall average inter-contact time over all considered node pairs is $18098sec$.

Figure 5 shows analytical and simulation results for an average delay per a given number of message copies under a heterogeneous network setting based on the *Infocom* trace and its homogeneous counterpart. Both results are similarly obtained as done in the previous test case. In the event-driven simulation, random contacts of each node pair are generated according to a Poisson process with its contact rate extracted from the *Infocom* trace for the heterogeneous setting or with rate $1/\bar{\mu}$ for its corresponding homogeneous setting. As shown in Figure 5, we again achieve the significant performance improvement (*2-copies* are enough) as similarly observed in the previous case. This performance gain becomes apparent, since the average inter-contact time is quite heterogeneous over different node pairs as depicted in Figure 6. Although we observed similar performance improvement using the sub-optimal forwarding policy for other heterogeneous network settings, we here omit them due to space constraint.

Note that the considerable performance gain is still shown through the delay upper bound of the sub-optimal two-hop forwarding policy. Moreover, it does not include the benefit of changing relay paths or choosing relay nodes on-the-fly upon encounter, as the forwarding policy works based on a predetermined set of relay nodes. In other words, an expected utility of each mobile node as a relay node is changing over time depending on *when* and *who* a source node encounters, and thus it can be taken into account in the design of new forwarding policy so as to minimize the message delivery delay. Therefore, with such a dynamic routing, the maximally achievable performance gain by exploiting the heterogeneity in mobile nodes' contact dynamics will be substantially higher than expected.

VI. Conclusion

In this paper we have demonstrated the significant performance gain obtained from exploiting the heterogeneity in mobile nodes' contact dynamics using the guaranteed delay bound of the sub-optimal two-hop forwarding policy. Thanks to the close-form expression of the guaranteed delay bound, we are able to show quantitative results for the performance improvement attainable through leveraging the underlying heterogeneity structure in the design of the forwarding policy, which cannot be captured in existing analytical studies based on the homogeneous network model. We expect that our analytical results will also complement the existing empirical studies on the design of forwarding/routing algorithms which exploit the underlying heterogeneity structure in MONs/DTNs.

Appendix A
Proof of Proposition 1

We will find an optimal solution $\vec{p}^*$ of the optimization problem (P1$'$) by obtaining a minimizer which achieves the lowest bound of the objective function in (P1$'$) under the constraint $\sum_{i=1}^{n} p_i \leq K$. By taking $\log$ function to the objective function in (P1$'$), it can be transformed as

$$\sum_{i=1}^{n} \log\left[p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)\right], \tag{20}$$

since $\log$ function is a monotone increasing function. From Jensen's inequality and concavity of $\log$, (20) is further lower bounded by

$$\sum_{i=1}^{n} \log[p_i \|f_i(T_{sd})\|_{q_i} + (1-p_i)] \geq \sum_{i=1}^{n} p_i \log \|f_i(T_{sd})\|_{q_i}. \tag{21}$$

Then, we want to minimize the RHS of (21) under the constraint $\sum_{i=1}^{n} p_i \leq K$, which in turn gives the lowest bound of (20). It is equivalent to solving the following simple linear programming problem:

$$(\mathbf{P1''}) \quad \begin{array}{ll} \textbf{minimize} & \sum_{i=1}^{n} p_i \log \|f_i(T_{sd})\|_{q_i} \\ \textbf{subject to} & \sum_{i=1}^{n} p_i \leq K, \end{array}$$

Recall the definition of a_i in (14), i.e., $a_i = \|f_i(T_{sd})\|_{q_i}$. $a_{[i]}$ is also the i^{th} smallest one among $a_1, \cdots, a_n$, i.e., $a_{[1]} \leq a_{[2]} \leq \cdots \leq a_{[n]}$. And, $c_1, \ldots, c_n$ is a permutation over $1, \ldots, n$ which satisfies $a_{c_l} = a_{[l]}$ for all $l = 1, \ldots, n$. Then, since $\log$ function is monotone increasing and the objective function in $(\mathrm{P1}'')$ is linear, it is easy to see that an optimal solution of $(\mathrm{P1}'')$ is $p_i = 1$ for $i \in \{c_1, \ldots, c_K\}$, otherwise $p_i = 0$. Hence, from (21) and the optimal solution of $(\mathrm{P1}'')$, we have

$$\sum_{i=1}^{n} \log\left[p_i a_i + (1-p_i)\right] \geq \sum_{i=1}^{n} p_i \log a_i \geq \sum_{l=1}^{K} \log a_{[l]}. \quad (22)$$

Note that the last lower bound in (22) is the lowest bound of (20) under the constraint $\sum_{i=1}^{n} p_i \leq K$, and the equality in (22) holds by the optimal solution of $(\mathrm{P1}'')$. Thus, the optimal solution of $(\mathrm{P1}'')$ is indeed the optimal solution $\vec{p}^*$ of $(\mathrm{P1}')$. This completes the proof. ■

Appendix B
The closed-form expression of $\mathbb{E}\{\tilde{D}_i\}$

Here, we derive the closed-form expression of $\mathbb{E}\{\tilde{D}_i\}$ as in (12). Recall that $f_0(t)$ is the complementary cumulative distribution function (ccdf) of an exponential random variable with rate λ_{sd} and $f_i(t)$ is the ccdf of the sum of two exponential random variables with rates λ_{sr_i} and $\lambda_{r_i d}$. Then, observe that by using binomial theorem, for $\lambda_{sr_i} \neq \lambda_{r_i d}$,

$$[f_i(t)]^K = \left[\frac{\lambda_{sr_i}}{\lambda_{sr_i} - \lambda_{r_i d}} e^{-\lambda_{r_i d} t} + \frac{\lambda_{r_i d}}{\lambda_{r_i d} - \lambda_{sr_i}} e^{-\lambda_{sr_i} t}\right]^K$$
$$= \frac{1}{(\lambda_{r_i d} - \lambda_{sr_i})^K} \sum_{j=0}^{K} \binom{K}{j} (-\lambda_{sr_i})^j e^{-j\lambda_{r_i d} t} \lambda_{r_i d}^{K-j} e^{-(K-j)\lambda_{sr_i} t},$$

and for $\lambda_{sr_i} = \lambda_{r_i d}$,

$$[f_i(t)]^K = (1 + \lambda_{sr_i} t)^K e^{-K\lambda_{sr_i} t} = \sum_{j=0}^{K} \binom{K}{j} (\lambda_{sr_i} t)^j e^{-K\lambda_{sr_i} t}.$$

Thus, we have for $\lambda_{sr_i} \neq \lambda_{r_i d}$,

$$\mathbb{E}\{\tilde{D}_i\} = \int_0^\infty f_0(t)[f_i(t)]^K dt$$
$$= \frac{1}{(\lambda_{r_i d} - \lambda_{sr_i})^K} \sum_{j=0}^{K} \binom{K}{j} \frac{(-\lambda_{sr_i})^j \lambda_{r_i d}^{K-j}}{\lambda_{sd} + j\lambda_{r_i d} + (K-j)\lambda_{sr_i}},$$

and for $\lambda_{sr_i} = \lambda_{r_i d}$,

$$\mathbb{E}\{\tilde{D}_i\} = \frac{1}{\lambda_{sr_i}} \sum_{j=0}^{K} \frac{K!}{(K-j)!(K + \lambda_{sd}/\lambda_{sr_i})^{j+1}}.$$

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